# Calculation of an Input Impedance of a Coax-fed Microstrip Circular Antenna

Jugoslav Joković, Tijana Dimitrijević, Nebojša Dončov, Bratislav Milovanović

*Abstract* - **In this paper, the integral cylindrical TLM method is used to determine an input impedance of a circular coax-fed patch antenna. Obtained input resistance and reactance are compared with results obtained using the integral TLM method in a rectangular grid, measured results and those calculated analytically based on the cavity model.**

*Keywords* - **TLM method, circular microstrip antenna, input impedance, cavity model**

## I. INTRODUCTION

Microstrip patch antennas are, in a variety of forms, used in numerous wireless communication applications. A microstrip patch antenna consists of an arbitrarily shaped metallic conductor etched on a grounded dielectric substrate. The properties of the substrate and its height have a significant influence on the performance of the antenna. The antenna can be excited in several ways which determine the achievable impedance bandwidth, the purity and direction of the radiated fields, the efficiency of the antenna, the ease of manufacturing of the antenna and its robustness. One of the excitation methods is probe feeding, where a probe extends through the ground plane and is connected to the patch conductor, typically soldered to it. The probe or feeding pin is usually the inner conductor of a coaxial line (coaxial feed). The probe feeding is considered as the most efficient feed technique due to the direct contact with the antenna and due to isolation from the patch, thus minimizing a spurious radiation [1,2,3].

In order to ensure any antenna to operate efficiently the maximum transfer of a power between the feeding system and the antenna should be provided. It can be reached when the input impedance of the antenna is matched to that of the feeding source impedance [2-5]. Thus, for achieving the optimum performance, the accurate calculation of the input impedance of the antenna is of significant importance.

For an analysis of microstrip antennas, there is a number of analytic/numerical methods proposed, which can be grouped as either approximate or full-wave techniques. The approximate techniques, such as one based on the cavity model [6], are useful in giving general trends of the patch antenna performance. These methods describe resonant frequency, input impedance, radiation pattern and bandwidth by simple equation design enabling efficient computation, and under certain conditions, can be relatively accurate. On the other hand, full-wave techniques, which are more complex and time consuming, give the most accurate results since they apply the Maxwell's equations to the problem and ensure the boundary conditions associated with the structure are satisfied [4].

The TLM method is a powerful and flexible numerical, full-wave method, and thus can play an important role in microwave circuits' design [7]. It can be applied before prototyping to provide estimation of system behaviour, or after to find where the design could be improved. In general, considering systems containing boundaries (internal or external), conducting structures (wires, microstrip etc.), and inhomogeneous media and losses it is of crucial importance to enable theirs accurate modelling. The TLM method enables all above features to be relevantly described, thanks to the development of various types of nodes and graded mesh [8 - 10], possibility of defining desired boundary conditions through the reflection coefficient [5], and incorporating the compact wire model [11]. Typically, commercial software packages use the TLM method based on the rectangular grid irrespective of the structure geometry. However, for describing structures of cylindrical geometry, such as circular patch antennas, the orthogonal polar mesh [12] is found to be more convenient since it enables precise modelling of boundaries coinciding with the coordinate axes. Recently, the TLM method based on the HSCN node [13] in a cylindrical grid has been enhanced by incorporation of the compact wire model, resulting in the integral cylindrical TLM method and corresponding self-written code *3DTLMcyl_cw* [14].

The purpose of this paper is to perform calculation of the input impedance of a circular patch antenna, as well as the optimum feed position determination, using the integral cylindrical TLM method. The input impedance of the considered circular patch antenna has been also calculated according to the simulations carried out in the TLM solver based on the rectangular grid. An analytical calculation of the input impedance of the given antenna, based on the cavity model [6, 15], improved to account for the location of the probe [16], has been also performed using the self-written MATLAB code. Numerical and analytic results are compared with the input impedance obtained from the measurement of the fabricated antenna in order to illustrate advantage of using the integral cylindrical TLM method for circular structures.

## II. CALCULATING THE INPUT IMPEDANCE

The equivalent circuit of a coax-fed microstrip patch antenna, with the feed presented by an equivalent reactance, $X_f$, is shown in Fig. 1 [2]. In general, the input impedance, $Z_{in,patch}$, of the patch antenna is complex and it includes both a resonant and a non-resonant part. Both the real and imaginary

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parts of the impedance vary as a function of frequency and exhibit symmetry about the resonant frequency. The reactance at resonance is equal to the average of sum of its maximum value and its minimum value [2].

The input impedance of the antenna can be controlled by varying the location of the feed from a high value when the probe is located close to the edge, to a low value when the probe is positioned near the centre [4]. At the resonant frequency it becomes resistive with zero or small input reactance. Since the input impedance is dependent on the location of the probe, an accurate determination of the input impedance of the antenna, and hence prediction of the optimum feed location, is important to provide the impedance matching between the feed and the radiating patch.

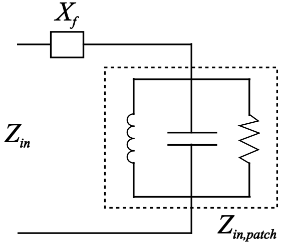


Fig. 1. Equivalent circuit of a probe-fed circular patch antenna

### A. The Cavity Model

According to the cavity model the circular microstrip antenna can be considered as a resonant cavity represented by a single resonant parallel R-L-C circuit, for a single isolated mode, as presented in Fig. 1.

The input impedance near the resonance is given by [17]

$$Z_{in}(f,\rho) = R_{in}(f,\rho) + jX_{in}(f,\rho), \tag{1}$$

$$R_{in}(f,\rho) = \frac{R_r(\rho)}{1+Q^2\left(\bar{f} - \bar{f}^{-1}\right)^2}, \tag{2}$$

$$X_{in}(f,\rho) = X_f(f) - \frac{R_r(\rho)Q\left(\bar{f} - \bar{f}^{-1}\right)}{1+Q^2\left(\bar{f} - \bar{f}^{-1}\right)^2}, \tag{3}$$

where $\bar{f} = f/f_r$ is the normalized frequency, $Q$ is the quality factor associated with system losses, $R_r(\rho)$ is the input resistance at resonance (resonance resistance), $\rho$ is the distance of the probe from the patch centre and $X_f$ is the feed reactance which accounts for the reactance contributed by the feed for the probe-fed microstrip patch antenna. The feed reactance is given by Harrington’s expression [18]

$$X_f(f) = \frac{\eta kh}{2\pi}\left(\ln\left(\frac{4}{kd}\right) - 0.577\right), \tag{4}$$

where $\eta$ is the intrinsic impedance of the medium, $k$ is the wave number, and $d$ is the diameter of the probe. However, since an accurate estimation of the feed reactance significantly depends on the location of the probe, the Harrington’s value, for a circular patch, should be multiplied with a new function given as [16]

$$F'(\rho) = \frac{J_n^2\left[ka\{1-(\rho/a)\}\right]}{J_n^2(ka)}, \tag{5}$$

where $a$ is the radius of the patch. Thus, the feed reactance for the circular patch can be calculated as

$$X_f(f,\rho) = X_f(f)\cdot F'(\rho). \tag{6}$$

### B. The TLM Method

To calculate the input impedance of the patch antenna using the TLM method the reflection coefficient has to be determined. Following the experimental approach that use an inner conductor of a coaxial guide as a probe, numerical characterization of EM field can be done by using the wire model for describing the inner conductor, introducing wire ports at the interface between wire probes and metallizations and calculating the scattering matrix. A model of the wire port incorporating, in general, a source voltage $V_g$ and impedance $R_g$ is shown in Fig.2. The TLM wire node defined at the wire port gives a wire current $I_1$, as an output of the TLM simulation which can be used to calculate the $S_{11}$ parameter, representing the reflection coefficient at the wire port. The wire port voltage $V_1$ can be expressed in terms of real source parameters $V_g$ and $R_g$ as follows [19]

$$V_1 = V_g - R_g I_1, \tag{7}$$

where $R_g$ is equal to the characteristic impedance $Z_0$ of the port.

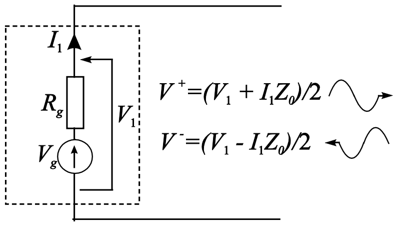


Fig. 2. Equivalent circuit of the TLM wire port

According to the expressions for incident and reflected voltage pulses at the wire port the reflection coefficient at the port can be determined as [19]

$$S_{11} = 1 - \frac{2Z_0 I_1}{V_g}. \tag{8}$$

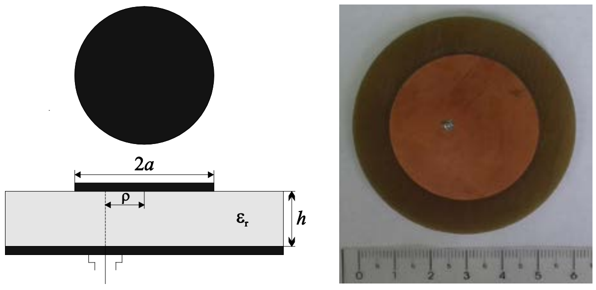
When the reflection coefficient is determined the input impedance can be calculated from the following expression

$$Z_{in}(f) = Z_0\frac{1+S_{11}(f)}{1-S_{11}(f)}. \tag{9}$$

## III. NUMERICAL RESULTS

A coax-fed microstrip circular patch antenna is fabricated on FR4 substrate with relative permittivity $\varepsilon_r$ = 4.2, loss tangent $\tan\delta$ = 0.02 and thickness $h$ = 1.5 mm (Fig.3). The radius of the circular patch is $a$ = 20 mm, whereas the radius of the substrate and ground plane is 30mm. The antenna is fed from a coaxial probe of diameter $d_0$ = 0.5 mm positioned at a distance $\rho$ = 4.5 mm from the patch centre.

The TLM model of the given microstrip antenna consists of a circular patch with the coaxial feed probe, inserted through the substrate, between the patch and the ground plane. The feed probe is excited with the source of $V_g$ = 1V and $R_g$ = 50Ω through the TLM wire port. Connection of both ends of the probe to teh perfectly conducting metal plates, used to describe the patch and the ground plane, is modelled by reflection coefficient equal to -1.. Due to handling with an open-boundary problem, the modelled region has been extended for 50% of the largest antenna dimension with the boundaries described by absorbing walls. A modelling process has been carried out by applying the TLM non-uniform networks in both cylindrical and rectangular grid with node spacing in the air filled area of 1mm. To maintain the time synchronization in the scattering process, the substrate is modelled by the network of nodes that are $\sqrt{\varepsilon_r}$ times smaller than nodes describing the air in the air-filled area.



a) b)

Fig. 3. Geometry of the circular coax-fed patch antenna, b) experimental model

Simulations have been conducted using the integral cylindrical TLM method for the position of the feed probe corresponding to the experimental model ($\rho = 4.5\,\text{mm}$). Obtained results of $S_{11}$ parameter and input impedance are compared with corresponding measured values in Figs. 4 and 5. These figures also show results obtained using the conventional TLM method in a rectangular grid as well as calculated by the cavity model explained previously. As can be seen, the input impedance calculated using the integral TLM cylindrical method shows the best agreement with the measurements compared to the cavity model and TLM method in the rectangular grid.

Since for the given feed position $\rho$=4.5 mm the fabricated antenna does not give minimum $S_{11}$ parameter at the operating frequency, further simulations have been carried out for various positions of the feed probe in order to assess the optimum position providing the matching between the input impedances of the antenna and the feed probe.

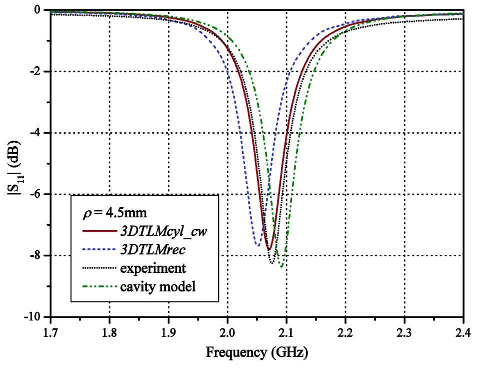
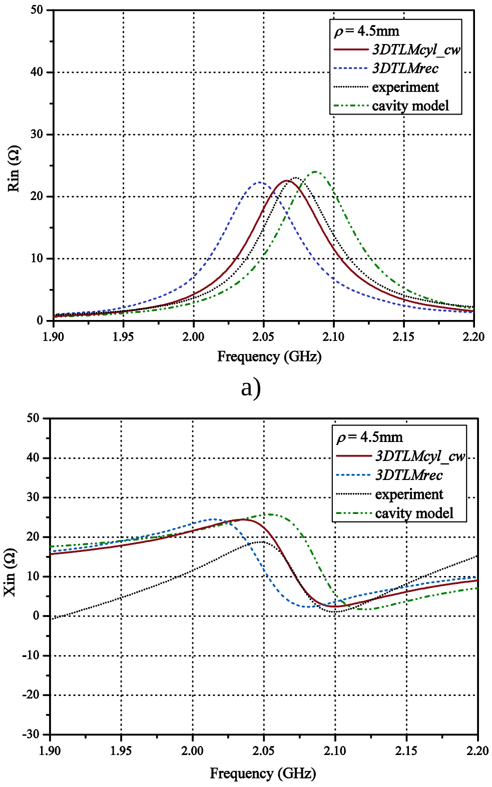


Fig. 4. $S_{11}$ parameter of the circular coax-fed patch antenna *for* $\rho_{opt}$ = 4.5mm



a)

b)

Fig. 5. Input impedance of a circular patch antenna for $\rho$ = 4.5mm: a) resistance, b) reactance

The obtained results, representing return loss versus the normalized feed position, are illustrated in Fig. 6. Obviously, the optimum feed position is found to be $\rho_{opt}$=7.5mm. Fig. 7. shows the input resistance changes by varying the position of the feed probe. When the optimum feed position has been established, simulations have been conducted using both meshes to obtain the reflection coefficient (Fig. 8) and to calculate the input impedance for this position (Fig. 9). In addition, $S_{11}$ parameter of the antenna has been measured for the optimum feed position. As can be seen from Fig. 8, an excellent agreement has been achieved between results simulated in the cylindrical grid and measurements.

## IV. CONCLUSION

This paper presents calculation of the input impedance of a circular patch antenna configuration using the TLM approach based on the cylindrical grid and enhanced with the compact wire model. Results in terms of the input resistance and reactance have shown better agreement with measurements in comparison to those determined using the TLM method in a rectangular grid and those calculated using the cavity model. According to the input impedance obtained for variable feed position, it has been found what feed position would give the best impedance matching between the antenna and the feed.

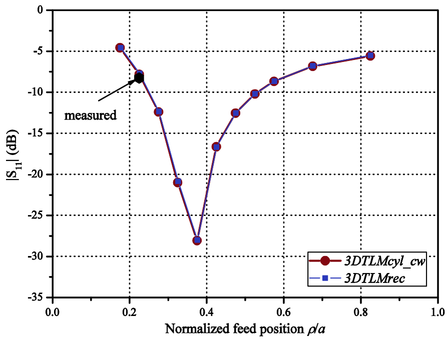


Fig. 6. $S_{11}$ parameter versus normalized feed position

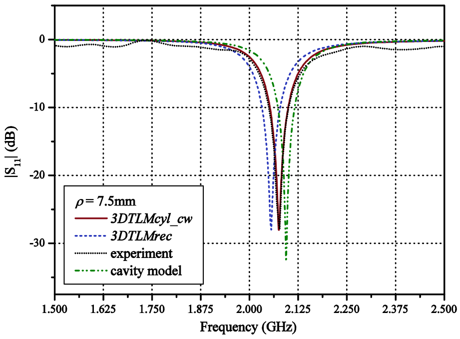


Fig. 8. $S_{11}$ parameter of the circular coax-fed patch antenna for $\rho_{opt}$ = 7.5mm

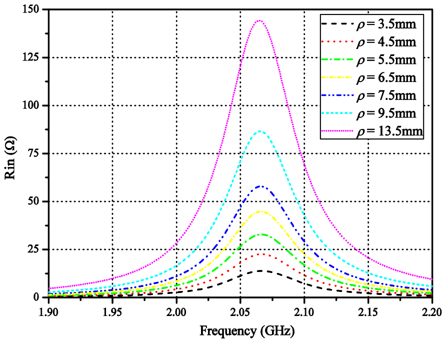


Fig. 7. Input resistance of a circular patch antenna for different probe position

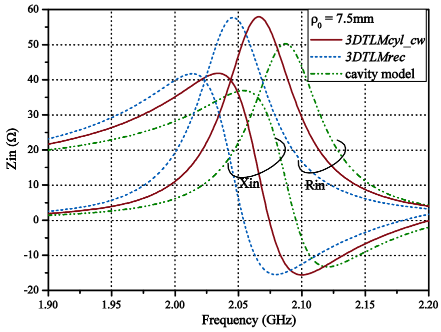


Fig. 9. Real and imaginary part of the input impedance at $\rho_{opt}$ = 7.5mm

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## REFERENCES

[1] C.A. Balanis, *Handbook of Microstrip Antennas*, John Wiley and Sons, New York, 1982.
[2] C.A. Balanis, *Antenna Theory, Analysis and Design*, John Wiley and Sons, New York, 2005.
[3] R. Garg, P. Bhartia, I. Bahl, A. Ittipiboon, *Microstrip Antenna Design Handbook*, Artech House, 2001.
[4] R. Waterhouse, *Microstrip Patch Antennas: A designer's Guide*, Kluwer Academic Publishers, London, UK, 2003.
[5] J. R. James and P. S. Hall, *Handbook of Microstrip Antennas*, Vols. 1 and 2, Peter Peregrinus, London, UK, 1989.
[6] A. G. Derneryd, "Analysis of the Microstrip Disk Antenna Element" IEEE Trans. Antennas Propagat., Vol. AP-27, No. 5, pp. 660–664, September 1979.
[7] C. Christopoulos, The Transmission-Line Modelling Method, IEE/OUP Press, 1995.
[8] D.A.Al.Mukhtar, J.E. Sitch, "Transmission-line matrix method with irregulary graded space", IEE proc. H,128, (6), pp.299-305, 1981.
[9] V. Trenkic, *The development and characterization of advanced nodes for TLM method*, Ph.D. Thesis, University of Nottingham, 1995.
[10] P.B.Johns, "A symmetrical condensed node for the TLM method", IEEE Trans. Microwave Theory tech, vol. MTT-35, no.4, pp.370-377, 1987.
[11] V.Trenkić, A.J.Wlodarczyk, R.A.Scaramuzza, "Modelling of Coupling Between Transient Electromagnetic Field and Complex Wire Structures", Int. Journal of Numerical Modelling, Vol.12, No.4, pp.257-273, 1999.
[12] H. Meliani, D. de Cogan, P.B. Johns, The use of Orthogonal Curvilinear Meshes in TLM Models, Int. Journal of Numerical Modelling, Vol.1, pp.221-238, 1988.
[13] R. Scaramuza, A.J. Lowery, "Hybrid Symmetrical Condensed Node for the TLM Method", Electronics letters, Vol.26, pp. 1947-1948, 1990.
[14] T. Ž. Dimitrijević, J. J. Joković, B. D. Milovanović, N. S. Dončov, "TLM Modelling of a Probe-coupled Cylindrical Cavity based on Compact Wire Model in the Cylindrical Mesh, International Journal of RF and Microwave Computer-Aided Engineering", John Wiley & Sons Inc., Wiley InterScience Periodicals, Vol.22, No. 2, pp. 184-192, 2012.
[15] F. Abboud, J.P. Damiano, A. Papiernik, "A New Model for Calculating the Input Impedance of Coax-fed Circular Microstrip Antennas with and Without Air Gaps", IEEE Trans. on Ant. And Prop., Vol.38, No.11, 1990.
[16] D. Guha, M. Biswas, J. Siddiqui, "Harrington's Formula Extended to Determine Accurate Feed Reactance of Probe-Fed Microstrip Patches", IEEE Ant. and Wireless Prop. Letters, Vol.6, 2007.
[17] Siddiqui, J.Y.; Guha, D., "Improved formulas for the input impedance of probe-fed circular microstrip antenna", Antennas and Propagation Society International Symposium, 2003. IEEE, vol. 3, pp:152 – 155, 22-27 June 2003.
[18] R.F. Harrington, *Time-Harmonic Electromagnetic Fields*, New York, McGraw-Hill, p.228, 1961.
[19] J. Jokovic, B. Milovanovic, N. Doncov, "Numerical and Experimental Analysis of Electromagnetic Field in a Probe Coupled Cylindrical Metallic Cavity", WSEAS Transaction on Communications, Issue 8, Vol.7, pp. 807-816, 2008.