Comparison of Novel Designed Class of CIC FIR Filter Functions with Classical CIC Filters

Biljana P. Stošić, Vlastimir D. Pavlović and Dejan N. Milić

Abstract – **This paper introduces a class of selective multiplierless Cascaded-Integrator-Comb (CIC) finite impulse response (FIR) filter functions where the CIC sections of different lengths give the improved frequency response characteristics. The novel class of CIC filter functions is designed only once, but by chosing values of the free integer parameters, various filter functions are obtained. Some design examples illustrate properties of the proposed filter class and comparisons, to well-known classical CIC filters, show a superiority of the novel filter class. Novel filter functions have minimum attenuation of 151.69 dB, 244.49 dB and 336.72 dB, respectively. The achieved improvements versus classical CIC filters are 15.01 dB, 20.83 dB and 26.08 dB, respectively.**

Keywords – **CIC filters, FIR filters, linear phase, multiplierless structure, selective filters.**

I. Introduction

Modern development of researches in pharmacy, biology and veterinary requires digital signal processing with integer coefficients of high-resolution. About the 80's, a measuring equipment up to 160 dB was used for processing of continuous signals. Digital data processing with higher resolution can analyze signals over 200 dB without the influence of undesirable parasitic effects. In many high-resolution scientific applications, highly selective FIR filters with integer coefficients will have an increasing hardware and software implementations.

The term "Cascaded-Integrator-Comb (CIC)" filters was first reported in 80's by E.B. Hogenauer [1]. Because of the disadvantages of a CIC FIR filter such as not flat passband and a high passband drop, it is of a great interest to improve magnitude response characteristic. The literature on the improving frequency response characteristics can be classified into several groups. For example, the authors from the first group [2-7] use a compensation filter in the cascade with the original filter. Then, some other use sharpening technique [8] or design new class of filter functions as shown in [9].

In the literature, the classical CIC filters are also known as $sinc^N$ filters. They are built as a cascade of N $sinc$ filters, each of finite length M. The use of these filters requires the calculation of all of the impulse response coefficients. In [10], simple straightforward recurrence relations were derived for fast computing of the impulse response coefficients of $sinc^N$ filters. A closed form expression for the first M coefficients has been presented in [11], while a recurrence formula has been given in [12].

In this paper, design of a novel class of CIC FIR filter functions which is based on cascading CIC sections of different lengths is described. For this class of modified CIC filters closed-form equations are proposed for computing the filter coefficients, as well as the study of frequency response characteristics through a specific set of parameter values is given. The non-recursive form can be used in order to provide the impulse response coefficients analytically. Design examples of the novel designed class of CIC filter functions are used to validate their characteristics by comparing them with those of the classical CIC filters under fair conditions: the same number of cascaded sections and the same group delay.

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II. Classical CIC Filter

The conventional CIC FIR filters are well known in literature. The normalized CIC FIR filter function of one section in z-domain is defined with

$$H(N,z)=\frac{1}{N}\cdot\left(1+z^{-1}+z^{-2}+\cdots+z^{-(N-1)}\right)=\frac{1-z^{-N}}{N\cdot(1-z^{-1})}. \quad (1)$$

A poor magnitude characteristic of the CIC filter composed of one section, Eq. (1), is improved by cascading several identical CIC filters. The classical CIC FIR filter function of normalized amplitude response characteristic, represented in the z-domain, is defined as

$$H(N,K,z)=\left(\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\right)^K, \quad (2)$$

where N is the decimation factor, and K is the number of sections (identical cascaded CIC filters of one section) [1].

The frequency response characteristic of CIC FIR filter function can be written in the form

$$H(N,K,z=e^{j\omega})=e^{-jK(N-1)\omega/2}\cdot\left(\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\right)^K. \quad (3)$$

III. Novel Class of CIC FIR Filter Functions

The novel class is design as cascade of four CIC FIR filter functions: one function $H(N-2,z)$, two functions $H(N,z)$, and one function $H(N+2,z)$, of the form given by (1), as well as seven cascaded non-identical CIC FIR filter sections which are repeated L times.

The filter function of normalized amplitude response characteristic of a designed novel class of CIC FIR filter functions can be written in non-recursive form as follows

$$H(N,K,L,z)=\left(\frac{1}{N-2}\sum_{r=0}^{N-3}z^{-r}\right)\left(\frac{1}{N}\sum_{r=0}^{N-1}z^{-r}\right)\left(\frac{1}{N}\sum_{r=0}^{N-1}z^{-r}\right)\cdot$$
$$\cdot\left(\frac{1}{N+2}\sum_{r=0}^{N+1}z^{-r}\right)\left[\left(\frac{1}{N-3}\sum_{r=0}^{N-4}z^{-r}\right)\left(\frac{1}{N-2}\sum_{r=0}^{N-3}z^{-r}\right)\left(\frac{1}{N-1}\sum_{r=0}^{N-2}z^{-r}\right)\right]^{L}\cdot$$
$$\cdot\left[\left(\frac{1}{N}\sum_{r=0}^{N-1}z^{-r}\right)\left(\frac{1}{N+1}\sum_{r=0}^{N}z^{-r}\right)\left(\frac{1}{N+2}\sum_{r=0}^{N+1}z^{-r}\right)\left(\frac{1}{N+3}\sum_{r=0}^{N+2}z^{-r}\right)\right]^{L} \quad (4)$$

where N and L are free integer parameters, and $K=7L+4$. This form is suitable for hardware realization, because it is unconditionally stable.

The recursive form of a novel class of CIC FIR filter functions with normalized amplitude response characteristic is

$$H(N,K,L,z)=$$
$$=\frac{1-z^{-(N-2)}}{(N-2)\cdot(1-z^{-1})}\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\frac{1-z^{-(N+2)}}{(N+2)\cdot(1-z^{-1})}\cdot$$
$$\left(\frac{1-z^{-(N-3)}}{(N-3)\cdot(1-z^{-1})}\frac{1-z^{-(N-2)}}{(N-2)\cdot(1-z^{-1})}\frac{1-z^{-(N-1)}}{(N-1)\cdot(1-z^{-1})}\right)^{L}\cdot$$
$$\left(\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\frac{1-z^{-(N+1)}}{(N+1)\cdot(1-z^{-1})}\frac{1-z^{-(N+2)}}{(N+2)\cdot(1-z^{-1})}\frac{1-z^{-(N+3)}}{(N+3)\cdot(1-z^{-1})}\right)^{L}$$

and $K=7L+4$. (5)

The frequency response of designed novel class of FIR filter functions is obtained by evaluating the filter function in the z -plane at the sample points defined by setting $z=e^{j\omega}$, where $\omega=2\pi\cdot f$ has units of radians per second. Using Euler's identity, it can be separated into a real-valued magnitude $A(N,K,L,\omega)$ and a real-valued phase angle $\varphi(N,K,L,\omega)$ for each frequency ω ,

$$H(N,K,L,z=e^{j\omega})=e^{j\cdot\varphi(N,K,L,\omega)}\cdot A(N,K,L,\omega) \quad (6)$$

where the parameter $K=7L+4$.

The normalized amplitude response characteristic of the proposed filter functions is defined in the form

$$A(N,K,L,\omega)=$$
$$=\frac{\sin((N-2)\omega/2)}{(N-2)\cdot\sin(\omega/2)}\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\frac{\sin((N+2)\omega/2)}{(N+2)\cdot\sin(\omega/2)}\cdot$$
$$\left(\frac{\sin((N-3)\omega/2)}{(N-3)\cdot\sin(\omega/2)}\frac{\sin((N-2)\omega/2)}{(N-2)\cdot\sin(\omega/2)}\frac{\sin((N-1)\omega/2)}{(N-1)\cdot\sin(\omega/2)}\right)^{L}\cdot$$
$$\left(\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\frac{\sin((N+1)\omega/2)}{(N+1)\cdot\sin(\omega/2)}\frac{\sin((N+2)\omega/2)}{(N+2)\cdot\sin(\omega/2)}\frac{\sin((N+3)\omega/2)}{(N+3)\cdot\sin(\omega/2)}\right)^{L}$$

and $K=7L+4$. (7)

The normalized magnitude response characteristic $\left|H(N,K,L,e^{j\omega})\right|$ is obtained as absolute value of the normalized amplitude response characteristic $A(N,K,L,\omega)$ given by (7).

The linear phase response characteristic of the proposed novel class of the modified CIC FIR filter has the form

$$\varphi(N,K,L,\omega)=-(N-1)\cdot K\cdot\omega/2+2\cdot\nu\cdot\pi, \qquad \nu=0,1,2,...$$

and $K=7L+4$. (8)

The constant group delay response characteristic of the proposed novel class of the modified CIC FIR filter functions is expressed as

$$\tau(N,K,L,\omega)=-d\varphi(N,K,L,\omega)/d\omega=(N-1)\cdot K/2,$$

and $K=7L+4$. (9)

It is independent of the frequency. For N being odd number, the group delay is an integer multiple of $K/2$. If N is even number, the group delay is equal to an integer plus half multiple of $K/2$.

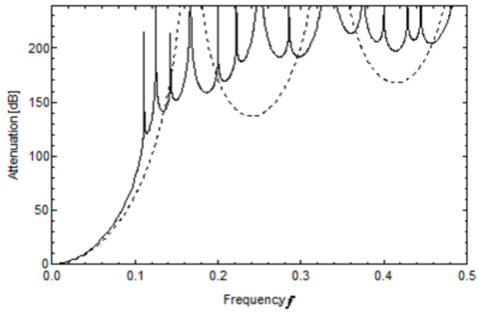
IV. FREQUENCY RESPONSE CHARACTERISTICS

The superiority of the novel designed class of CIC filter functions has further been established by comparing these novel CIC FIR architectures with existing classical CIC structures. For fair comparison with the existing classical CIC filters, this paper considers the novel filter functions $H(N,K,L,z)$ from Eq. (5) and the classical CIC filters $H(N,K,z)$ from Eq. (2) of the same values of the parameter N, the same number of cascaded sections K and the same value of group delay. Generaly, the main task of the comparison was to vary the free parameters and to compare obtained filter characteristics.

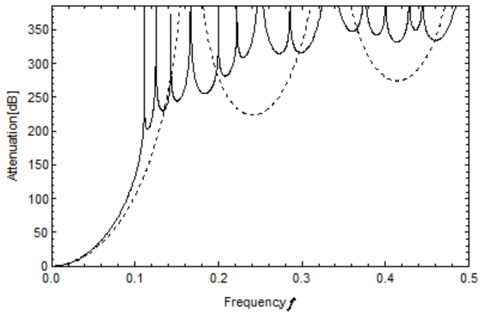
A detailed analysis of magnitude response characteristics, as well as their comparison with the classical filter functions is presented in Figs. 1-4.

The normalized magnitude response characteristics in dB, defined for the classical CIC filters as $\alpha_{CIC}(f)=-20\cdot\log\left|H(N,K,e^{j2\pi f})\right|$ and for the novel class of CIC FIR filter functions as $\alpha(f)=-20\cdot\log\left|H(N,K,L,e^{j2\pi f})\right|$, versus normalized frequency $f=\omega/(2\pi)$, are depicted on Fig. 1. The maximum attenuation in the passband is $\alpha_{\max}=0.28\,\mathrm{dB}$. The classical CIC FIR filter functions and the designed novel class of CIC FIR filter functions have the same number K of cascaded sections with the difference that the CIC filters have an identical sections in all cascades, and the designed novel class has a cascade-connected CIC filter sections of different lengths. Also, they have the same level of constant group delay, as well as number of delay elements, but the novel first designed class gives higher insertion losses in stopband, as well as it has higher selectivity.

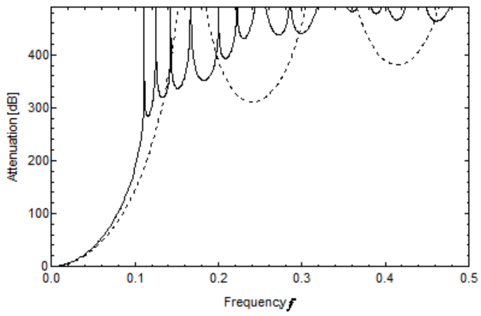
Zooms of the normalized magnitude response characteristics of classical CIC filter and proposed class of CIC FIR filter functions are given in Fig. 2. The novel class of CIC filter functions has two peaks in the transition area of the classical filter (on frequency between the passband f_{cp} and stopband f_{cs} cut-off frequencies). Note that attenuation of the novel class in the stopband area is higher then attenuation of the classical CIC filter in the stopband area. In case of $K=11$ (Fig. 3*a*), classical CIC filter has attenuation of 136.68 dB and novel class 151.69 dB. It is achieved improvement of 15.01 dB or approximately about 11 %. In case of $K=18$ (Fig. 3*b*), it is achieved improvement of 20.83 dB or approximately about 9 %. In case of $K=25$ (Fig. 3*c*), it is achieved significant improvement of 26.08 dB or approximately about 8 %.



(a) Classical CIC filter for $K=11$ (dashed line), novel class of CIC filter for $K=11$, $L=1$ (solid line)



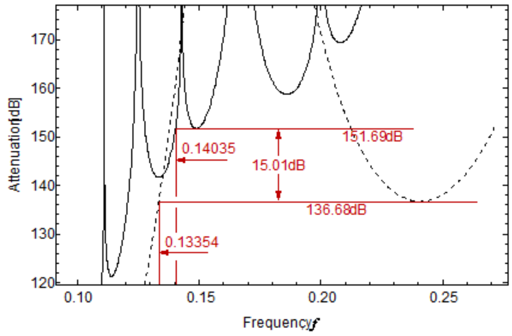
(b) Classical CIC filter for $K=18$ (dashed line), novel class of CIC filter for $K=18$, $L=2$ (solid line)



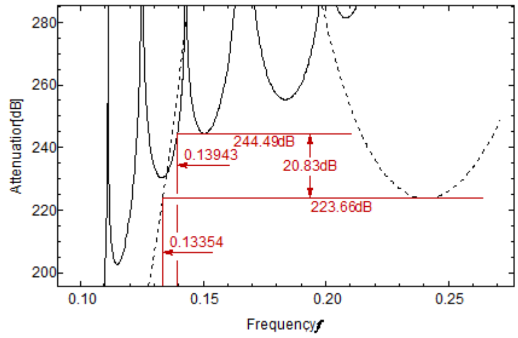
(c) Classical CIC filter for $K=25$ (dashed line), novel class of CIC filter for $K=25$, $L=3$ (solid line)

Fig. 1. Comparison of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and proposed novel class of CIC FIR filter functions (solid lines), for $N=6$

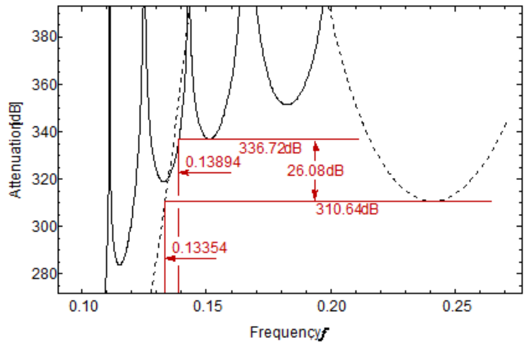
Fig. 3 presents two-dimensional (2D) contour plots of normalized magnitude response characteristics (overall and lower frequency part zoomed) of the classical CIC filters and the proposed novel class of CIC FIR filter functions. As the value of the parameter N increases, as well as the normalized frequency, the benefits of the proposed filter class become less apparent, and the characteristics closely resemble that of the classical CIC filters. Therefore, it can be concluded that the proposed filter class is more efficient in lower part of frequency range and for smaller values of parameter N.



(a) Classical CIC filter for $K=11$ (dashed line), novel class of CIC filter for $K=11$, $L=1$ (solid line)



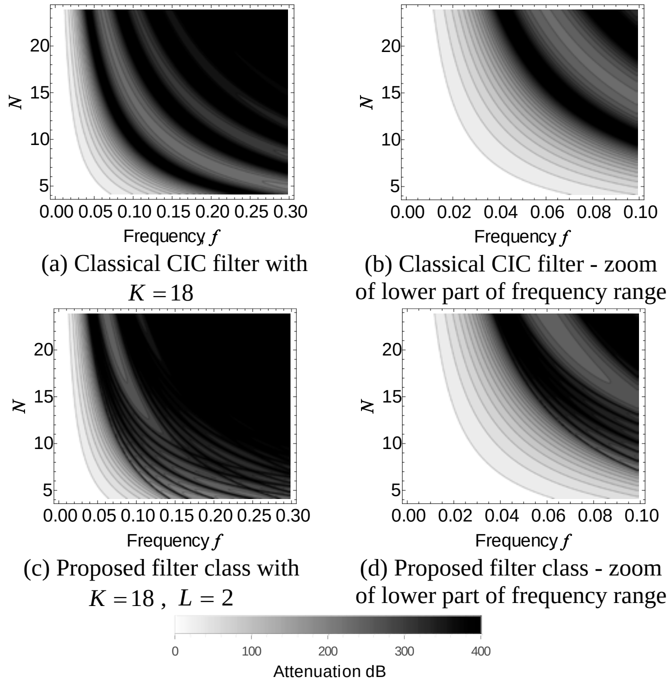
(b) Classical CIC filter for $K=18$ (dashed line), novel class of CIC filter for $K=18$, $L=2$ (solid line)



(c) Classical CIC filter for $K=25$ (dashed line), novel class of CIC filter for $K=25$, $L=3$ (solid line)

Fig. 2. Zooms of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and proposed novel class of CIC FIR filter functions (solid lines), for $N=6$

In Fig. 4, three-dimensional (3D) plot of normalized magnitude response characteristic of a novel class of CIC FIR filter functions is shown. It is shown a normalized magnitude response in frequency domain as a function of parameter $N\in\{4-17\}$, for case of $K=18$. It is worth to noting that with the increase in the value of the parameter N the passband becomes narrower, as is expected. The number of transfer function zeros is increased and this is clearly visible in branching of high loss regions in magnitude response characteristics, especially for the smaller values of the parameter N and towards higher frequencies.



(a) Classical CIC filter with $K = 18$

(b) Classical CIC filter - zoom of lower part of frequency range

(c) Proposed filter class with $K = 18$, $L = 2$

(d) Proposed filter class - zoom of lower part of frequency range

Fig. 3. 2D contour plots of magnitude frequency response characteristics for classical and proposed first class of CIC FIR filters for $N \in \{4-24\}$ and $K = 18$

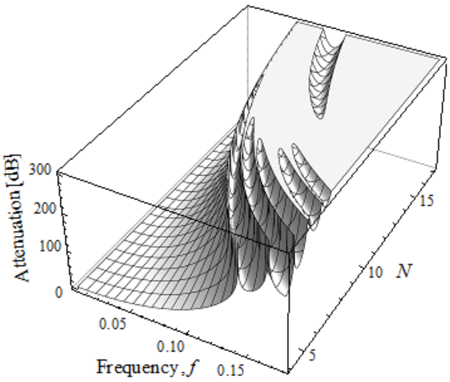


Fig. 4. 3D plot of normalized attenuation response characteristic in dB of the proposed first class of CIC FIR filter for $N \in \{4-17\}$, and $K = 18$ obtained for $L = 2$

V. Conclusion

This paper proposes a novel class of CIC FIR filter functions. Design with subfilters of different lengths has been considered here.

Comparative study of the performances of novel class is made with that of well-known classical CIC filters. From the study, it is evident that the suggested novel class is very useful for modifying the magnitude frequency response characteristics of filters without changing the group delay of classical CIC filters.

As an application example of the novel filter classes, the sigma-delta ($\Sigma\Delta$) analog-to-digital (A/D) converters can be observed. Some modified filter structures for $\Sigma\Delta$ A/D converters published earlier are given in [13-14]. The application of filter sharpening technique to the CIC and the modified comb filters is shown in [15-16].

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