Performance Analysis of RF/FSO System With Interference at the Relay

Aleksandra M. Cvetkovic[1], Milica I. Petkovic[1] and Jelena A. Anastasov[1]

Abstract – **In this paper we investigate outage probability of the mixed radio frequency and free space optical (RF/FSO) system in the presence of multiple interferers at the relay. The system performance of dual-hop RF/FSO system with both the amplify-and-forward and decode-and-forward protocols is analyzed. Based on brief analytical evaluations, numerical results are obtained and influences of atmospheric turbulence, interference and type of relay on outage performance are discussed.**

Keywords – **Co-channel interference, Free space optical system, Outage probability, Relay.**

I. INTRODUCTION

Free space optical (FSO) communications are recently proposed as a solution for the last-mile problem due to many offered benefits [1]. The FSO technology is based on the optical signal transmission through atmosphere without requiring optical fiber as a medium, which provides low-cost and savings in economic resources. The FSO represents promising alternative technology for indoor and outdoor applications, since it provides the speed of optical fiber with the benefits of wireless systems. Furthermore, the utilization of FSO systems does not require securing spectrum licenses, which is the major advantage compared to radio frequency (RF) systems. The FSO transmission requires the optical line-of-sight and its quality is greatly dependent on atmospheric conditions, like rain, snow and fog. Due to mentioned properties, the FSO technology found a number of applications, including building-to-building connectivity, overall data communications, network redundancy and temporary connectivity [2].

The performance of wireless systems can be improved by using relaying technology. In this way, the extension of the coverage area and increase of the capacity of the wireless networks is accomplished [3]. Using the benefits of both RF and FSO technologies, the mixed dual-hop RF/FSO relaying systems were proposed in [4].

Two kinds of relaying exist: decode-and-forward (DF) and amplify-and-forward (AF) [5]. Assuming DF relaying, the signal at relay is first decoded after direct detection, converted to an optical one via subcarrier intensity modulation (SIM) scheme, and next retransmit via FSO channel. Compared to DF, the AF system is simpler since any decoding is not employed. The AF relay only amplifies the received signal with a proper gain and forwards it to the destination. Depending how relay gain is determined, AF relays can be classified as: fixed and variable gain relaying [5]. Papers [4], [6]-[9] analyzed mixed RF/FSO relaying systems with fixed gain, i.e. the relay gain is determined by the statistical channel state information (CSI). Furthermore, the RF/FSO relaying system with variable gain is observed in [10], where the relay monitors instantaneous CSI of first RF link, so the signal is amplified according to corresponding CSI.

In this paper we analyzed the mixed RF/FSO system with both AF and DF relays. Due to the non-interfering nature of FSO link, only relay node is corrupted by co-channel interferences. The end-to-end outage performance for proposed systems is observed, while the fading in RF channels of the first hop and the interferences is described by Rayleigh distribution. The atmospheric turbulence of the FSO link is modelled by gamma-gamma distribution. Based on the equivalent signal-to-interference-noise ratio (SINR), the outage probability expressions are derived for both AF and DF relaying systems.

[1]Aleksandra M. Cvetković, Milica I. Petković and Jelena A. Anastasov, are with the University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mails: aleksandra.cvetkovic@elfak.ni.ac.rs, milica.petkovic@elfak.ni.ac.rs, jelena.anastasov@elfak.ni.ac.rs

II. SYSTEM AND CHANNEL MODEL

Considered mixed dual-hop RF/FSO relay system is presented in Fig. 1. The source node S communicates with the relay node R via RF link. The second hop is the FSO link, which operates between the node R and the destination node D. It is considered that the level of the additive white Gaussian noise at the relay is neglecting small compared to level of co-channel interference (CCI), as in all interference-limited fading environments.

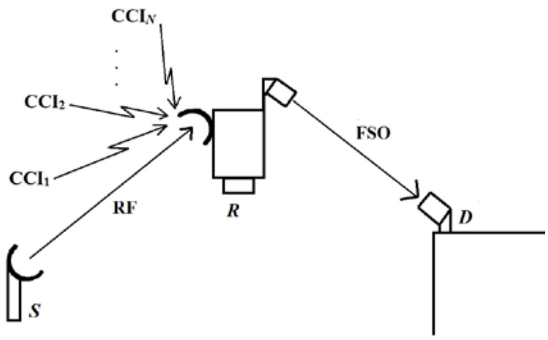

Fig. 1. System model of a mixed dual-hop RF/FSO relay system

The relay is corrupted by N co-channel interferers, each with an average power of P_i, i=1,2,...,N, so the received RF signal at the relay node is

$$r_R = h_{SR}s_0 + \sum_{i=1}^{N} h_i s_i \tag{1}$$

where s_0 is the RF signal transmitted from the node S with an average power P_0, h_{SR} is the RF fading amplitude and h_i is the co-channel interference fading amplitude, both modeled by Rayleigh distribution. The SIM scheme converts the RF signal to the optical for transmission.

A. System with CSI-assisted AF relay

In the mixed AF RF/FSO system, the signal at the relay is multiplied with a gain G, and then retransmitted to the destination. Afterwards, the RF signal is converted to the optical one as

$$r_{opt} = G(1 + m r_R) \tag{2}$$

where m represents the modulation index. It is assumed that m=1. After removing the dc bias by a bandpass filter, the photodetector converts the received optical signal to electrical, which is given by

$$r_R = h_{RD} G r_R + n_D = h_{RD} G\left(h_{SR} s_0 + \sum_{i=1}^{N} h_i s_i\right) + n_D \tag{3}$$

where $n_D(t)$ is additive white Gaussian noise over FSO link with zero mean and variance $\sigma_D{}^2$ The overall SINR at the destination, for the case when AF relay is implemented, can be written as

$$\gamma_{eq}^{AF} = \frac{|h_{SR}|^2 |h_{RD}|^2 G^2 P_0}{|h_{RD}|^2 G^2 \sum_{i=1}^{N} |h_i|^2 P_i + \sigma_D^2} = \frac{|h_{SR}|^2 P_0 \dfrac{|h_{RD}|^2 P_R}{\sigma_D^2}}{\dfrac{|h_{RD}|^2 P_R}{\sigma_D^2} \sum_{i=1}^{N} |h_i|^2 P_i + \dfrac{P_R}{G^2}} \tag{4}$$

where P_R is the average transmitted optical power at the output of the relay.

The gain of AF relay depends on CSI of RF hop and CSI from all received interferences at the relay (assuming that the effect of noise ignored) as [11]

$$G^2 = \frac{P_R}{|h_{SR}|^2 P_0 + \sum_{i=1}^{N} |h_i|^2 P_i} \tag{5}$$

The overall SINR of RF/FSO system can be expressed as [11]

$$\gamma_{eq}^{AF} = \frac{\gamma_1 \gamma_2}{\gamma_3(\gamma_2 + 1) + \gamma_1} \tag{6}$$

where $\gamma_1 = |h_{SR}|^2 P_0$, $\gamma_2 = \dfrac{|h_{RD}|^2 P_R}{\sigma_D^2}$ and $\gamma_3 = \sum_{i=1}^{N} |h_i|^2 P_i$.

We considered that fading envelope of RF link has Rayleigh distribution so probability density function (pdf) of γ_1 is given by [12]

$$p_{\gamma_1}(\gamma_1) = \frac{1}{\mu_1} \exp\left(-\frac{\gamma_1}{\mu_1}\right) \tag{7}$$

where μ_1 represents the average signal power.

It is assumed that the interfering signals also experience the Rayleigh fading, as independent and identically distributed with average power μ_3, so pdf of γ_3 is [12]

$$p_{\gamma_3}(\gamma_3) = \frac{1}{\Gamma(N)\mu_3^N} \gamma_3^{N-1} \exp\left(-\frac{\gamma_3}{\mu_3}\right) \tag{8}$$

The FSO link is under the influence of atmospheric turbulence which is modeled by gamma-gamma distribution. The instantaneous SNR, γ_2, has pdf given by [4]

$$p_{\gamma_2}(\gamma_2) = \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)\mu_2^{(\alpha+\beta)/4}} \gamma_2^{(\alpha+\beta)/4-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{\gamma_2}{\mu_2}}}\right) \tag{9}$$

where $K_v(.)$ is the vth-order modified Bessel function of the second kind [13, eq. (8.432.2)] and μ_2 represents the average SNR of R-D link. The parameters α and β can be related to atmospheric conditions. If the plane wave propagation and zero inner scale is assumed, the parameters α and β can be expressed as [1]

$$\alpha = \left(\exp\left[\frac{0.49\sigma_R^2}{\left(1 + 1.11\sigma_R^{12/5}\right)^{7/6}}\right] - 1\right)^{-1} \tag{10}$$

$$\beta = \left(\exp\left[\frac{0.51\sigma_R^2}{\left(1 + 0.69\sigma_R^{12/5}\right)^{5/6}}\right] - 1\right)^{-1} \tag{11}$$

where σ_R^2 is the Rytov variance. For simplification, turbulence strength is counted by σ_R .

B. System with DF relay

The outage probability of the system with DF relay is defined as the probability that the instantaneous SIR of the first hop or SNR of the second hop falls below the given threshold γ_{th}. For observed system, the equivalent SINR is [11]

$$\gamma_{eq}^{DF} = \min(\gamma_1 / \gamma_3, \gamma_2) \tag{12}$$

In order to calculated the outage performance, it is necessary to define the cumulative distribution function (cdf) of the instantaneous SNR of the second hop as [14]

$$F_{\gamma_2}(\gamma_{th}) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{2,1}\left[\frac{\alpha\beta\gamma_{th}^{1/2}}{\mu_2^{1/2}}\middle|\begin{matrix} 1 \\ \alpha \quad \beta \quad 0 \end{matrix}\right] \tag{13}$$

The cdf of the instantaneous SIR of the first hop is determined as [15]

$$F_{\gamma_{SIR}}(\gamma_{th}) = 1 - \left(1 + \frac{\gamma_{th}}{\rho}\right)^{-N} \tag{14}$$

where $\rho = \mu_1 / \mu_3$ is average signal-to-single interference (SIR) of the RF hop.

III. OUTAGE PROBABILITY

The outage probability is defined as the probability that the instantaneous equivalent SINR falls below a predetermined threshold, γ_{th} [3]

$$\Pr(\gamma_{eq} < \gamma_{th}) = F_{\gamma_{eq}}(\gamma_{th}) \tag{15}$$

A. Outage probability for system with CSI-assisted AF relay

Outage probability for the RF/FSO system with CSI-assisted AF relay can be derived as [5]

$$F_{\gamma_{eq}}^{AF}(\gamma_{th}) = \int_0^\infty \int_0^\infty \Pr\left(\gamma_1 \le \frac{\gamma_{th}(y+1)z}{y-\gamma_{th}}\right) p_{\gamma_2}(y) p_{\gamma_3}(z) \mathrm{d}y\mathrm{d}z \quad (16)$$

where $p_{\gamma_2}(x)$ and $p_{\gamma_3}(x)$ are pdfs given by (9) and (8), respectively and cdf of γ_1 is $F_{\gamma_1}(\gamma_{th}) = 1 - \exp(-\gamma_{th}/\mu_1)$. Using $\omega = y - \gamma_{th}$, the previous equation can be rewritten as

$$\begin{aligned} F_{\gamma_{eq}}^{AF}(\gamma_{th}) &= 1 - \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)\Gamma(N)\mu_2^{(\alpha+\beta)/4}\mu_3^N} \\ &\times \int_0^\infty \int_0^\infty \exp\left(-\frac{\gamma_{th}(\omega+\gamma_{th}+1)z}{\omega\mu_1}\right)(\omega+\gamma_{th})^{(\alpha+\beta)/4-1} \\ &\times K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{\omega+\gamma_{th}}{\mu_2}}}\right) z^{N-1} \exp\left(-\frac{z}{\mu_3}\right) \mathrm{d}\omega\mathrm{d}z \end{aligned} \quad (17)$$

Integral in (17), the one on *z*, can be solved applying [13, (3.351.3)], resulting in

$$\begin{aligned} F_{\gamma_{eq}}^{AF}(\gamma_{th}) &= 1 - \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)\mu_2^{(\alpha+\beta)/4}\mu_3^N} \int_0^\infty (\omega+\gamma_{th})^{(\alpha+\beta)/4-1} \\ &\times \left(\frac{1}{\mu_3} + \frac{\gamma_{th}(\omega+\gamma_{th}+1)}{\omega\mu_1}\right)^{-N} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{\omega+\gamma_{th}}{\mu_2}}}\right) \mathrm{d}\omega \end{aligned} \quad (18)$$

Because of unavailable closed-form of remaining integral in (18), we used numerical integration in *Mathematica* software package for further analysis.

B. Outage probability for system with DF relay

The outage probability of dual-hop DF relay mixed RF/FSO system is the probability that either one of the two hops is in outage, and can be determined using

$$F_{\gamma_{eq}}^{DF}(\gamma_{th}) = 1 - \left(1 - F_{\gamma_{SIR}}(\gamma_{th})\right)\left(1 - F_{\gamma_2}(\gamma_{th})\right) \quad (19)$$

where $F_{\gamma_{SIR}}(\gamma_{th})$ is the cdf given by (14) and $F_{\gamma_2}(\gamma_{th})$ is the cdf of instantaneous SNR of FSO link given by (13).

IV. NUMERICAL RESULTS

Numerical results for outage probability of RF/FSO system with interference at the relay, obtained using (18) for AF relay and (19) for DF relaying system, are shown in this Section.

In Fig. 2. numerical results for outage probability of mixed RF and FSO system with both AF and DF relay are illustrated for different atmospheric turbulence condition. Results are presented in the function of $\mu_2=\rho$, for single dominant interference at the relay (N=1) and for γ_{th} = -5 dB. The system has better performance in weak turbulence conditions (σ_R = 0.8). With worse atmospheric conditions (σ_R = 2 and σ_R = 5), outage probability increases and system performance is degraded for both types of relay. As it was expected, the better performance is achieved for the system with DF relay compared to the system with AF relay. The difference in outage probability for those systems is more expressed in better turbulence conditions.

Outage performance versus the average SIR over RF link for different number of interferences at AF relay is presented in Fig. 3. The outage probability increases when number of present interferences at the relay increases. At the high values of ρ, the impact of the number of interference is more pronounced for greater values of the average SNR per FSO hop. We can notice that when the average SNR of FSO link increases, the system performance becomes better.

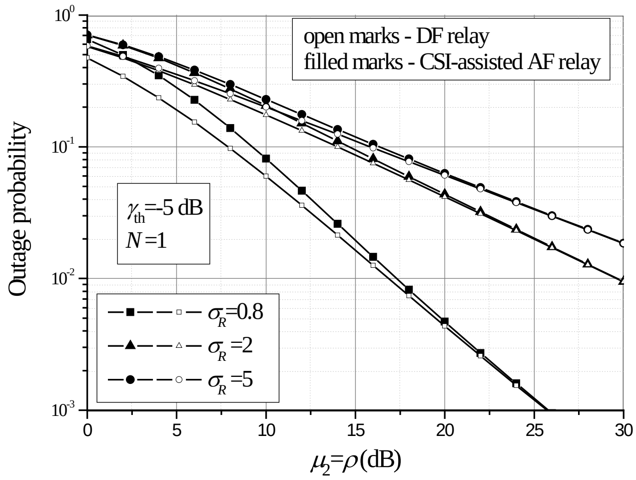

Fig. 2. Outage probability dependence on $\mu_2=\rho$ for different turbulence condition

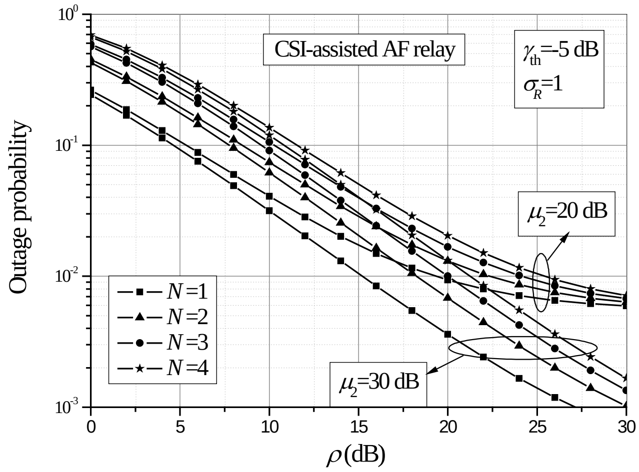

Fig. 3. Outage probability versus the average SIR over RF link for different number of interferences

In Fig. 4. outage dependence on average SIR per RF link for different values of the average SNR of FSO hop are shown. Results are presented for both AF and DF relays. System with DF relays outperforms system with AF relays at lower values of the average SIR of RF link and the average SNR of FSO link. In addition, the outage threshold occurs at higher values of ρ and the type of implemented relay has no influence on the system performance. At high values of ρ and μ_2 the systems with AF and DF relays have the same outage probability.

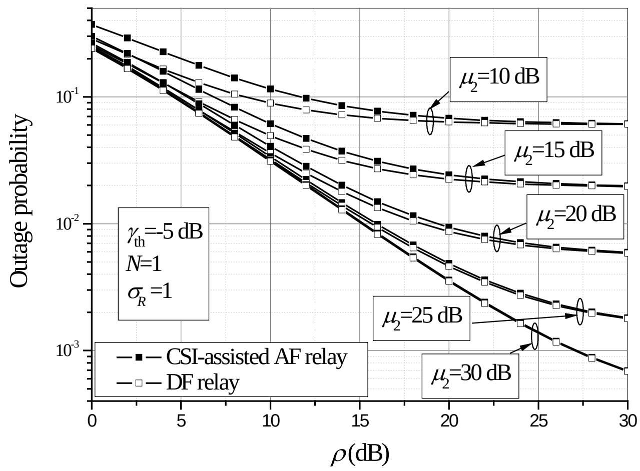

Fig. 4. Outage probability versus the average SIR over RF link for different values of the average SNR of FSO hop

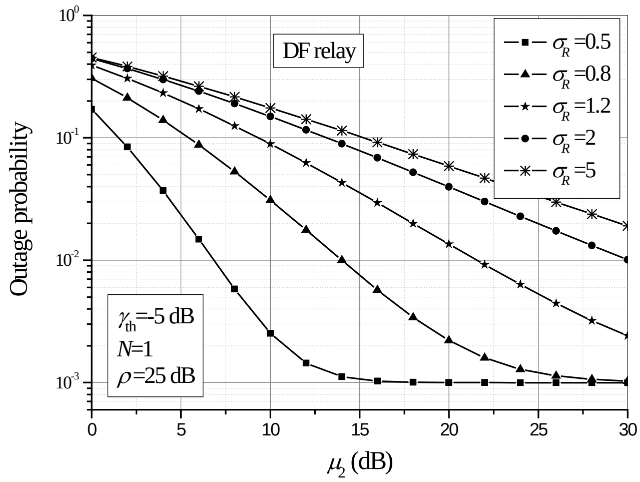

Fig. 5. Outage probability versus the average SNR over FSO link for different turbulence condition

Outage probability for DF relaying system as a function of average SNR over FSO link for different turbulence condition is presented in Fig. 5. If the turbulence condition is better, i.e. σ_R decreases, the outage probability also decreases. Furthermore, the outage floor appears at greater values of average SNR per FSO link. When σ_R takes a smaller value, the outage floor appears at lower values of μ_2.

V. Conclusion

A dual-hop system where RF and FSO links were connected with relay which is corrupted by co-channel interferences has been analyzed. The RF link and interference experience the Rayleigh fading, while the FSO link is under the atmospheric turbulence modelled by gamma-gamma distribution. The expressions for outage probability have been derived for CSI-assisted AF and DF relaying RF/FSO system. The numerical results are presented and compared. The effects of turbulence strength, type of relays and the number of interferers have been analyzed and discussed.

Acknowledgement

This work was supported by Ministry of science and technology development of Republic of Serbia (grants III 44006, TR-32028 and TR-32052).

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