Algorithms for APSK Constellation Optimization

Lidia Jordanova[1], Lyubomir Laskov[2] and Dobri Dobrev[3]

Abstract – **In this paper two algorithms for optimizing the parameters of APSK constellations are presented. A mathematical model is synthesized which is applicable for both algorithms. The optimal parameters for 16APSK constellations derived by both algorithms are determined. Dependencies of the symbol error probability on the E_s/N_0 ratio are given using the optimized parameters of the APSK constellations studied.**

Keywords – **APSK constellation, Minimum Euclidean distance, SER, E_s/N_0.**

I. Introduction

The requirement to the contemporary communication systems for providing higher rate of the transmitted data demands the application of modulation methods with a greater spectral efficiency. When M-ary modulation technique of higher order is used then both the spectral efficiency and the bit rate are increased but in turn the channel noise immunity is decreased. These contradictious requirements have to be taken into account when modulation technique is selected for satellite DVB systems.

In the satellite systems except the noise and the interference, the channel nonlinearity also causes a problem. The nonlinearity exists because of the fact that the working mode of the board power amplifier is chosen near the saturation point in order to reach the maximum level of the transmitted signal. The nonlinear signal distortions, similarly to the noise and interference in the radio channel, are the reason for the increase of the error probability.

The research done shows that the noise immunity of a linear AWGN channel when using APSK and QAM modulations with equal order of manipulation, is almost the same [1]. The APSK modulation, however, provides a greater resistance to nonlinear distortion in the radio channel [2,4], which is the reason why this modulation is most widely used in the second-generation satellite DVB systems.

The concept of the APSK modulation was put forward 40 years ago [3], but only later were its advantages appreciated in the cases when the radio channel is nonlinear [4]. In [1] are given APSK constellations whose parameters are optimized according to the criterion of maximum mutual information. Exactly these constellations are accepted in the standard DVB-S2.

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The aim of this paper is to study the impact of the APSK signal parameters on the noise immunity of the radio channel and to derive their optimal values with which a minimal symbol error probability is reached.

II. Mathematical Models of the Used Algorithms

The APSK constellation consists of N number of concentric circles, where the k-th circle contains n_k number of signal points. Each of the circles in the constellation is characterized by a primary phase shift φ_k and radius r_k. For convenience, instead the radiuses are used their retios relative to the radius of the innermost circle - γ. The APSK modulation is usually denoted as n_1-n_2-…APSK.

The aim of the parameter optimization of the APSK constellation is to achieve maximum radio channel noise immunity without any essential deterioration of the power effectiveness. There are several known algorithms for optimization of APSK constellations [5,6], whose major differences are related to the criterion chosen. The studies presented in this paper are based on two of these algorithms. In the first algorithm, the constellation parameters are chosen so as to maximize the minimum Euclidean distance. The aim of the optimization which is done by the second algorithm is to provide minimum symbol error probability.

After processing the basic expressions, given in [7], the following dependence for determination of symbol error probability P_s was obtained:

$$P_s \le \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,\, j\ne i}^{M} P(s_i \to s_j) = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,\, j\ne i}^{M} \frac{1}{2}\, erfc\left(\frac{d_{ij}}{2\sqrt{N_0}}\right). \quad (1)$$

In this expression, $P(s_i \rightarrow s_j)$ denotes the probability that instead of the i-th symbol the j-th one is accepted, M is the modulation order, N_0 is the noise power density and d_{ij} is the Euclidean distance between the i-th и j-th points of the constellation. The value of the error complementary function is obtained by the formula [8]

$$erfc(x) \approx \frac{1}{x\sqrt{\pi}} \cdot \exp\left(-x^2\right). \quad (2)$$

In order to determine the Euclidean distance between two points which define the i-th and j-th positions of the APSK signal vector, we can use the cosines theorem, i.e.

$$d_{ij} = \sqrt{r_{p(i)}^2 + r_{q(j)}^2 - 2.r_{p(i)}.r_{q(j)}.\cos\Theta_{ij}}\ , \quad (3)$$

where $r_{p(i)}$ and $r_{q(j)}$ are the radiuses of the circles where the two points are located and Θ_{ij} is the angle between the signal vectors studied. The value of Θ_{ij} is derived by the formula [6]

$$\Theta_{ij} = \left| \left(\varphi_p - \varphi_q\right) + 2\pi\left(\frac{i-1}{n_p} - \frac{j-1}{n_q}\right) \right|, \qquad (4)$$

where φ_p and φ_q denote the relative phase shifting of the signal points located on the p-th and q-th circles, n_p and n_q is the number of these points.

The relation between the radiuses of the circles in the APSK constellation and the energy per symbol E_s can be described by the following dependency:

$$E_s = \frac{n_1.r_1^2 + n_2.r_2^2 + \ldots + n_N.r_N^2}{M} = \frac{r_1^2(n_1 + n_2.\gamma_1^2 + \ldots + n_N.\gamma_{N-1}^2)}{M} \qquad (5)$$

where $\gamma_i = r_{(i+1)}/r_1$.

The mathematical model described is applicable for both algorithms and allows to optimize the following parameters of APSK constellations: the number of phase states n_k of the signal vector with amplitude r_k, the ratios between the amplitudes of the signal vector $\gamma_{(k-1)} = r_k/r_1$ and the relative phase shifting of the symbol points φ_k, where $k = 1, 2, \ldots N$.

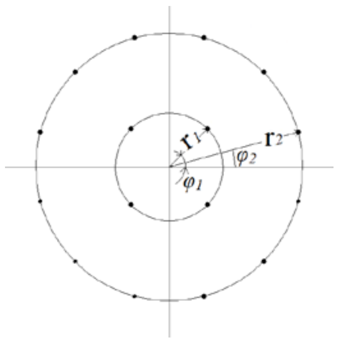

Fig. 1. 4-12APSK constellation

In this paper are given only the results derived when using both algorithms for the optimization of 16APSK constellation parameters. As it is evident in Fig. 1, the signal vector in this case has two amplitude (r_1 and r_2) and 12 phase states, and the initial phase shifting of the symbol points located on the inner and outer circles, is respectively φ_1 and φ_2. The parameters which undergo optimization are the number of symbol points on the inner and outer circles of the 16APSK constellation n_1 and n_2, their initial phase shifting φ_1 and φ_2 and the radius ratio of the two circles $\gamma_1 = r_2/r_1$.

III. Parameters of 16APSK constellation obtained by the first algorithm

For the optimization of the 16APSK constellation parameters, by the first algorithm we need to find the maximum value of the minimum Euclidean distances $d_{\min}$, and for this purpose the expressions (3), (4) and (5) are used. The study is conducted for different combinations of the parameters n_1, n_2 and φ_1, and for each of them is defined the functional dependency of $d_{\min}$ on the rest parameters of the APSK constellation – γ_1 and φ_2.

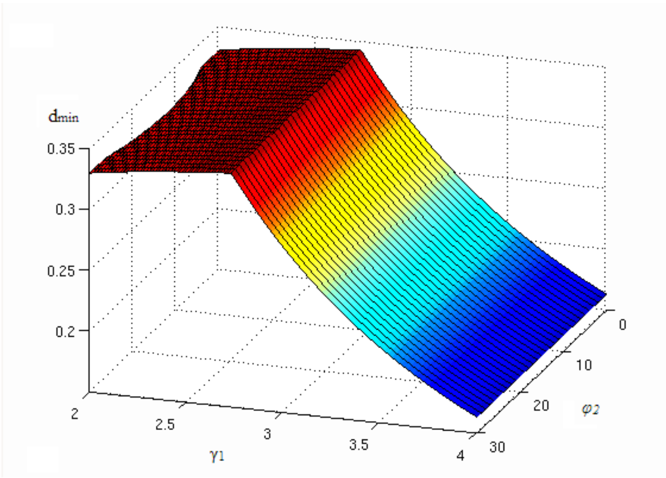

Fig. 2. Minimum Euclidean distances with 4-12APSK in a dependency on γ_1 and φ_2

In Fig. 2 are shown the results of a 16APSK constellation simulation study, where is accepted that $n_1 = 4$, $n_2 = 12$ and $\varphi_1 = 45^{\circ}$ and the energy per symbol is limited, i.e. $E_s = 1$. The analysis of the results obtained shows that the maximum Euclidean distance is provided when $\gamma_1 = 2.73$, and the impact of the parameter φ_2, which is not very prominent, can be ignored, when $\gamma_1 \geq 2{,}08$. As a result, for the 4-12APSK constellation in question, the following values of the parameters studied are chosen as optimal: $\gamma_1 = 2.73$ and $\varphi_2 = 0$.

Analogous studies have also been conducted for other 16APSK constellations, and the parameters of some of them are shown in Table I. For comparison, in the same table are given the parameters of the standard 16APSK constellation, which is used in the DVB-S2 systems with LDPC rate $R_{LDPC} = 8/9$.

Table I
Parameters of the optimal 16APSK constellations

Constellation Number (CN)	n_1, n_2	γ_1	φ_1, deg	φ_2, deg
CN 1	3, 13	3.61	60	0
CN 2	4, 12	2.73	45	0
CN 3	5, 11	2.17	36	0
CN 4	6, 10	2.00	30	0
DVB-S2	4, 12	2.6	45	15

The dependencies of the symbol error rate (SER) on the energy per symbol to noise power density radio (E_s/N_0) for the optimized 16APSK constellations are shown in Fig. 3. In Table II are given the values of E_s/N_0 parameter which are necessary for providing SER = 10^{-1}, 10^{-2} and 10^{-3}.

As it is evident in Table II, the best radio channel noise immunity can be provided when the third APSK constellation is used, where $n_1 = 5$, $n_2 = 11$, $\gamma_1 = 2.17$, $\varphi_1 = 36^{\circ}$ and $\varphi_2 = 0^{\circ}$.

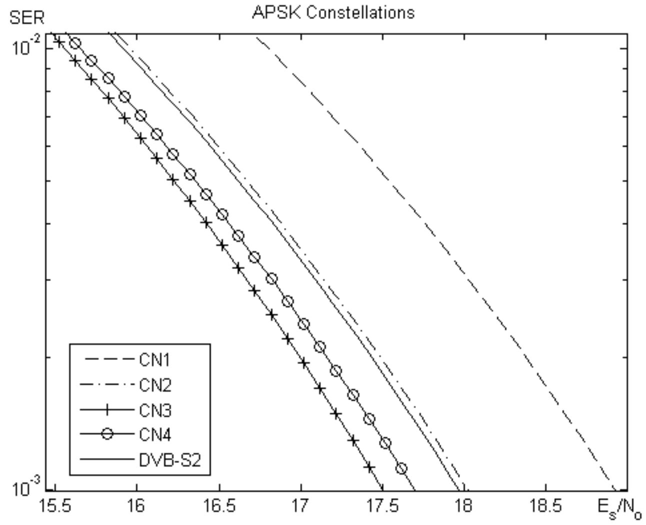

Fig. 3. SER characteristics of 16APSK channels

In comparison with the constellation which is standard for DVB-S2, the achieved benefit is 0.36 dB (for SER = 10^{-2}) and 0.46 (for SER = 10^{-3}).

TABLE II
CALCULATED VALUES OF THE PARAMETER E_S/N_0, dB

SER	10^{-1}	10^{-2}	10^{-3}
CN 1	13.080	16.803	18.926
CN 2	12.540	15.958	18.026
CN 3	12.357	15.558	17.507
CN 4	12.371	15.654	17.703
DVB-S2	12.528	15.913	17.969

IV. OPTIMIZATION OF 16APSK CONSTELLATION UNDER THE CRITERIA FOR A MINIMUM SER

The aim of the study presented in this section is to derive the parameters of the 16APSK constellation, where a minimum of the functional dependency which is presented by expression (1), is reached. We have used the same combinations of the parameters n_1, n_2 and φ_1 as in the previous section and for each of them is derived the functional dependency of SER on the parameters γ_1 and φ_2 of the APSK constellation.

The results of the conducted simulation study for the 4-12APSK constellation, obtained when $E_s/N_0 = 15$ dB, are shown in Fig. 4. In this case, the symbol error ratio is minimum, when the ratio of the radiuses of the outer and inner circles $\gamma_1 = 2.5$. As the impact of the initial phase shifting of the signal points located on the outer circle of the APSK constellation is ignorable small, it is accepted that $\varphi_2 = 0$.

Analogous studies are conducted for the rest of the 16APSK constellations and the derived results are given in Table III. It is evident that the values of the parameter γ_1 are very close to the ones derived in the previous study.

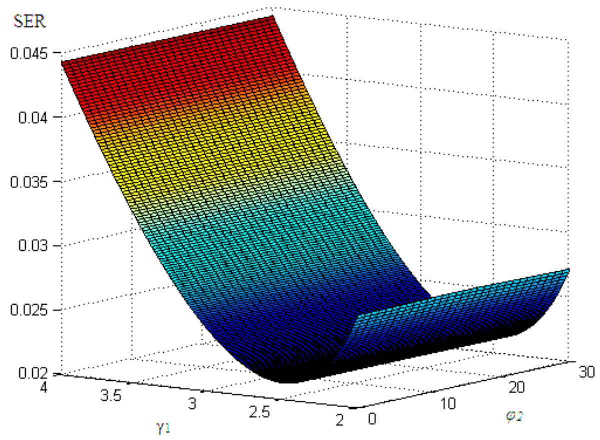

Fig.4. Dependencies of the symbol error rate for 4-12APSK on the parameters γ_1 and φ_2

TABLE III
PARAMETERS OF THE OPTIMAL 16APSK CONSTELLATIONS

Constellation Number (CN)	n_1, n_2	γ_1	φ_1, deg	φ_2, deg
CN 1	3, 13	2.86	60	23
CN 2	4, 12	2.5	45	0
CN 3	5, 11	2.25	36	6
CN 4	6, 10	2.07	30	0

In Fig. 5 are shown the dependencies of SER on the E_s/N_0 where the optimized parameters of the 16APSK constellations studied are used. The values of the parameter E_s/N_0, for which the symbol error rate is 10^{-1}, 10^{-2} and 10^{-3}, are given in Table IV.

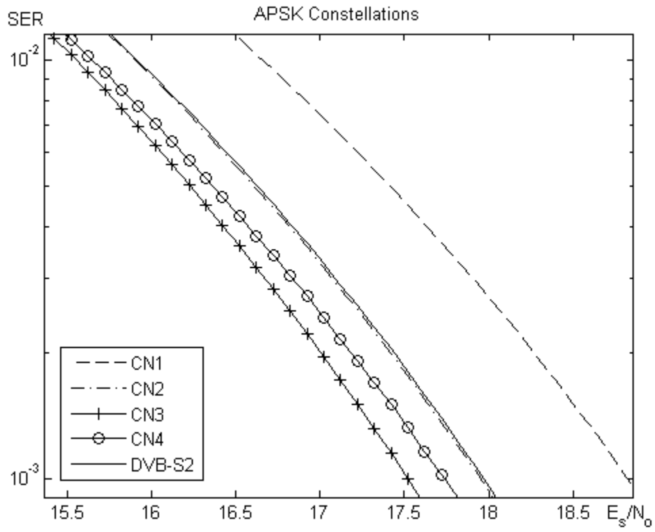

Fig. 5. SER characteristics of 16APSK channels

In this case, too, the highest noise immunity of the radio channel is achieved when the third APSK constellation is used with parameters $n_1 = 5$, $n_2 = 11$, $\gamma_1 = 2.25$, $\varphi_1 = 36^o$ and $\varphi_2 = 6^o$. In comparison with the standard for the DVB-S2 constellation, the benefit achieved is approximately 0.4 dB.

TABLE IV
CALCULATED VALUES OF THE PARAMETER E_S/N_0, dB

SER	10^{-1}	10^{-2}	10^{-3}
CN 1	12.978	16.656	18.826
CN 2	12.532	15.899	17.950
CN 3	12.328	15.546	17.521
CN 4	12.329	15.642	17.735
DVB-S2	12.528	15.913	17.969

V. CONCLUSION

The algorithms presented in this paper allow the optimization of the amplitude and phase states of the M-APSK signal vector, so as to achieve maximum noise immunity of the radio channel. The results of the studies conducted show that the parameters of the APSK constellations which are derived by both algorithms, are very close. For the example used in this paper, which is related to 16APSK constellations, by the optimization carried out with the first algorithm are provided values for the parameter E_s/N_0, which are from 0,36 dB to 0,46 dB smaller than those allowed in the DVB-S2 standard.

ACKNOWLEDGEMENT

The research described in this paper is supported by the Bulgarian National Science Fund under the contract No DDVU 02/74/2010.

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