Outage Performance of Dual-hop AF Relaying System in Weibull-gamma Fading Environment

Jelena A. Anastasov, Aleksandra M. Cvetković, Daniela M. Milović and Dejan N. Milić

Abstract – **In this paper, brief outage performance analysis of dual-hop relaying system over composite Weibull-gamma fading channels is presented. Upper bound SNR approximation is used for analytical and numerical evaluations of outage probability. New lower bound for outage performance for proposed system with variable gain amplify-and-forward (AF) relay is derived. Appropriate comparison of the approximation and the exact results is obtained. Effects of fading and shadowing phenomena, outage threshold and average signal-to-noise ratio per hop are explained by numerical analysis.**

Keywords – **CSI - assisted AF relays, dual-hop system, fading channel, outage probability, shadowing**

I. INTRODUCTION

Implementation of relaying technology can extend the cell coverage and reduce no signal area. On the other hand, relay networks redirect high intensity traffic in order to establish a balanced network load [1]. In wireless systems, utilization of relays is one of the ways to improve the system performance, by reducing the multi-path fading and shadowing effects [1-3].

The effect of fading caused by multipath propagation is often modelled by Rayleigh, Ricean, Nakagami-*m* and Weibull distributions [4]. When propagation channel is affected only by fading (short distances), due to reflection, diffraction and scattering, the average signal power is assumed to have constant value. At the large distance, the obstructions between transmitter and receiver diminish signal power which often results in the variation of the average power, i.e. shadowing effects [4]. The random fluctuations of average signal power are often modelled by the lognormal distribution [5]. Composite fading models based on the lognormal shadowing distribution are very inconvenient for analytical manipulations because of unavailable closed-form solution of those problems [6], [7]. Therefore, a gamma distribution may be used as a good approximation for the lognormal distribution while a K (i.e. Rayleigh/gamma) and generalized-K (GK) (i.e. Nakagami-m/gamma) distributions [6], [7] are mathematically more tractable and convenient for performance evaluations. On the other hand, Weibull fading model gives an excellent fit to measured data of multipath fading in urban propagation environment. Also, Weibull fading model unifies Rayleigh fading model and under required circumstances describes channel with deeper fading conditions [8].

Based on achievable performance gain, relaying systems can be classified as: decode-and-forward (DF) and amplify-and-forward (AF) relays [2]. DF relays process digital signals and require more complex and more expensive practical realization compared to AF relays. AF relays process analog signals by amplifying transmitted signal and retransmiting it to the destination. Furthermore, channel state information (CSI) − assisted AF relays (or variable gain AF relay) and relays with fixed-gain are two subcathegories of AF relays [2], [3].

Analysis of dual-hop system with CSI-assisted AF relays over channels corrupted by fading and shadowing phenomena, simultaneously, was presented in [9]-[11]. In [9], considering upper-bound signal-to-noise ratio (SNR) approximation some performance evaluations over GK fading channels were performed. Also, system with variable AF dual-hops with Ricean fading and shadowing was analyzed in [10]. The exact and bounded expressions of outage and bit error performance were presented in this paper. In [11], the noise- and interference-limited AF relaying systems were discussed over composite extended generalized-K fading links. In this paper, we analyse outage performance of dual-hop AF relaying systems in Weibull-gamma fading environment. This composite fading/shadowing model describes a variety of real channel conditions unlike other fading models do. To the best of authors' knowledge, AF relaying systems performances in given fading environment were not analysed in the literature.

The main focus in this paper is on the analysis of CSI-assisted AF relaying systems as low-cost and simpler to implement for practical realization. We derive analytical result for evaluating lower-bounded outage probability. Based on this approximation, numerical results were obtained and simulation validity was confirmed. The derived analytical expression is compared to the exact result and appropriate conclusions are given.

II. PERFORMANCE EVALUATION

In a typical dual-hop wireless communication system, the source, *S*, and destination terminals, *D*, communicate over the CSI-assisted AF relay. In that case, signal at the relay terminal, *R*, can be presented as

$$r_R(t) = \alpha_1 r_S(t) + n_R(t), \qquad (1)$$

where α_1 is the fading envelope over the *S*-*R* channel, $r_S(t)$ is the desired signal and $n_R(t)$ is the additive white Gaussian noise (AWGN) at the relay. The signal $r_R(t)$ is amplified and

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re-transmitted to the destination. The received signal at the destination has the following form

$$r_D(t) = \alpha_2 G(\alpha_1 r_S(t) + n_R(t)) + n_D(t), \qquad (2)$$

where α_2 is the fading envelope over the *R-D* channel. The AWGN at the destination is denoted by n_D. The average power of this AWGN is denoted by N_2. The gain of the CSI-assisted relay is set to [2]

$$G^2 = \frac{1}{\alpha_1^2 + N_1}, \qquad (3)$$

where E[$|n_R(t)|^2$]=N_1, (E[.] denotes expectation).

The overall SNR at the receiving side can be expressed as [3]

$$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}, \qquad (4)$$

where $\gamma_i = \alpha_i^2 / N_i$, *i*=1, 2, are the SNRs at the first and the second hop, respectively. Exact result for outage probability of the observed system, over Weibull-gamma channels, is defined as probability that instantaneous overall SNR falls below a predetermined threshold. The exact expression can not be obtained in a closed form and it is not easy for mathematical computation. Therefore, the upper bound, γ_{bound}, of end-to-end SNR, γ_{eq}, is introduced as [9], [11]

$$\gamma_{eq} \le \gamma_{bound} = \min(\gamma_1, \gamma_2). \qquad (5)$$

As a matter of fact, this approximation chooses the hop with the weakest SNR to determine system performance. This bound used for performance evaluations in many papers e.g. [9], [10], is shown to be sufficiently accurate at mid-high average SNR range. The cumulative distribution function (cdf) of independent fading/shadowing channels, for the proposed approximation, has the form [9]

$$P_b = F(\gamma_{th}) = 1 - \prod_{i=1}^{2} \left(1 - F_{\gamma_i}(\gamma_{th})\right), \qquad (6)$$

where $F_{\gamma_i}(\gamma_{th})$, *i*=1, 2, is the cdf of SNR at the first and the second hop, respectively.

We assume that both hops experience Weibull-gamma fading. When the desired signal envelope, *R*, is Weibull distributed, the corresponding probability density function (pdf) is given by [12]

$$p_{R/y}(R/y) = \beta \left(\frac{\Gamma(1 + 2/\beta)}{y} \right)^{\beta/2} R^{\beta-1} \exp\left(-\left(R^2 \Gamma(1 + 2/\beta)/y\right)^{\beta/2} \right), \quad R \ge 0 \qquad (7)$$

where β is the multipath fading severity parameter, $\Gamma(.)$ denotes the Gamma function [13, eq. (8.3107/1))], and $y = \mathrm{E}(R^2)$ is the average fading power. As mentioned before, when shadowing effects are present in the propagation channel, *y* is also random process. For the proposed scenario *y* is described by the gamma pdf as [12]

$$p_y(y) = \frac{y^{\alpha-1} \exp(-y/\Omega)}{\Gamma(\alpha)\Omega^{\alpha}}, \quad y \ge 0, \qquad (8)$$

where α is the shadowing severity parameter and $\Omega = \mathrm{E}(y^2)$. In the propagation channel where fading and shadowing occur simultaneously, the average PDF of the WG random variable can be determined as

$$p_R(R) = \int_0^{\infty} p_{R/y}(R/y) p_y(y) dy. \qquad (9)$$

After substituting (7) and (8) in (9), and representing the exponential functions in terms of the Meijer's G functions according to [14, eq. (01.03.26.0004.01)] and [13, eq. (9.312)], the resulting integral has the following form

$$p_R(R) = \int_0^{\infty} \frac{\beta \Gamma(1 + 2/\beta)^{\beta/2} R^{\beta-1}}{\Gamma(\alpha)\Omega^{\beta/2}} y^{\alpha-1-\beta/2} \times H_{0,1}^{1,0}\left(\frac{y}{\Omega} \middle| \begin{matrix} - \\ (0,1) \end{matrix} \right) H_{0,1}^{1,0}\left(\left(\frac{R^2 \Gamma(1 + 2/\beta)}{y} \right)^{\beta/2} \middle| \begin{matrix} - \\ (0,1) \end{matrix} \right) dy, \qquad (10)$$

with $H_{p,q}^{m,n}\left(z \middle| \begin{matrix} (a_i, A_i)_{1,p} \\ (b_j, B_j)_{1,q} \end{matrix} \right)$ denoting the Fox's *H* function, where *p*, *q*, *m*, *n* are integers such that $0 \le m \le q$, $0 \le n \le p$; $a_i, b_j \in$ C, where C is the set of complex numbers, and $A_i, B_j \in \mathrm{R}^+ = (0,\infty)$, ($i = 1,..., p$; $j = 1,..., q$) [4].

Now, by applying [14, eq. (07.34.21.0012.01)] and [4, eqs (2.1.3), (2.1.4)], after some mathematical manipulations, the pdf of the desired signal envelope is derived as [16, eq. (4)]

$$p_R(R) = \frac{R^{\beta-1}}{\Gamma(\alpha)\Omega^{\beta/2}} \left(\Gamma\left(1 + \frac{2}{\beta}\right) \right)^{\beta/2} H_{0,2}^{2,0}\left(\left(\frac{R^2 \Gamma(1 + 2/\beta)}{\Omega} \right)^{1/2} \middle| \begin{matrix} - \\ (0, 1/\beta), (\alpha - \beta/2, 1/2) \end{matrix} \right). \qquad (11)$$

Since the instantaneous SNR per symbol of *i*th receiving branch is $\gamma_i = R_i^2 E_s / N_0$, the corresponding average input SNR will be $\bar{\gamma}_i = \Omega_i E_s / \alpha N_0$. Based on this, we derive the pdf of the instantaneous SNRs in the following form

$$p_{\gamma_i}(\gamma) = \frac{\gamma^{\beta_i/2-1}}{2\Gamma(\alpha_i)\bar{\gamma}_i^{\beta_i/2}} \left(\alpha_i \Gamma\left(1 + \frac{2}{\beta_i}\right) \right)^{\beta_i/2} H_{0,2}^{2,0}\left(\left(\frac{\gamma \Gamma(1 + 2/\beta_i)}{\bar{\gamma}_i} \right)^{1/2} \middle| \begin{matrix} - \\ (0, 1/\beta_i), (\alpha_i - \beta_i/2, 1/2) \end{matrix} \right). \qquad (12)$$

The cdf of SNRs can be readily evaluated as $F_{\gamma_i}(\gamma)=\int_0^{\gamma} p_{\gamma_i}(u)du$ using [15, eqs. (2.8.17), (2.1.9)] which results into

$$F_{\gamma_i}(\gamma)=\frac{\gamma^{\beta_i/2}}{2\Gamma(\alpha_i)\bar{\gamma}_i^{\beta_i/2}}\left(\alpha_i\Gamma\left(1+\frac{2}{\beta_i}\right)\right)^{\beta_i/2} H_{1,3}^{2,1}\left(\left(\frac{\gamma\alpha_i\Gamma(1+2/\beta_i)}{\bar{\gamma}_i}\right)^{1/2}\middle|\begin{matrix}(1-1/\beta_i,1/2)\\(0,1/\beta_i),(\alpha_i-\beta_i/2,1/2),(-\beta_i/2,1/2)\end{matrix}\right) \quad (13)$$

Substituting (13) in (6) the outage probability of AF dual-hop system can be approximately obtained.

III. Numerical Results

The following numerical results based on the analytical approximation are evaluated in *Mathematica* software package, considering the relation between Fox's *H* function and Meijer's *G* function, given in [17, p. 531, eq. (22)]. Simulation results are obtained in *Matlab* software package, and these results correspond to the exact outage probability results.

The outage performance versus average SNR values($\bar{\gamma}_1=\bar{\gamma}_2=\bar{\gamma}$) is presented in Fig. 1 for different Weibull fading parameters. Presented curves are obtained based on derived approximate expression (6) and compared with the simulations based on exact approach. Very good agreement of approximation and simulation results can be noticed at middle and high SNR values. When Weibull fading parameter increases, the channel fading conditions are better and the outage probability decreases, as expected. Also, it is noticable that approximation and simulation results match better at low SNR values over channel with deeper fading.

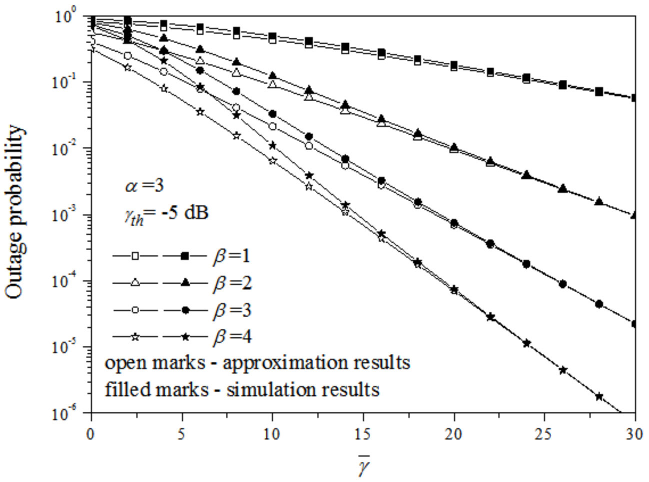

Fig. 1. Outage probability versus average SNR per both hops for different values of fading parameter

A mismatch of the simulated and approximation outage probability results for different values of average SNR can be noticed in Fig. 2. When average SNR is 10dB, the relative error between simulation value of outage probability and its approximation when β=2.8 is 34% while the the relative error when β=0.8 (deeper fading) is 8%,. This example confirms conclusion mentioned in description of the previous figure, namely, better agreement between approximation and simulation results are expected in channels with deeper fading. Also, it is noticable that considered mismatch is smaller for higher average SNR values. For example, when average SNR is 20 dB the relative error between simulations and analytical results for β=0.8 and β=2.8 is the same and equals 7%.

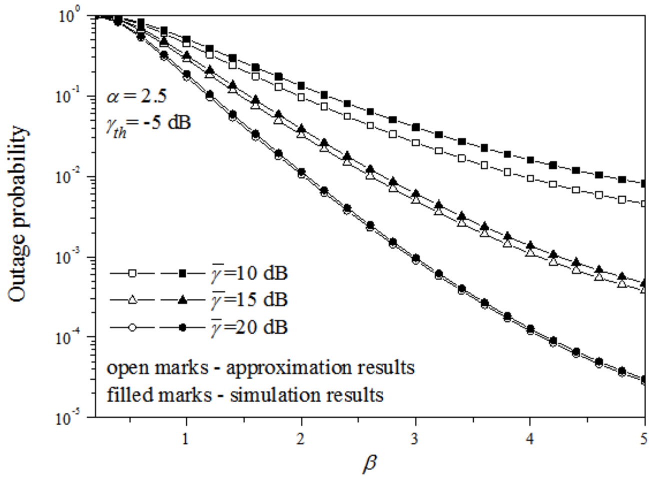

Fig. 2. Outage probability versus fading parameter for different average values of SNR

In Fig. 3. the lower bound outage probability in function of the average SNR for various values of fading and shadowing is obtained. As expected, the outage probability decreases with the shadowing parameter α increasing (lighter shadowing environment). Also, it can be seen that the effect of shadowing parameter, α, on outage probability is weaker in environment with deeper fading (for lower values of β). For example, when β=2.5 difference between lower bounded values of outage probability for α=1.8 and α =4.5 is 0.027, and when β=0.8 this difference in outage performance for the same values of parameter α is smaller and equals 0.00265.

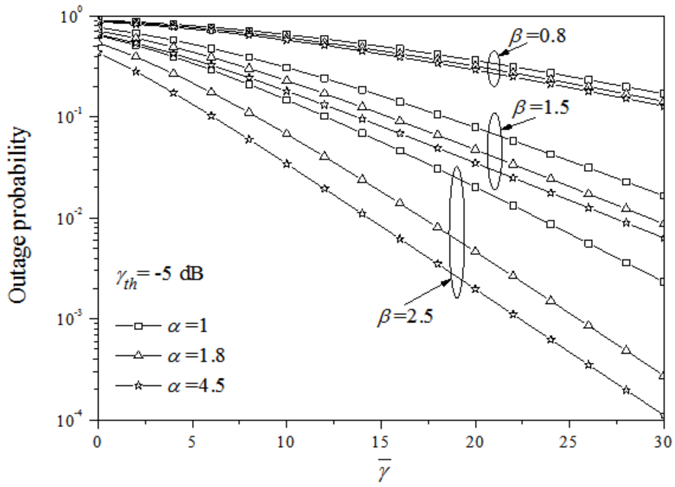

Fig. 3. Outage probability versus average SNR per both hops for different values of shadowing and fading parameters

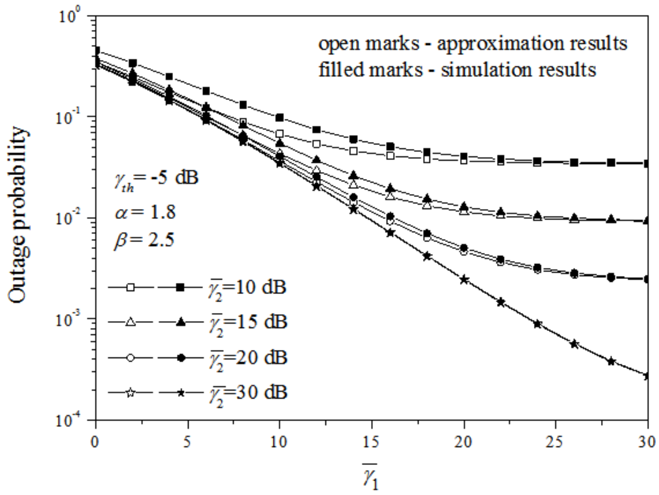

Fig. 4. Outage probability versus first hop average SNR for different values of second hop average SNR

Fig. 4. shows the outage probability as the function of average SNR on the first hop versus the average SNR on the second hop. Beside the numerical results based on the analytical approach, simulation results are also presented in this figure. We can notice that for high values of average SNR at the first hop, values of outage probability tend to irreducible outage floor. We can also see that outage floor depends on average second hop SNR value. For smaller values of average SNR at the second hop, the outage floor appears at lower values of average SNR at the second hop. The outage floor can be also numerically evaluated considering derived expression (6).

IV. Conclusion

Outage performance analysis of CSI-assisted AF dual-hop relaying system based on derived analytical result was realized in this paper. The propagation environment was described as Weibull-gamma fading environment. Influences of fading and shadowing conditions over propagation channels on outage probability were discussed. The obtained results showed good agreement of analytical approximation method and simulations at mid-high SNR values. Also, better agreement of approximation and simulation results is noticed with fading/shadowing parameters decreasing, namely for worst channel conditions.

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