Performance Analysis of SIM-FSO System over Gamma-Gamma Atmospheric Channel

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Abstract – **This paper presents the performance analysis of free space optical (FSO) system with subcarrier intensity modulation (SIM) employing multiple binary modulation techniques. The FSO channel experiences atmospheric turbulence modelled by the Gamma-Gamma distribution. Expressions for outage probability and bit error rate (BER) are theoretically derived and numerical results are presented and validated by Monte Carlo simulations. Furthermore, improvement of the BER performance is accomplished by using convolutional codes (CC), which is established by Monte Carlo simulations. The effects of propagation distance, turbulence strength, type of modulation and coding gain are discussed.**

Keywords – **Atmospheric turbulence, bit error rate, convolutional codes, Free space optical, Gamma-Gamma distribution, outage probability.**

I. INTRODUCTION

Optical communication systems are gaining in importance since a license-free signal transmission is provided in contrast to traditional radio frequency systems. Since the implementation of fibre-optics have proved to be complex and quite expensive, free-space optical (FSO) systems have found its purpose as a simple and low-cost, non-interfering and high speed technology. Due to its advantages, FSO represents an appropriate solution for the last-mile problem. However, optical signal transmission through free space suffers from atmospheric turbulence, which occurs as a result of the refractive index variations due to the random changes in atmospheric temperature, altitude and pressure. Although many statistical models have been proposed in order to determine the turbulence strength, the Gamma-Gamma distribution has been approved as the most convenient because it provides excellent matching between theoretical and experimental results [1 - 3].

In order to be more cost effective, commercial FSO systems usually apply the intensity-modulation/direct detection (IM/DD) with the on-off keying (OOK) scheme [2]. Still, the main flaw of OOK is the requirement for adaptive threshold setting for demodulation. For that reason, the subcarrier intensity modulation (SIM) technique has been suggested as alternative for further improvement of FSO system. When SIM is applied at the transmitting part, an electrical subcarrier information signal is first premodulated by one of already known type of RF modulator. Next, the output of modulator is used to modulate the intensity of the laser source, which forwards the signal via free space. At the reception, an optical signal is detected by DD and converted to an electrical by photodetector. The final data stream is obtained by signal demodulation by corresponding RF demodulator [2]. Paper [4] analyses the BER of optical communication systems with either OOK or SIM phase-shift keying (PSK) over log-normal turbulence channel convenient in weak turbulence conditions. Also, this paper includes convolutional codes (CC) to improve system performance. The BER performance of FSO system employing binary PSK (BPSK) over gamma-gamma and negative exponential channels is analysed in [5]. The BER and outage probability analysis of SIM FSO system employing differential phase shift keying (DPSK) in negative exponential atmospheric turbulence environment is done in [6], while [7] derived the error rate expressions using a series expansion approach of SIM FSO systems with PSK, DPSK, and non-coherent frequency-shift keying (NFSK).

In this paper, we derive novel closed-form outage probability and BER expressions in terms of Meijer G-functions of FSO system employing SIM with various modulations. Atmospheric turbulence is modelled by the Gamma-Gamma distribution. The obtained analytical results are validated by Monte Carlo simulations. Furthermore, CC are used in addition to further improve system performance [8]. The results are achieved by Monte Carlo simulations.

The rest of the paper is organized as follows. The system and channel model are described in Section II, while outage probability analysis is given in Section III. BER expressions for different types of modulations are derived in IV. Numerical results and discussions are presented in V with concluding remarks given in VI.

II. SYSTEM AND CHANNEL MODEL

The SIM FSO system with CC is presented in Fig 1. First, the information bits are encoded by CC with the code rate of C_R=0.5, and modulated by electrical modulator whose output is denoted by $s(t)$. It is next forwarded to the optical intensity modulator. The optical signal is fed to the transmitting telescope, which determines direction and the size of the optical beam. Since SIM scheme is applied, the intensity of optical signal is dependent on $s(t)$, which is DC-level shifted in order to ensure the non-negativity requirement. Therefore, the intensity of the transmitted optical signal is

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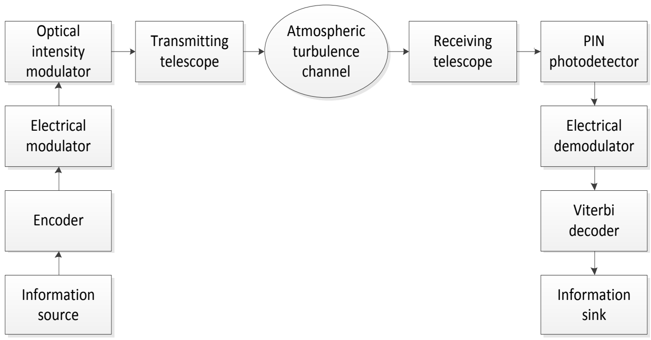

Fig. 1. Block diagram of SIM FSO system with CC through atmospheric turbulence channel

$$I_t(t) = P_t\left(1 + ms(t)\right), \quad (1)$$

where P_t represents the average transmitted optical power and m is the modulation index ($0<m<1$), that provides the laser operates without over-modulation induced clipping.

Further, the optical signal is transmitted over FSO channel under the influence of Gamma-Gamma atmospheric turbulence. After direct detection at the destination, the optical signal is converted to the electrical one by PIN photodetector. Next, the DC bias is filtered out and the signal is demodulated by the corresponding electrical demodulator and decoded by maximum likelihood decoding using the Viterbi algorithm. Finally, data stream is given by:

$$r(t) = \eta R m P_t I_{GG} s(t) + n(t) \quad (2)$$

where R represents detector responsivity, η denotes optical-to-electrical conversion coefficient, $n(t)$ is the additive white Gaussian noise with zero mean and variance σ_n^2 and I_{GG} is the maximal attenuation due to Gamma-Gamma atmospheric turbulence.

The instantaneous electrical signal-to-noise ratio (SNR) can be defined as

$$\gamma = \frac{(\eta R m P_t I_{GG})^2}{\sigma_n^2} \quad (3)$$

while the average electrical SNR is defined as

$$\mu = \frac{(\eta R m P_t E[I_{GG}])^2}{\sigma_n^2} = \frac{(\eta R m P_t)^2}{\sigma_n^2} \quad (4)$$

where E[.] denotes the statistical expectation. Note that $E[I]=1$ since I is normalized.

The channel is under the influence of the atmospheric turbulence modelled by the Gamma-Gamma distribution, which is convenient in wide range of turbulence conditions. Hence, the probability density function (PDF) of I_{GG} is defined as [1-3]

$$p_{I_{GG}}(I_{GG}) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I_{GG}^{(\alpha+\beta)/2-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I_{GG}}\right) \quad (5)$$

where $\Gamma(.)$ is the gamma function [9, (8.310.1)] and $K_\nu(.)$ is the νth-order modified Bessel function of the second kind [9, (8.432)]. The parameters α and β are the parameters related to the atmospheric conditions. The plane wave propagation and zero inner scale are assumed, so the parameters α and β are defined as [3, (3)]

$$\alpha = \left(\exp\left[0.49\sigma_R^2 \Big/ \left(1+1.11\sigma_R^{12/5}\right)^{7/6}\right] - 1\right)^{-1}$$

$$\beta = \left(\exp\left[0.51\sigma_R^2 \Big/ \left(1+0.69\sigma_R^{12/5}\right)^{5/6}\right] - 1\right)^{-1} \quad (6)$$

The Rytov variance σ_R^2 is given by [3, (1)]

$$\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6} \quad (7)$$

where $k=2\pi/\lambda$ is the wave-number, λ is the wavelength, L is the propagation distance, and C_n^2 denotes the index of refraction structure parameter, here used as the metric of turbulence strength.

Now, using simple power transformation of the random variables, the PDF of γ is found as

$$p_\gamma(\gamma) = \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)\mu^{(\alpha+\beta)/4}} \gamma^{(\alpha+\beta)/4-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{\gamma}{\mu}}}\right) \quad (8)$$

III. OUTAGE PROBABILITY

The outage probability represents the probability that the instantaneous SNR falls below a predetermined outage threshold γ_{th}. For the considered FSO system, outage probability is found as

$$P_{out}(\gamma_{th}) = \Pr\left[\gamma < \gamma_{th}\right] = \int_0^{\gamma_{th}} p_\gamma(t)dt$$

$$= \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)\mu^{(\alpha+\beta)/4}} \int_0^{\gamma_{th}} t^{(\alpha+\beta)/4-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{t}{\mu}}}\right)dt \quad (9)$$

where Pr[.] denotes probability. This integral is solved by representing modified Bessel function of the second kind in terms of Meijer G-function [9, (9.301)] using [10, (03.04.26.0009.01)], and afterwards using [10, (07.34.21.0084.01)] and [10, (07.34.16.0001.01)]. The outage probability expression for the system under the investigation is derived as

$$P_{out}(\gamma_{th}) = \frac{2^{\alpha+\beta-2}}{\pi\Gamma(\alpha)\Gamma(\beta)} G_{1,5}^{4,1}\left(\frac{\alpha^2\beta^2\gamma_{th}}{16\mu}\middle| \begin{matrix}1\\ \kappa\end{matrix}\right) \quad (10)$$

where $\kappa = \alpha/2, \quad (\alpha+1)/2, \quad \beta/2, \quad (\beta+1)/2, \quad 0$.

IV. BER ANALYSIS

This section analyses the BER performances of FSO system employing SIM with various types of modulations.

A. SIM-BPSK

The BER expression of SIM-BPSK FSO system can be derived using well-known method by averaging over the instantaneous electrical SNR [11]:

$$P_{b|\gamma} = \frac{1}{2}\int_0^\infty Q\left(\sqrt{2\gamma}\right) f_\gamma(\gamma) d\gamma \qquad (11)$$

where $Q(.)$ is the Gaussian Q-function defined as $Q(x) = \left(1/\sqrt{2\pi}\right)\int_0^\infty \exp\left(-t^2/2\right)dt$. Also, it is related to the complementary error function erfc(.) [9, (8.250.4)] by $\mathrm{erfc}(x) = 2Q\left(\sqrt{2}x\right)$.

The integral in (11) is represented in terms of Meijer G-function using [10, (06.27.26.0006.01)] and [10, (03.04.26.0009.01)], respectively, as

$$P_b = \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{4\sqrt{\pi}\Gamma(\alpha)\Gamma(\beta)\mu^{(\alpha+\beta)/4}}\int_0^\infty x^{(\alpha+\beta)/4-1} G_{1,2}^{2,0}\left(x \middle| \begin{matrix} 1 \\ 0, \quad 1/2 \end{matrix}\right)$$

$$\times G_{0,2}^{2,0}\left(\alpha\beta\mu^{-1/2}x^{1/2} \middle| \begin{matrix} - \\ (\alpha-\beta)/2, \quad (\beta-\alpha)/2 \end{matrix}\right)dx \qquad (12)$$

In order to solve the integral in (12), [10, (07.34.21.0013.01)] is applied and [10, (07.34.16.0001.01] is used for transformation of Meijer G-function. Finally, the average BER of SIM-BPSK FSO system is derived as:

$$P_b = \frac{2^{\alpha+\beta-3}}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} G_{2,5}^{4,2}\left(\frac{\alpha^2\beta^2}{16\mu} \middle| \begin{matrix} 1, \quad 1/2 \\ \kappa \end{matrix}\right) \qquad (13)$$

B. Binary modulations

Following approach done in [12], the average BER expressions can be derived for different types of binary modulations: binary frequency shift keying (BFSK), BPSK, binary DPSK (DBPSK) and binary NFSK (NBFSK). Assuming Gamma-Gamma turbulence channel, the BER expression is given by[12, (12)]

$$P_b = \frac{q^p}{2\Gamma(p)}\int_0^\infty e^{-q\gamma}\gamma^{p-1} F_\gamma(\gamma) d\gamma \qquad (14)$$

where $F_\gamma(\gamma)$ is the CDF of the instantaneous SNR given as outage probability by (10), and p and q denote the parameters given in Table I for different type of binary modulations [12]. Substituting (10) into (14), the BER expression has a form:

TABLE I

modulation	(p, q)
BFSK	(0.5, 0.5)
BPSK	(0.5, 1)
NBFSK	(1, 0.5)
DBPSK	(1, 1)

$$P_b = \frac{q^p 2^{\alpha+\beta-2}}{2\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(p)}\int_0^\infty \gamma^{p-1}e^{-q\gamma} G_{1,5}^{4,1}\left(\frac{\alpha^2\beta^2\gamma}{16\mu_2} \middle| \begin{matrix} 1 \\ \kappa \end{matrix}\right)d\gamma \qquad (15)$$

This integral is solved by representing exponential function in terms of Meijer G-function by [10, (01.03.26.0004.01)] and using [10, (07.34.21.0011.01)] as:

$$P_b = \frac{2^{\alpha+\beta-3}}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(p)} G_{2,5}^{4,2}\left(\frac{\alpha^2\beta^2}{q16\mu_2} \middle| \begin{matrix} 1, \quad 1-p \\ \kappa \end{matrix}\right) \qquad (16)$$

It can be noticed that applying corresponding parameters for BPSK modulation, expressions obtained by (16) is equal to (13).

V. NUMERICAL RESULTS

On the basis of derived outage probability and BER expressions, numerical results are presented. It is assumed that the value of the optical wavelength is 1.55 μm. The values of index refraction are determined according to turbulence strength: $C_n^2 = 6\times10^{-15}$ in weak, $C_n^2 = 2\times10^{-14}$ in moderate and $C_n^2 = 5\times10^{-14}$ in strong turbulence conditions [13].

Fig. 2 shows the outage probability dependence on average SNR for different propagation distances. With longer distance between transmitter and receiver, the optical signal suffers more damage, which will be reflected in worse system performance. As it is expected, lower values of outage threshold lead to the better outage performance.

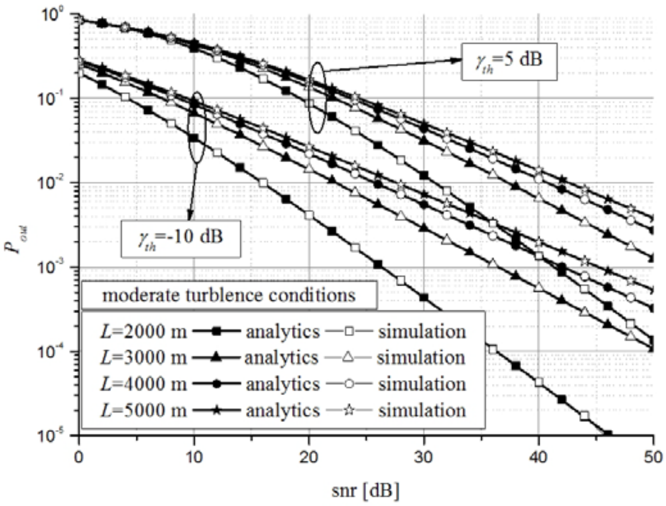

Fig. 2. Outage probability dependence on the average SNR for different values of propagation distance and outage threshold

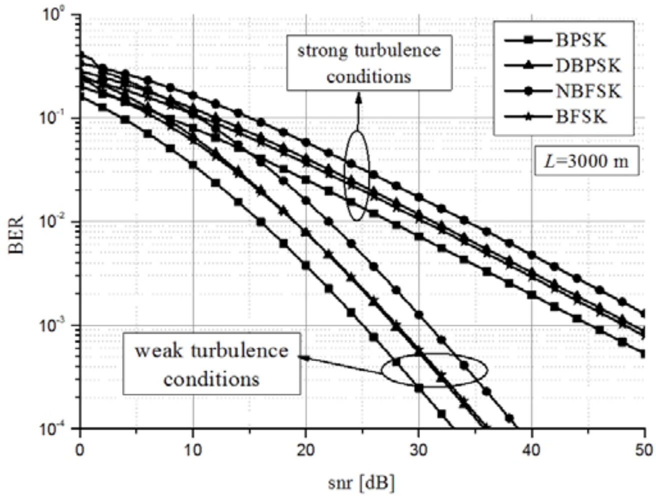

Fig. 3. BER dependence on the average SNR for different types of binary modulations in various turbulence conditions

The BER dependence on average SNR in weak and strong turbulence conditions is presented in Fig. 3. The SIM FSO system employs different binary modulation techniques: BFSK, BPSK, DBPSK and NBFSK. The system has best performance when SIM-BPSK is applied, and worst with SIM-NBFSK. The DBPSK and NBFSK show similar BER performance, especially in weak turbulence conditions. Also, BER performance is better in weak conditions, which corresponds to clear air terms.

Fig. 4 shows the BER dependence on average SNR of FSO system with SIM-BPSK and SIM-DPSK. Further, CC are applied and the improvement of system performance is noticed. The coding gain is most noticeable when CC (2,1,7) is used both modulation schemes, especially for SIM-DPSK.

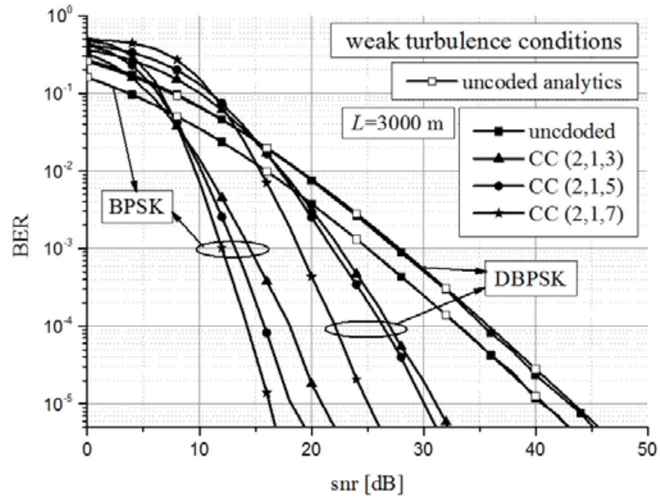

Fig. 4. BER dependence on the average SNR with CC

VI. Conclusion

In this paper the new closed-form outage probability and BER expressions of FSO system employing SIM with different types of modulations have been derived. The FSO channel is under the influence of atmospheric turbulence, which is modelled by Gamma-Gamma distribution. Numerical results are confirmed by Monte Carlo simulations. The effects of propagation distance, turbulence strength and type of modulation are presented and discussed. In addition, the CC is employed as the improvement of the BER performance, which is demonstrated by Monte Carlo simulations.

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