Analysis of the SC Macrodiversity Reception in the Presence of Gamma Shadowed Nakagami-m Fading

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Abstract – **In this paper an analysis of selection combining (SC) macrodiversity system reception in correlated Gamma shadowing environment will be presented. The system consists of three maximal ratio combining (MRC) microdiversity combiners with an arbitrary number of input branches. The signal envelope is exposed to a short-term fading, modeled by Nakagami-m distribution. Closed form results are obtained for the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) at the reception. Outage probability (OP) of the macrodiversity system will be observed, in order to determine the influence of different parameters such as correlation level and short-term fading severity on reception performance as well as the optimum number of branches.**

Keywords – **Macrodiversity, Shadowed fading channel, Correlation, Probability density function, Cumulative density function.**

I. Introduction

The performance of a wireless communication system mainly depends on the wireless environment. The wireless channel is unpredictable, and therefore an analysis of wireless systems is difficult. However, there is a simple analysis of the wireless channel using a phenomenon known as fading. In wireless communication, fading is deviation of the signal amplitude and it can vary with time or radio frequency. Furthermore, fading can also be a consequence of the multipath signal propagation and due to its nature is described as a random process.

The total fading in the channel is a complex combination of a short and long-term fading [1]. The long-term fading refers to a slower variation of the mean signal value. This type of fading occurs in mobile terrestrial and satellite communication systems and it is called shadowing, too. The long-term fading occurs due to the specific propagation environment (high-rise buildings, vegetation...). Since the location, size and dielectric properties of objects which are located in the signal path, as well as changes in reflecting surfaces and objects that scatter and attenuate signal are unknown and random facts, statistical models are used for fading describing. The most common models found in literature for describing this type of fading are lognormal and gamma distribution [1]. By the short-term fading it is considered short signal amplitude variations due to multipath propagation of the signal which occurs due to reflection, diffraction and scattering from objects in the environment [2], [3]. This type of fading can be described using many distributions such as Rician and Rayligh, and one of the mostly used is also Nakagami-m distribution. This distribution has a wide range of applicability and it is especially suitable for channel modeling in mobile terrestrial wireless systems.

Wireless communication system may be strongly degraded by the influence of fading. As a result, many techniques were developed and one of the most common that can improve system’s performance is diversity technique. This technique does not need the increase of transmission power or additional bandwidth employment [4]. The most effective way to improve the reliability of the transmission is a technique called spatial diversity. This technique mitigates the effect of short-term fading, and thus can be applied to a microdiversity system implemented as series of separated antennas at the reception.

When the diversity system is applied to a single base station on the small terminals with multiple antennas, due to a lack of space between the antennas, there is a correlation between the signals received by input branches of microcombiner. The correlation at the macro level is a common phenomenon that has proved to be significant in many wireless communication systems and its influence will be discussed in this paper, while the correlation at micro level will be considered to be negligible. Channels at micro level are not correlated if the distance between antennas is $\lambda/2$ (for example, if the frequency is 900 MHz, sufficient distance is 16.67cm) [5].

In a single mobile station (MS) and observed base stations at any given moment, components often exhibit a link correlation described in [6]. The level of correlation depends on several factors such as the distance between the base stations, the relief of the surrounding terrain and the angle at which it receives the signal. In cellular radio systems, the correlation of the links between the mobile station and base stations affects the coverage area and the characteristics of the interference. In this paper we analyze the influence of several channel parameters to the system performance and the optimal number of antennas at micro level.

II. System model

Considered system model consists of SC macrodiversity system composed of three geographically distributed MRC microdiversity receivers with an arbitrary number of input branches, implemented at single base stations and it is shown in Fig. 1. Moreover, we consider that the branches at macro level are subjected to the correlated gamma shadowing, while at micro level branches operate in the presence of Nakagami-m fading.

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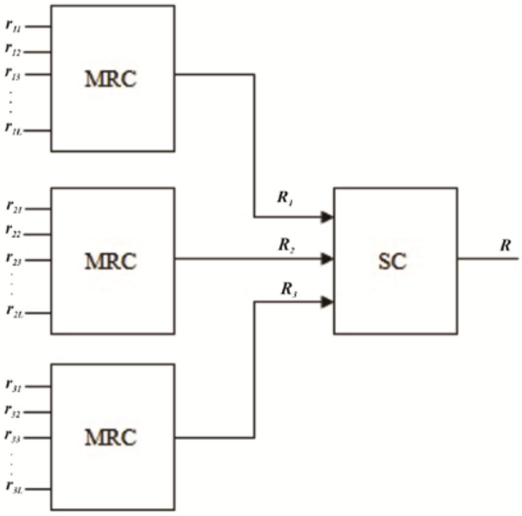

Fig. 1. System model

Signal received by the i^{th} antenna at the j^{th} base station in the presence of Nakagami-m fading can be described with the following distribution [7]:

$$p_{r_{ij}}\left(r_{ij}\right)=\frac{2m^m r_{ij}^{m-1}}{\Gamma(m)\Omega_j^m}\exp\left(-\frac{m}{\Omega_j}r_{ij}^2\right) \;,\; i=\overline{1,L},\; j=1,2,3. \quad (1)$$

In the previous equation m represents Nakagami-m fading severity parameter and it can take a value $m \geq 0.5$. With increase of parameter m, the fading severity decreases. The average signal power at the j^{th} base station is described with Ω_j while $\Gamma(\cdot)$ is Gamma function.

The signal envelope at the output of the j^{th} MRC combiner with L branches is equal to:

$$R_j=\sum_{i=1}^{L} r_{ji}^2 \quad (2)$$

This signal envelope can be described with the following conditional PDF [7]:

$$p_{R_j}\left(R_j / y_j\right)=\frac{R_j^{M-1}M^M}{\Gamma(m)y_j^M}\exp\left(-\frac{M}{y_j}R_j\right) \;,\; j=1,2,3. \quad (3)$$

Where $y_j = L\Omega_j$ is the total input power and M is the parameter that describes Nakagami-m fading influence, defined with:

$$M_j=\frac{m_j L^2}{q_j}. \quad (4)$$

Parameter q_j refers to the exponentially correlation coefficient ρ_{mj} between input channels at micro level and it is described with the following equation:

$$q_j = L+\frac{2\rho_{mj}}{1-\rho_{mj}}\left[L-\frac{1-\rho_{mj}^L}{1-\rho_{mj}}\right] \quad (5)$$

Since we consider that branches at the micro level are not correlated, $\rho_{mj}=0$, so finally $M_j = m_j L$.

The correlated shadowing environment at the macro level is usually described with the correlated Gamma distribution [8]:

$$p_{y_1 y_2 \ldots y_N}\left(y_1 y_2 \ldots y_N\right)=\frac{y_1^{\frac{c-1}{2}} y_N^{\frac{c-1}{2}}}{\rho^{\frac{c-1}{2}(N-1)}(1-\rho)^{N-1} y_0^{N+c-1}\Gamma(c)}$$

$$\times\exp\left(-\frac{y_1+y_N+(1+\rho)\sum_{i=2}^{N-1}y_i}{y_0(1-\rho)}\right)$$

$$\times\prod_{i=1}^{N-1} I_{c-1}\left(\frac{2\sqrt{\rho}}{y_0(1-\rho)}\sqrt{y_i y_{i+1}}\right), \quad N=3 \quad (6)$$

where ρ is the correlation coefficient between the branches at the macro level, c represents the order of gamma shadowing and y_i are random variables of the total input power. y_0 is related to the average power of y_1, y_2 and y_3, while $I_n(\cdot)$ is the first kind and n^{th} order modified Bessel function [9].

After signal processing at the micro and macro level, probability density function (PDF) at the macrodiversity system output is defined by [8]:

$$p_R(R)=\int_0^\infty dy_1\int_0^{y_1} dy_2\int_0^{y_1} p_{R_1}\left(R_1 / y_1\right)p_{y_1 y_2 y_3}\left(y_1 y_2 y_3\right)dy_3$$

$$+\int_0^\infty dy_2\int_0^{y_2} dy_1\int_0^{y_2} p_{R_2}\left(R_2 / y_2\right)p_{y_1 y_2 y_3}\left(y_1 y_2 y_3\right)dy_3$$

$$+\int_0^\infty dy_3\int_0^{y_3} dy_1\int_0^{y_3} p_{R_3}\left(R_3 / y_3\right)p_{y_1 y_2 y_3}\left(y_1 y_2 y_3\right)dy_3 \quad (7)$$

while the cumulative density function of the signal (CDF) is defined as [8]:

$$F_R(R)=\int_0^\infty dy_1\int_0^{y_1} dy_2\int_0^{y_1} F_{R_1}\left(R_1 / y_1\right)p_{y_1 y_2 y_3}\left(y_1 y_2 y_3\right)dy_3$$

$$+\int_0^\infty dy_2\int_0^{y_2} dy_1\int_0^{y_2} F_{R_2}\left(R_2 / y_2\right)p_{y_1 y_2 y_3}\left(y_1 y_2 y_3\right)dy_3$$

$$+\int_0^\infty dy_3\int_0^{y_3} dy_1\int_0^{y_3} F_{R_3}\left(R_3 / y_3\right)p_{y_1 y_2 y_3}\left(y_1 y_2 y_3\right)dy_3 \quad (8)$$

where is conditional cumulative density function of the MRC receiver output signal defined as:

$$F_{R_i}(R_i / y_i) = \int_0^{R_i} p_{R_i}(R_i / y_i) dR_i \quad (9)$$

Now after substituting Eq.(3) and Eq.(6) into Eq.(7), and Eq.(3), Eq.(6) and Eq.(9) into Eq.(8), taking for simplicity that system is symmetric ($\rho_i = \rho, m_i = m, c_i = c, L_i = L,\ i = \overline{1,3}$), the first order statistical measures (PDF and CDF) can be presented respectively as:

$$p_R(R) = 6 \cdot \sum_{k=0}^{+\infty}\sum_{s=0}^{+\infty}\sum_{i=0}^{+\infty}\sum_{j=0}^{+\infty} \frac{M^M \cdot R^{M-1}}{\Gamma(M)\rho^{c-1}(1-\rho)^2 y_0^{c+2}\Gamma(c)\Gamma(k+c)k!}$$

$$\times \frac{\left(2\sqrt{\rho}\right)^{2(k+s+c-1)}}{\Gamma(s+c)s!\,2^{2(k+s+c-1)}\left[y_0(1-\rho)\right]^{2(k+s+c-1)+i+j}(s+c)(s+c+1)_j}$$

$$\times \frac{(1+\rho)^i}{(k+s+c)(k+s+c+1)_i} \cdot \left[\frac{M \cdot R \cdot y_0(1-\rho)}{3+\rho}\right]^{\frac{v_1}{2}}$$

$$\times K_{v_1}\left(2\sqrt{M \cdot R \frac{3+\rho}{y_0(1-\rho)}}\right) \quad (10)$$

where is $v_1 = -M + 2k + 3c + 2s + i + j$, and:

$$F_R(R) = 6 \cdot \sum_{z=0}^{+\infty}\sum_{k=0}^{+\infty}\sum_{s=0}^{+\infty}\sum_{j=0}^{+\infty}\sum_{i=0}^{+\infty} \frac{(M \cdot R)^{z+M}}{\Gamma(M)M(M+1)}$$

$$\times \frac{\left(2\sqrt{\rho}\right)^{2(k+c+s-1)}}{\rho^{c-1}(1-\rho)^2 y_0^{c+2}\Gamma(c)\left[y_0(1-\rho)\right]^{2(k+c+s-1)+j+i}\Gamma(k+c)k!\,\Gamma(s+c)}$$

$$\times \frac{(1+\rho)^i}{s!\,2^{2(k+s+c-1)}(s+c)(s+c+1)_j(k+s+c)(k+s+c+1)_i}$$

$$\times \left(\frac{M \cdot R \cdot y_0(1-\rho)}{3+\rho}\right)^{\frac{v_2}{2}} K_{v_2}\left(2\sqrt{\frac{3+\rho}{y_0(1-\rho)} M \cdot R}\right) \quad (11)$$

where is $v_2 = -z - M + 2k + 3c + 2s + i + j$. In previous equations $K_v(z)$ is modified Bessel function of the second kind while $(a)_n$ is the Pochhammer symbol.

Outage probability (P_{out}) is defined as a probability that the achieved level of received signal is less than the threshold γ sufficient for the satisfactory reception. It can be described using cumulative density function as:

$$P_{out} = P_r(R < \gamma_{th}) = \int_0^{\lambda_{th}} p_R(t)dt = F_R(\gamma_{th}) \quad (12)$$

III. NUMERICAL RESULTS

In this section will be presented the behaviour of the first-order statistical measures at the output of described macrodiversity system. Obtained results, using the previous mathematical analysis for y_0=1 and various values of system's parameters will be graphically presented.

First of all, probability density function of the output signal is shown on Fig. 2 for the different values of shadowing severity and correlation coefficient among macro level branches.

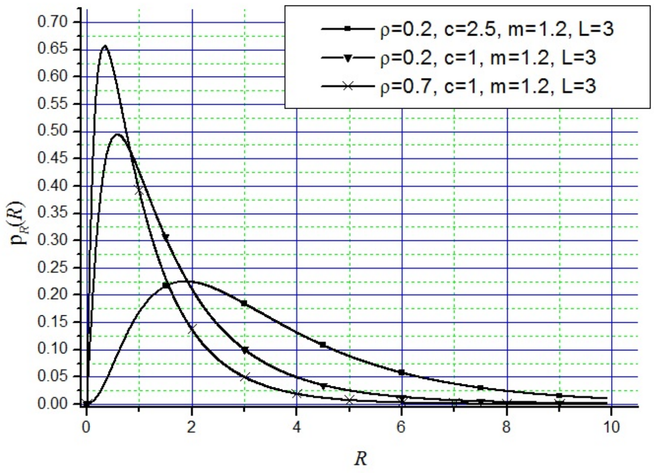

Fig. 2. Probability density function at the output of the macrodiversity system depending on the shadowing severity and correlation among the branches.

In Fig. 3 it is shown the outage probability at the output of the macrodiversity system depending on the various system parameters, including shadowing order, short-term fading severity and the correlation among branches at macro level.

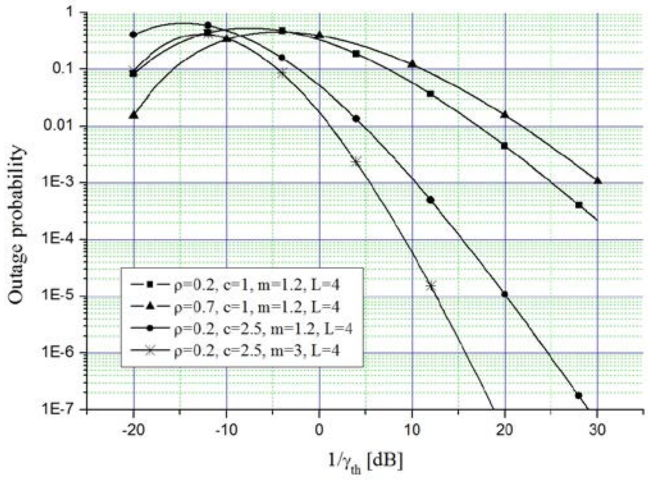

Fig. 3. Outage probability at the output of the macrodiversity system depending on the various system parameters.

It can be seen that with the increase of correlation coeficient ρ between the branches at the macro level, the outage probability is increasing, i.e. system performance is getting worse. Moreover, it can be seen that with increasing the order of gamma shadowing parameter c and with increasing the severity of Nakagami-m fading, the outage probability is getting lower, i.e. system performance is increasing.

Fig. 4 shows the outage probability for fixed high short-term fading severity and shadowing order, depending on the number of branches at micro level..

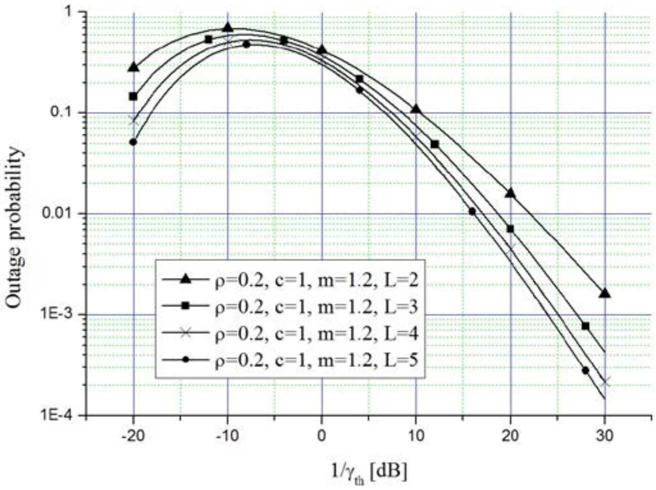

Fig. 4. Outage probability at the output of the macrodiversity system depending on the number of branches at micro level for strong fading influence.

By observing Fig. 4 it can be clearly seen that with the increase of the number of branches *L* at the micro level, the outage probability is decreasing, i.e. the system performance is getting better.

In. Fig. 5 is presented the influence of increasing the number of branches at micro level when the input signal is not subjected to a strong impact of fading at the both micro and macro level.

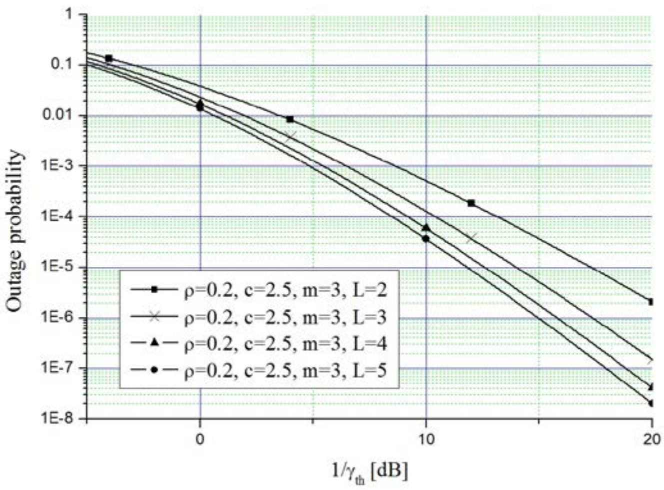

Fig. 5. Outage probability at the output of the macrodiversity system depending on the number of branches at micro level for low fading influence.

From Fig. 5 it is evident that as the number of input branches at the micro level increases, a significant decrease of the outage probability is provided. In the case when the input microcombiner has 2, 3, 4 or 5 antennas, the outage probability for $1/\gamma_{th}$=15 dB is 3.7E-5, 5.2E-6, 1.8E-6, 9.8E-7, respectively. If we fix the outage probability at 10^{-5}, it can be seen that required normalized threshold for achieving this probability for 2, 3, 4 and 5 antennas at the micro level is 17.30 dB, 13.97 dB, 12.58 dB and 11.78 dB, respectively. As it was expected, with increasing the number of antennas, relative gain is being reduced. Consequently, with increasing the number of antennas at micro level from 4 to 5 obtained relative gain is less than 1dB so it is not suitable for typical application.

IV. Conclusion

In this paper, we discussed performance of the SC macrodiversity receiving system in the presence of Nakagami-m short-term fading at the micro level. The system is exposed to the correlated gamma shadowing environment at the macro level. System performance was discussed using probability density function and outage probability. Based on the rapidly converging infinite-series expressions obtained for probability density function and cumulative density function, numerical results were presented graphically. It is shown that with the increase of number of branches at micro level the outage probability of whole macrodiversity system decreases which means that the output accuracy increases. On the other hand, due to simplicity it is preferred the small number of branches. Therefore, as results of compromise between the complexity and the quality, we suggest that *L* takes value up to 4, depending on the long-term shadowing severity. Further increasing the number of branches at micro level would not provide the same increase of quality so it is not suitable for typical application.

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