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Probability of Collision in a Cooperative Relay Diversity Scheme in Nakagami Fading Channel

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Abstract – **In this paper we consider the probability of collision of a cooperative diversity scheme with *M* relays in Nakagami fading channel. The influence of fading channel parameters, as well as the influence of system's parameters on the probability of collision is investigated. The analytical results obtained in this paper will be compared to the simulation results, and it will be shown that the results match very well.**

Keywords – **Relaying, Diversity, Cooperative transmission, Nakagami fading.**

I. Introduction

The increasing demand for the capacity of wireless systems, dictate the development of new system configurations and new protocols. Cooperative diversity protocols may significantly improve the performance of wireless communication systems, and such protocols are analyzed in detail during the last decade [1], [2], [3]. Using these protocols it is possible to create additional paths between the source and destination using other users' terminals which serve as intermediate relay nodes. The case of multiple relays is considered in [4] and [5]. Paper [4] studied the case of orthogonal transmission from the source and from the relay. In [5], this orthogonality constraint was relaxed, the source and relay are allowed to transmit simultaneously, and it was shown that in this way a significant performance improvement could be achieved at the cost of higher complexity decoder.

In [6], a simple scheme with multiple relays that selects the best relay between source and destination based on instantaneous channel measurements is proposed. The proposed scheme requires no knowledge of the topology or its estimation. The technique is based on signal strength measurements rather than distance and requires a small fraction of the channel coherence time. Additionally, the algorithm itself provides for the necessary coordination in time and group formation among the cooperating terminals. Also, the simplicity of the technique allows immediate implementation in existing radio hardware. Paper [6] considers the probability of collision, the probability that two or more relays transmit the information from source to destination, in Rayleigh and Rician fading channel.

In this paper we derive the expression for the collision probability in Nakagami channel, for the cooperative diversity scheme given in [6]. Analytical results, from this paper, will be shown to be very close to the simulation results.

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II. System Model

The block diagram of the system, proposed in [6], is shown in Fig. 1. The system selects one relay among a set of M relays, based on a measurement which relay provides the "best" end-to-end path between source and destination. The wireless channel a_{si} between source and each relay i, as well as the channel a_{id} between relay i and destination affect performance. These parameters model the propagation environment between any communicating terminals and change over time, with a rate that macroscopically can be modeled as the Doppler shift, inversely proportional to the channel coherence time. Opportunistic selection of the "best" available relay involves the discovery of the most appropriate relay, in a distributed and "quick" fashion, well before the channel changes again. In that way, topology information at the relays (specifically location coordinates of source and destination at each relay) is not needed.

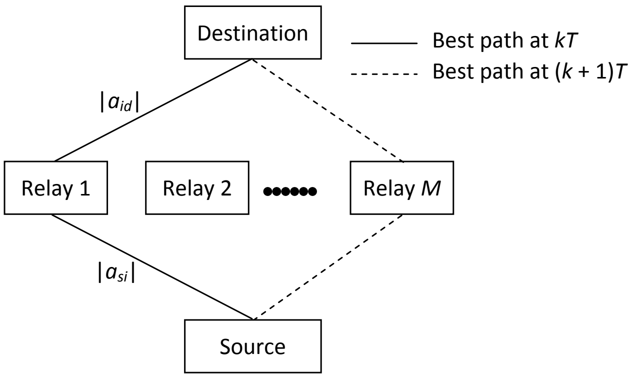

Fig. 1. System model with source, destination, and multiple relays

More specifically, the relays overhear a single transmission of a ready-to-send (RTS) packet and a clear-to-send (CTS) packet from the destination. From these packets, the relays assess how appropriate each of them is for information relaying. The transmission of RTS from the source allows for the estimation of the instantaneous wireless channel a_{si} between source and relay i, at each relay i. Similarly, the transmission of CTS from the destination allows for the estimation of the instantaneous wireless channel a_{id} between relay i and destination at each relay i.

Since communication among all relays should be minimized for reduced overall overhead, a method based on time was selected: as soon as each relay receives the CTS packet, it starts a timer from a parameter h_i based on the instantaneous channel measurements a_{si}, a_{id}. The timer of the relay with the best end-to-end channel conditions will expire first. That relay transmits a short duration flag packet, signaling its presence. All relays, while waiting for their timer to reduce to zero (i.e., to expire), are in listening mode. As soon as they hear another relay to flag its presence or forward information (the best relay), they back off.

The channel estimates a_{si}, a_{id} at each relay, describe the quality of the wireless path between source-relay-destination, for each relay i. Since the two hops are both important for end-to-end performance, each relay should quantify its appropriateness as an active relay, using a function that involves the link quality of both hops. Two functions are used in [6]: under Policy I, the minimum of the two is selected, while under Policy II, the harmonic mean of the two is used. Policy I selects the "bottleneck" of the two paths while Policy II balances the two link strengths and it is a smoother version of the first one. For Policy I:

$$h_i = \min\{|a_{si}|^2, |a_{id}|^2\} \tag{1}$$

and for Policy II:

$$h_i = \frac{2}{\dfrac{1}{|a_{si}|^2} + \dfrac{1}{|a_{id}|^2}} \tag{2}$$

The relay that maximizes function h_i is the one with the "best" end-to-end path between initial source and final destination. After receiving the CTS packet, each relay i will start its own timer with an initial value T_i, inversely proportional to the end-to-end channel quality h_i, according to the following equation:

$$T_i = \frac{\lambda}{h_i}, \tag{3}$$

where λ is a constant and has the units of time.

III. Performance Analysis

The probability of having two or more relay timers expire "at the same time" is zero. However, the probability of having two or more relay timers expire within the same time interval c, and therefore cause a collision, is nonzero and was evaluated in [6]. It was shown that the probability of collision is equal to

$$P_c = 1 - M(M-1)\int_c^{\infty} f(x)[1-F(x)]^{M-2} F(x-c)dx \tag{4}$$

where $f(x)$ is the probability density function (pdf), and $F(x)$ is the cumulative distribution function (cdf) of random variables $T_i, i = 1,...,M$, where M is the number of relay nodes. The cumulative distribution function and probability density function are related to respective distributions of h_i in the following way:

$$F(t) \equiv \mathrm{cdf}_{T_i}(t) = 1 - \mathrm{cdf}_{h_i}(\lambda / t) \tag{5}$$

$$f(t) \equiv \mathrm{pdf}_{T_i}(t) = \frac{d}{dt}F(t) = \frac{\lambda}{t^2}\mathrm{pdf}_{h_i}(\lambda / t) \tag{6}$$

In case of Nakagami channel, it will be assumed that $|a_{si}|$ and $|a_{id}|$, $i = 1,...,M$, are independent (but not identically distributed) Nakagami random variables with the following pdfs, respectively:

$$p_1(x) = \frac{2m_1^{m_1} x^{2m_1-1}}{\Gamma(m_1)\Omega_1^{m_1}} e^{-\frac{x^2 m_1}{\Omega_1}} \tag{7}$$

$$p_2(x) = \frac{2m_2^{m_2} x^{2m_2-1}}{\Gamma(m_2)\Omega_2^{m_2}} e^{-\frac{x^2 m_2}{\Omega_2}} \tag{8}$$

where $m_1, m_2, \Omega_1, \Omega_2$ are pdf's parameters, and $\Gamma(\cdot)$ is Gamma function.

The probability density functions of $|a_{si}|^2$ and $|a_{id}|^2$ are given by a Gamma distribution:

$$p_{k,sq}(x) = \frac{m_k^{m_k} x^{m_k-1}}{\Gamma(m_k)\Omega_k^{m_k}} e^{-\frac{x \cdot m_k}{\Omega_k}},\ k = 1,2 \tag{9}$$

A. Policy I

In case of Policy I, the minimum of the variables $|a_{si}|^2$ and $|a_{id}|^2$ is determined. The probability density function of the minimum of variables $|a_{si}|^2$ and $|a_{id}|^2$ is:

$$p_{\min}(x) = p_{1,sq}(x)(1 - P_{2,sq}(x)) + p_{2,sq}(x)(1 - P_{1,sq}(x)) \tag{10}$$

where $P_{k,sq}, k = 1,2$ are cumulative distribution functions of $|a_{si}|^2$ and $|a_{id}|^2$:

$$P_{k,sq}(x) = \int_0^x p_{k,sq}(y)dy = \frac{\gamma(m_k, m_k \cdot x / \Omega_k)}{\Gamma(m_k)} \tag{11}$$

where $\gamma(m,x)$ is incomplete Gamma function, defined by

$$\gamma(m,x) = \int_0^x y^{m-1} e^{-y} dy \tag{12}$$

Finally:

$$f(t) = \frac{\lambda}{t^2} p_{\min}\left(\frac{\lambda}{t}\right) \tag{13}$$

$$F(t) = 1 - \int_0^{\lambda/t} p_{\min}(u)du \tag{14}$$

B. Policy II

In case of Policy II, a harmonic mean of random variables $|a_{si}|^2$ i $|a_{id}|^2$ is used. In this case, we will consider that random variables have identical distributions. Probability density function of the harmonic mean of two Gamma random variables with identical distributions

$$p_\gamma(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} \quad (15)$$

is derived in [7] and is equal to

$$p_{\gamma H}(x) = \frac{\sqrt{\pi}\beta^{-\alpha}}{\Gamma^2(\alpha)} \left(\frac{x}{2}\right)^{\alpha-1} e^{-\frac{2x}{\beta}} \Psi\left(\frac{1}{2}-\alpha, 1-\alpha, \frac{2x}{\beta}\right) \quad (16)$$

where $\Psi(\cdot,\cdot,\cdot)$ is confluent hypergeometric function defined in [8]. The cumulative distribution functions of the harmonic mean of two Gamma random variables is also derived in [7] and is equal to

$$P_{\gamma H}(x) = \frac{\sqrt{\pi}x}{2^{2\alpha-2}\Gamma^2(\alpha)\beta} G_{23}^{21}\left(\frac{2x}{\beta}\middle| \begin{matrix} 0, \alpha-1/2 \\ \alpha-1, 2\alpha-1, -1 \end{matrix}\right) \quad (17)$$

where $G_{pq}^{mn}(\cdot)$ is Meyer G-function defined in [9]. In this case, Gamma distribution (15) is defined by (9), and therefore in (16) and (17) variables are

$$\alpha = m$$
$$\beta = \frac{\Omega}{m} \quad (18)$$

where $\Omega_1 = \Omega_2 = \Omega$, and $m_1 = m_2 = m$.

Now, from equations (5) and (6) we get

$$f(t) = \frac{\lambda}{t^2} p_{\gamma H}\left(\frac{\lambda}{t}\right) \quad (19)$$

$$F(t) = 1 - P_{\gamma H}\left(\frac{\lambda}{t}\right) \quad (20)$$

IV. Numerical Results

In this section we will show the probability of collision, determined both analytically using equations (4), (13), (14), (19), and (20), and by computer simulation. Simulation results are determined using Monte-Carlo simulation, and random variable with Nakagami probability density function is generated using a method from [10].

Fig. 2 shows the probability density function (10) and the same pdf obtained by Monte-Carlo simulation. It is clear that there is an excellent match between the analytical result in (10) and the Monte Carlo simulation.

The probability of collision as a function of fading parameter *m* is shown in Fig. 3. Other fading parameters are $\Omega_1 = \Omega_2 = 1$. The collision probability rises with the increase of fading parameter *m*. For higher *m* fading is less severe, and each node's channels are more equal, which results in higher probability of collision. As expected, it can be seen that the higher the number of relay nodes, the higher the probability of collision.

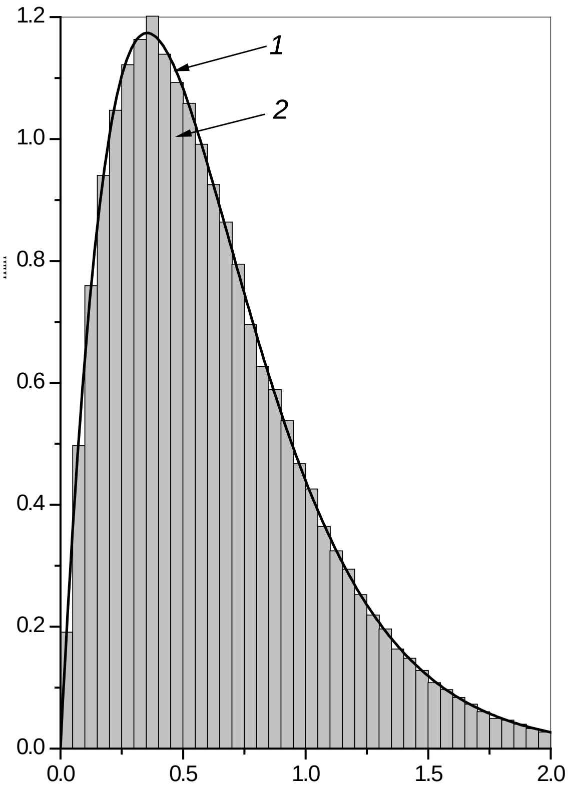

Fig. 2. Comparison between the analytical results (*1*) and the Monte Carlo simulation (*2*) for pdf (10), and $\Omega_1 = \Omega_2 = 1$, $m_1 = m_2 = 2$, 200000 iterations

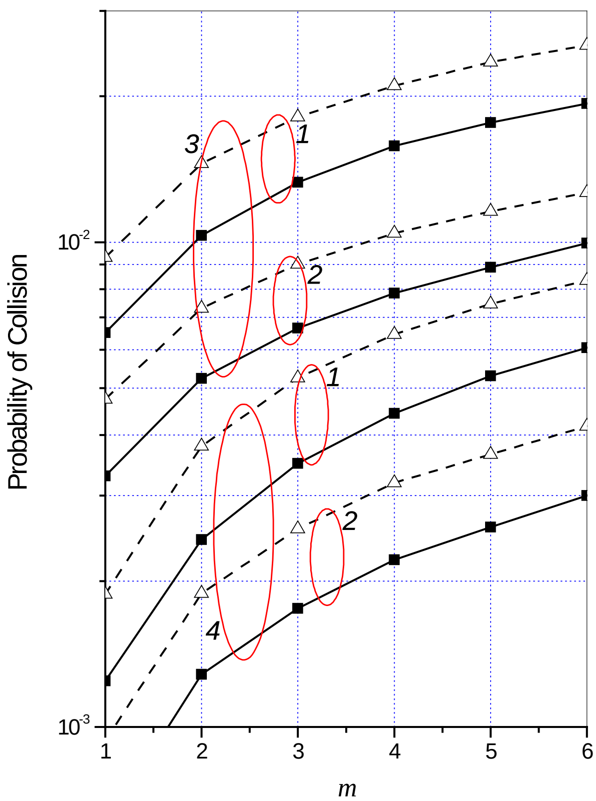

Fig. 3. Probability of collision as a function of *m*, for $m_1 = m_2 = m$
(*1*) λ / c = 200, (*2*) λ / c = 400, (*3*) M = 6, (*4*) M = 2
Policy I – Analytic results (solid line), simulation (squares)
Policy II – Analytic results (dashed line), simulation (triangles)

As in paper [6] for Rayleigh and Rician fading, here Fig. 3 confirms that Policy I has better performance than Policy II. It can also be seen that analytic and simulation results are very close to each other.

V. Conclusion

An analysis of a cooperative diversity system in the presence of Nakagami fading is considered in this paper. It was shown that the analytical results are in great accordance with the ones obtained by Monte-Carlo simulation. Also, the results show that the probability of collision increases with the number of relay nodes, as well as with parameter *m*.

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