Level Crossing Rate of Nakagami-*m* Signal Envelope Subjected to Gamma Shadowing

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Abstract – **In this paper signal envelope subjected simultaneously to Nakagami-*m* multipath fading and Gamma shadowing is considered. Multipath Nakagami-*m* fading causes signal envelope variation and Gamma shadowing causes signal envelope power evaluation. Average level crossing rate of considered signal envelope is calculated. Numerical results are presented graphically to show the influence of Nakagami-*m* fading severity and Gamma shadowing severity on average level crossing rate. Obtained results can be used in performance analysis of wireless communication system in the presence multipath Nakagami-*m* fading and Gamma shadowing.**

Keywords **– Level crossing rate (LCR), Nakagami-*m*, Gamma shadowing.**

I. INTRODUCTION

Short term fading and long term fading degrades outage probability of wireless communication system and limits system capacity and spectral efficiently [6]. Received signal suffers simultaneously to long term fading and short term fading, resulting in degradation of system performance of communication system. Short term fading is result of multipath propagation due to reflection, refraction and scattering of radio wave, causing signal envelope variation. Long term fading is result of large obstacles between transmitter and receiver, causing signal envelope power variation [5]. There are more distributions that can be used to describe signal envelope variation in fading channel depend on system nonlinearity, line-of-sight between transmitter and receiver and the number of clusters propagation environment. Rayleigh distribution can be used to describe small scale signal envelope variation in linear, non line-of-sight multipath fading environment with one cluster. In linear line-of-sight multipath fading environment, small scale signal envelope variation can be described by using the Rician statistical model [8]. Nakagami-*m* distribution describes small scale signal envelope variation in linear, non line-of-sight multipath fading condition with more clusters. In non linear channels, small scale signal envelope variation can be analyzed by using Weibull and α-μ distributions. The α-μ distribution has two parameters. The parameters α is related to nonlinearity of environment and μ is associated to the number of clusters in environment. Large scale signal envelope power variation in shadowing fading environment can be described by using log-normal distribution or Gamma distribution. The first order statistics or random variable is probability density function, cumulative distribution function, moment generating function and moments. The second order statistics are joint probability density function of random variable and it’s first derivative, the joint probability density function of random variable at two time instants, average level crossing rate and average fade duration. The average level crossing rate can be calculated as average value of the first derivative of random variable. There are more works considering average level crossing rate of random process and system performance of wireless communication system subjected to long term fading and short term fading.

In paper [1], macrodiversity SC receiver with two microdiversity MRC receivers subjected simultaneously to Nakgami-*m* multipath fading and Gamma shadowing is considered. Closed form expressions for average level crossing rate of SC receiver output signal envelope and average fade duration of proposed system are evaluated.

In paper [2], macrodiversity system with two microdiversity MRC receivers affected to Rician multipath fading and Gamma shadowing is analyzed. The second order statistics of wireless communication system are calculated in closed form expressions.

In paper [3], level crossing rate and average fade duration of wireless communication system with SC receiver operating over Rician fading channel in the presence of co-channel interference subjected to Rayleigh multipath fading are calculated.

In paper [4], the ratio of product of two random variables and random variable is considered. The product of two random variables can represent desired signal envelope affected to two multipath fading. The random variable in the denominator of the ratio can represent co-channel interference envelope affected to multipath fading. The average level crossing rate of considered ratio is determined.

In this paper the signal envelope subjected to Nakgami-*m* short-term fading and Gamma long term fading is considered. Joint probability density function of this signal envelope and it’s first derivative is derived. This expression is used for evaluation of the average level crossing rate of signal

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envelope affected simultaneously to Nakagami-*m* multipath fading and Gamma shadowing. The numerical results are presented graphically to show the influence of Nakagami-*m* severity parameter and Gamma severity parameter on average level crossing rate. To the best of author's knowledge, level crossing rate of signal envelope subjected simultaneously to Nakagami-m short term fading and Gamma long term fading is not reported in open technical literature. Obtained results can be applied in performance analysis and designing of wireless communication system in the presence of Nakagami-*m* multipath fading and Gamma shadowing fading.

II. LEVEL CROSSING RATE OF GAMMA SHADOWED NAKAGAMI-M MULTIPATH FADING ENVELOPE

Squared Nakagami-*m* random variable can be written as sum of 2μ independent Gaussian random variables [9]:

$$x^2 = x_1^2 + x_2^2 + \cdots + x_{2\mu}^2, \tag{1}$$

where x_i, i=1, 2, …, $\mathbf{2}\mu$ independent Gaussian random variables with zero mean and variance σ^2. The first derivative of Nakagami-*m* random variables x is:

$$\dot{x} = \frac{1}{x}\left(x_1\dot{x}_1 + x_2\dot{x}_2 + \cdots + x_{2\mu}\dot{x}_{2\mu}\right). \tag{2}$$

The first derivative of Gaussian random variable is Gaussian random variable with zero mean and variance [10]:

$$\sigma_{\dot{x}_1}^2 = 2\sigma^2\pi^2 f_m{}^2, i = 1, 2, ..., 2\mu, \tag{3}$$

where f_m is maximal Doppler frequency. The linear transformation of Gaussian random variables is Gaussian random variable. Therefore, the first derivative of Nakagami-*m* random variable $\dot{x}$ has conditional Gaussian distribution with mean:

$$\overline{\dot{x}} = \frac{1}{x}\left(x_1\overline{\dot{x}_1} + x_2\overline{\dot{x}_2} + \cdots + x_{2\mu}\overline{\dot{x}_{2\mu}}\right) = 0, \tag{4}$$

since $\overline{\dot{x}_1} = \overline{\dot{x}_2} = \cdots = \overline{\dot{x}_{2\mu}} = 0$.

The variance of $\dot{x}$ is:

$$\sigma_{\dot{x}}^2 = \frac{1}{x^2}\left(x_1^2\sigma_{\dot{x}_1}^2 + x_2^2\sigma_{\dot{x}_2}^2 + \cdots + x_{2\mu}^2\sigma_{\dot{x}_{2\mu}}^2\right). \tag{5}$$

After substituting (3) in (5), the expression for variance of $\dot{x}$, becomes:

$$\sigma_{\dot{x}}^2 = \frac{2\sigma^2\pi^2 f_m^2}{x^2}\left(x_1^2 + x_2^2 + \cdots + x_{2\mu}^2\right) =$$

$$= 2\sigma^2\pi^2 f_m^2 = \pi^2 f_m^2 \frac{\Omega}{m}, \tag{6}$$

where *Ω* is power of Nakagami-*m* random variable and *m* is Nakagami-*m* fading severity parameter. Joint probability density function of Nakagami-*m* random variable and the first derivative of Nakagami-*m* random variable is:

$$p_{x\dot{x}}\left(x, \dot{x}/\Omega\right) = p_x\left(x/\Omega\right)p_{\dot{x}}\left(\dot{x}/\Omega\right), \tag{7}$$

since random variables x and $\dot{x}$ are independent random variables and Gamma shadowing causes signal envelope power variation. The conditional probability density function of x is:

$$p_x\left(x/\Omega\right) = \frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-\frac{m}{\Omega}x^2}, \tag{8}$$

where *Ω* is average square value of *x* and *m* is Nakagami-*m* parameter and *Γ(m)* is Gamma function. The distribution of $\dot{x}$ is:

$$p_{\dot{x}}\left(\dot{x}/\Omega\right) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}}}. \tag{9}$$

After substituting (8) and (9) in (7), the expression for conditional joint probability density function of x and $\dot{x}$ becomes:

$$p_{x\dot{x}}\left(x, \dot{x}/\Omega\right) = \frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-\frac{m}{\Omega}x^2} \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}^2}}}. \tag{10}$$

The random variable *Ω* follows Gamma distribution:

$$p_\Omega(\Omega) = \frac{\Omega^{c-1}}{\beta^c\Gamma(c)} e^{-\frac{\Omega}{\beta}}. \tag{11}$$

The joint probability density function of x and $\dot{x}$ can be calculated by averaging the expression (10), over *Ω*:

$$p_{x\dot{x}}(x, \dot{x}) = \int_0^\infty d\Omega\, p_{x\dot{x}}\left(x, \dot{x}/\Omega\right)p_\Omega(\Omega) = \int_0^\infty d\Omega \frac{1}{\Gamma(m)} \cdot$$

$$\cdot\left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-\frac{m}{\Omega}x^2} \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}^2}}} \frac{\Omega^{c-1}}{\beta^c\Gamma(c)} e^{-\frac{\Omega}{\beta}}. \tag{12}$$

Average level crossing rate of x can be calculated as average value of the first derivative of x [11]:

$$N_x = \int_0^\infty dx\, p_{x\dot{x}}(x, \dot{x})\dot{x} = \int_0^\infty d\dot{x}\,\dot{x}\int_0^\infty d\Omega \frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \cdot$$

$$\cdot\, x^{2m-1} e^{-\frac{m}{\Omega}x^2} \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}^2}}} \frac{\Omega^{c-1}}{\beta^c\Gamma(c)} e^{-\frac{\Omega}{\beta}} =$$

$$= \int_0^\infty d\Omega \frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-\frac{m}{\Omega}x^2} \frac{\Omega^{c-1}}{\beta^c \Gamma(c)} \cdot$$

$$\cdot e^{-\frac{\Omega}{\beta}} \int_0^\infty d\dot{x}\, \dot{x} \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} = \frac{1}{\Gamma(m)} m^m x^{2m-1} \cdot$$

$$\cdot \frac{1}{\beta^c\Gamma(c)} \int_0^\infty d\Omega\, \Omega^{-m+c-1} e^{-\frac{mx^2}{\Omega}-\frac{\Omega}{\beta}} \frac{1}{\sqrt{2\pi}}\sigma_{\dot{x}}, \quad (13)$$

since:

$$\int_0^\infty d\dot{x}\, \dot{x} \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} = \frac{1}{\sqrt{2\pi}}\sigma_{\dot{x}}. \qquad (14)$$

The expression (13), becomes:

$$N_x = \frac{m^m x^{2m-1}}{\sqrt{2\pi}\Gamma(m)\beta^c\Gamma(c)} \int_0^\infty d\Omega\, \Omega^{-m+c-1} e^{-\frac{mx^2}{\Omega}-\frac{\Omega}{\beta}} \cdot$$

$$\cdot \pi f_m \frac{\Omega^{1/2}}{m^{1/2}} = \frac{m^{m-1/2} x^{2m-1} \pi f_m}{\sqrt{2\pi}\Gamma(m)\beta^c\Gamma(c)} \cdot$$

$$\cdot \int_0^\infty d\Omega\ \Omega^{-m+c-1} e^{-\frac{mx^2}{\Omega}-\frac{\Omega}{\beta}}.$$

(15)

By using the integral [7]:

$$\int_0^\infty dx\, x^{r-1} e^{-\frac{a}{x}-bx} = \left(\frac{a}{b}\right)^{\frac{r}{2}} K_r\left(2\sqrt{ab}\right), \qquad (16)$$

where $K_r(y)$ is Bessel function of the second kind, the previously expression becomes:

$$N_x = \frac{m^m x^{2m-1}}{\sqrt{2\pi}\Gamma(m)\beta^c\Gamma(c)} \left(m x^2 \beta\right)^{\frac{c-m+1/2}{2}} \cdot$$

$$\cdot K_{c-m+1/2}\left(2\sqrt{\frac{mx^2}{\beta}}\right). \qquad (17)$$

In figure 1, average level crossing rate is plotted versus signal envelope for several values of Nakagami-*m* fading severity *m*, Gamma shadowing severity *c* and signal envelope power *b*. For lower values of signal envelope average level crossing rate increases as signal envelope increases. For higher values of signal envelope, average level crossing rate decreases as signal envelope increases. Maximal value of average level crossing rate increases as Gamma shadowing severity decreases. System performances are better for lower values of average level crossing rate. The influence of signal envelope on average level crossing rate is greater for higher values of Nakagami-*m* fading several.

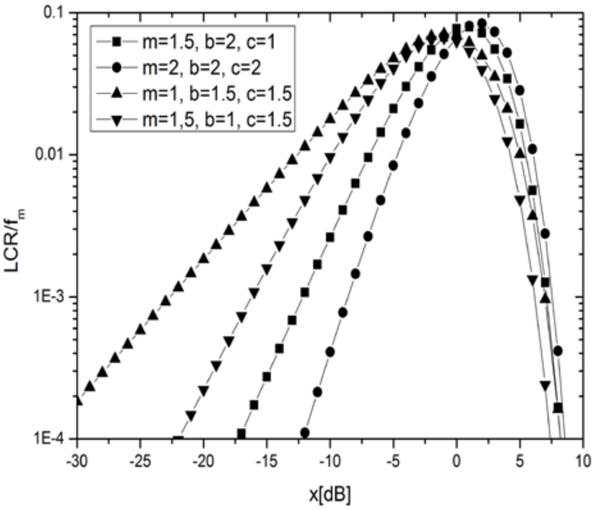

Fig. 1. LCR for different parameters *m, c* and *b*

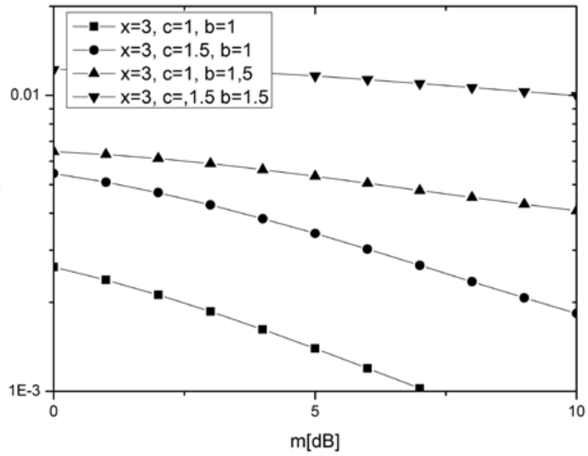

Fig. 2. LCR for different parameters *x, c* and *b*

In figure 2, average level crossing rate is shown versus Nakagami-*m* fading severity for several values of signal envelope *x*, Gamma shadowing severity *c* and signal envelope power *b*. The influence of Nakagami-*m* fading severity on average level crossing rate is greater for lower values of Gamma shadowing severity *c*. Average level crossing rate increases as Gamma shadowing severity decreases. System performances are better for higher values of Nakagami-*m* fading severity and Gamma shadowing severity parameter.

III. CONCLUSION

Obtained results can be used for performance analysis of wireless communication system subjected to multipath fading and shadowing. Received signal experiences Nakagami-*m* short term fading and Gamma long term fading resulting in system performance degradation. Small scale Nakagami-*m* multipath fading causes signal envelope variation and long scale Gamma shadowing causes signal envelope power variation. Received signal envelope has conditional Nakagami-*m* probability density function. The average square value of received signal envelope follows Gamma distribution. Probability density function of received signal envelope is derived by averaging conditional Nakagami-*m* described envelope over Gamma distribution. In this paper joint probability density function of received signal envelope and it's first derivative is calculated. This expression is used for evaluation of average level crossing rate of received signal envelope in the presence short term fading and long term fading. Average level crossing rate is derived in closed form expression. Average level crossing rate is calculated as average value of the first derivative of signal envelope. Level crossing rate is the second order statistics of random variable envelope. Numerical results are presented graphically to show the influence of short term fading severity parameter and long term fading severity parameter on average level crossing rate. System performances are better for lower values of average level crossing rate of received signal envelope. Average level crossing rate of received signal envelope increases as short term fading severity parameter decreases. Influence of short term fading severity parameter on average level crossing rate is greater for higher values of long term fading severity parameter. The system performances are better for higher values of long term fading severity parameter.

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