iCEST 2014 Serbia, Niš, June 25 - 27, 2014

ANN Models for DOA Estimation of Correlated Signals using Circular Antenna Array

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Abstract – **This article investigates performance of artificial neural networks (ANNs) in detection of correlated signals. In order to cover the whole azimuth plane, circular antenna array is employed at the receiver. Separate ANN configurations are developed for detection of two and three incoming signals, and their results are analysed with regard to distance between sources and degree of correlation. The estimation results are compared with the reference data and the MUSIC algorithm.**

***Keywords* – Artificial neural networks, Circular antenna array, Correlated signals, DOA estimation.**

I. INTRODUCTION

Direction of arrival (DOA) estimation of correlated signals is often required in practical applications. Due to the multipath propagation of RF signals, a receiving array collects direct signals from sources as well as fractionally delayed multipath components that have bounced off from some nearby reflective surfaces.

Detection of correlated signals is not always an easy task. Standard algorithms for detection of uncorrelated signals [1], [2] could be modified in order to provide detection of correlated sources as well. The drawback of these algorithms is that they include additional calculations to decorrelate signals. This procedure is useful but, on the other hand, it reduces the effective aperture of the antenna array obtaining the results of lower resolution [3].

In this paper, artificial neural networks (ANNs) are employed to provide DOA estimates of correlated signals. Due to their universal approximation capability and parallel processing of input data, estimation results are provided almost instantaneously. In this way, complex calculations over spatial covariance matrices are avoided. Application of ANNs in detection of correlated signals does not degrade resolution of DOA estimates. In the paper, we analyze detection of two and three EM signals, at different mutual distances, azimuth positions and with various correlation coefficients. Therefore, three levels of generalization of ANN models are achieved. Advantage over MUSIC algorithm is illustrated on several examples of detecting closely spaced and strongly correlated sources.

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II. DATA MODEL

A uniform circular array (UCA), composed of N equally spaced omnidirectional elements, is illustrated in Fig 1. The radius of the circle is r while the distance between adjacent elements in the array is $d=\lambda/2$ (where λ is the wavelength of impinging signals). The angular position of the n-th element of the array is given by $\gamma_i=2\pi i/N$, i=0, 1,..., N-1. DOA of the kth incoming signal is determined by the azimuth and elevation angles φ_k and θ_k, respectively. The array manifold vector $\boldsymbol{a}_k(\varphi_k, \theta_k)$, k=1, 2, ..., K is given by

$$\boldsymbol{a}_k(\varphi_k,\theta_k)=[e^{j(2\pi/\lambda)r\sin\theta_k\cos(\varphi_k-\gamma_0)} \; e^{j(2\pi/\lambda)r\sin\theta_k\cos(\varphi_k-\gamma_1)} \; \dots \\ \dots \; e^{j(2\pi/\lambda)r\sin\theta_k\cos(\varphi_k-\gamma_{N-1})}]^T \qquad (1)$$

Further, a N x K steering matrix **A** and signal model can be defined as follows

$$\mathbf{A}(\varphi,\theta)=[\boldsymbol{a}(\varphi_1,\theta_1) \; \boldsymbol{a}(\varphi_2,\theta_2) \; \dots \; \boldsymbol{a}(\varphi_K,\theta_K)] \qquad (2)$$

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t) \qquad (3)$$

where **n**(t) is the $N\times1$ noise vector, **x**(t) is the $N\times1$ data vector and **s**(t) is the $K\times1$ signal vector given by

$$\mathbf{s}(t)=[s_1(t) \; s_2(t)\dots s_K(t)]^T=[\rho_1 \; \rho_2\dots\rho_K]^T s_l(t) \qquad (4)$$

where ρ_i denotes the relative amplitude and phase between the i-th and l-th source (ρ_l=1).

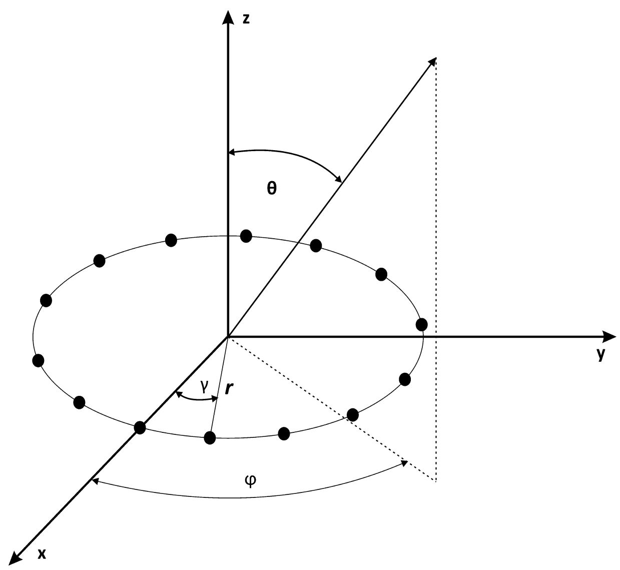

Fig. 1 Uniform circular antenna array (UCA)

The spatial covariance matrix can be estimated as follows

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{ASA}^H + \sigma^2\mathbf{I} \quad (5)$$

where $E\{\}$ is the expectation operator, H denotes the complex conjugate transpose operation, σ^2 is the noise variance, **I** is the identity matrix, **S** is $K\times K$ signal covariance matrix given by

$$\mathbf{S} = E[\mathbf{s}(t)\mathbf{s}^H(t)] = E[|s_l(t)|^2]\mathbf{cc}^H \quad (6)$$

where $\mathbf{c}=[\rho_1\ \rho_2\ .\ .\ .\ \rho_K]^T$. It can be observed that the signal subspace of matrix **R** is of rank one instead of K, and the noise subspace is orthogonal to **Ac** instead of the columns of **A.** This implies the failure of the subspace based method when the matrix **R** is used in this form.

III. DOA ESTIMATION BY RBF NEURAL NETWORK

Application of Radial Basis Function Neural Networks (RBF-NN) in the area of DOA estimation is based on the inverse mapping to the one that an antenna array performs. That is the mapping G: $\mathbf{C}^N\rightarrow\mathbf{R}^K$ from the space of antenna array outputs $\{\mathbf{x}(t)=[x_1(t)\ x_2(t)\ \ldots\ x_N(t)]^T\}$to the space of DOAs $\Phi=[\varphi_1\ \varphi_2\ \ldots\ \varphi_K]^T$. The input data of RBF-NN is the spatial covariance matrix **R** of the antenna array outputs, and DOAs of users' signals are neural network responses.
The DOAs from the received signal **x** can be estimated as follows

$$\varphi=f(\mathbf{x}) \quad (7)$$

An RBF-NN approximates the reverse mapping function $f(\mathbf{x})$ as a weighted sum of the radial basis function

$$f(\mathbf{x}) = \sum_{j=1}^{J}\omega_{jm}\exp\left(-\frac{\|\mathbf{x}-\mathbf{c}_j\|^2}{2\sigma_j^2}\right) \quad (8)$$

where $\mathbf{c}_j$ is the central vector of the jth neuron, σ_j^2 is the spread while ω_{jm} is a weight. These parameters are determined through the training process of the RBF-NN. In the equation (8), $\|\cdot\|$ is the Euclidean norm and J denotes the number of hidden neurons [4].

An RBF-NN is composed of input, hidden and output layers. Number of neurons in the input layer of the network depends on the dimensionality of **R** matrix. Since **R** is an $N \times N$ square matrix and having in mind that ANNs cannot operate with complex numbers, there should be $2N^2$ neurons in the input layer of the network. In the case of correlated signals, upper triangular part of matrix **R** is used to represent signals at the array output. Therefore, all inputs are organized into a N^2-element vector **z**. Before it is applied to the input layer of the RBF-NN, the input vector **z** is normalized with its norm, $\mathbf{z}_{\text{norm}}=\mathbf{z}/\|\mathbf{z}\|$. The jth neuron in the hidden layer has the nonlinear response that can be written as follows

$$h_j(\mathbf{z}_{norm}) = b_{1j}\exp\left(-\frac{\|\mathbf{z}_{norm}-\mathbf{c}_j\|^2}{2\sigma_j^2}\right) \quad (9)$$

where b_{1j} represents a real coefficient. Using this expression the output of the RBF-NN can be calculated by

$$g_k(\mathbf{z}_{norm}) = \sum_{j=1}^{J}b_{1j}\omega_{kj}\exp\left(-\frac{\|\mathbf{z}_{norm}-\mathbf{c}_j\|^2}{2\sigma_j^2}\right)+b_{2k} \quad (10)$$

where b_{2k} is a bias of the output layer while ω_{kj} is the weight between the kth neuron in the output layer and jth neuron in the hidden layer.

IV. RESULTS

We consider DOA estimation of two and three correlated sources, respectively. Since the number of outputs of an RBF-NN is fixed, separate configurations are developed using the incorporated function provided by the neural network toolbox of MATLAB [5]. The training set is prepared of the training vectors corresponding to different positions of EM sources. The input to the network is the upper triangular part of matrix **R**, organized into a vector, while the desired outputs are azimuth positions of sources. Test samples are used to evaluate the performance of the developed neural networks. The test set is formed from data that have not been involved in the training process. However, these data must belong to the same distribution as the training data.

To collect data for the training and test set, it is assumed that a 10-element circular antenna array is positioned at the receiver. Signals that have to be detected are narrowband with SNR of 20 dB. To estimate spatial covariance matrix 1024 signal snapshots are used. The correlation of signals is varied between 0 and 1.

A. Detection of two correlated sources

Let's suppose that two EM sources are located at different azimuth positions from -180° to 180°. Mutual distance between sources is varied as follows: 2°, 5°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°, 100°, 110° and 120°. The starting positions of two sources are -180° and -180°+*dist*, where *dist* denotes the distance between sources. The sources are further moved towards 180°, in steps of 1°. Correlation between signals is introduced through the correlation coefficient which took values 0, 0.5 and 1, respectively. To validate the performance of trained RBF-NN models, a test set is formed using mutual distances 3°, 15°, 35°, 55°, 75° and 95°, and step of 1°. The correlation coefficient is varied as follows 0.1, 0.4 and 0.7. Accordingly, the networks are trained with 6420 samples and tested with 3090 samples.

Following the previously described procedure, a number of neural models are developed and among them, the best model is chosen for further analysis. In our case, the network with correlation coefficient of 0.999 and average error of 0.58% has demonstrated the best performance. The hidden layer of the network is composed of 498 neurons.

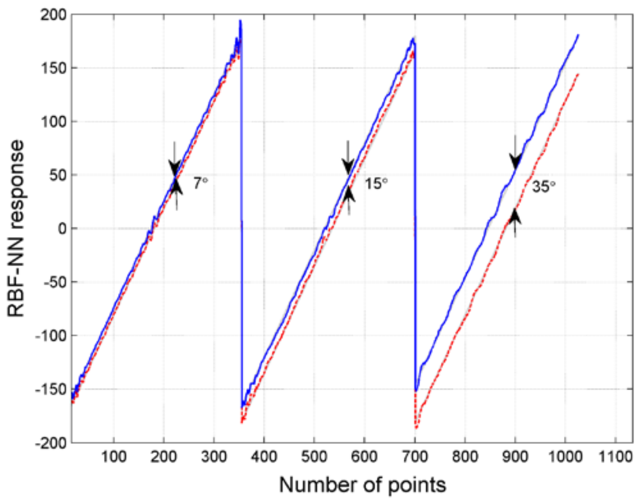

Fig. 2 Response of the RBF-NN model for detection of two signals (-- source 1, - source 2), ρ=1

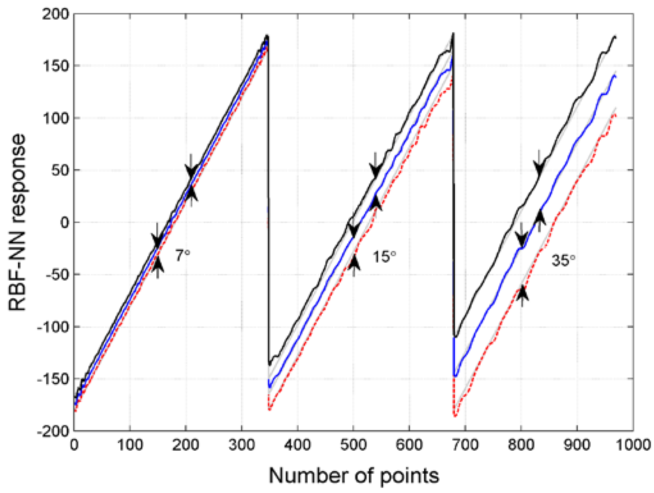

Fig. 4 Response of the RBF-NN model for detection of three sources (-- source 1, - source 2, - source 3), ρ=1

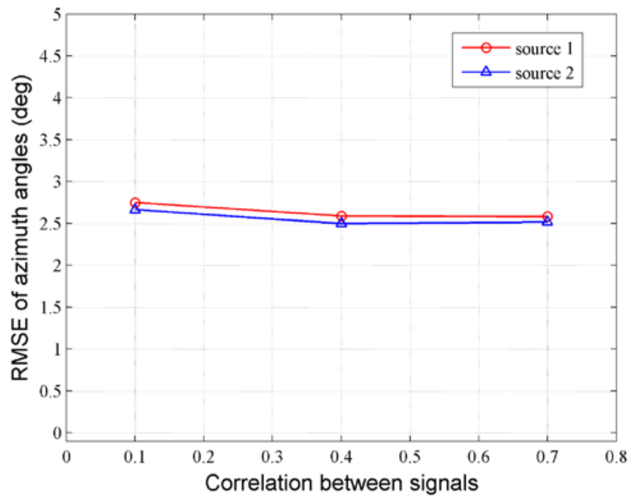

Fig. 3 RMSE versus correlation between signals

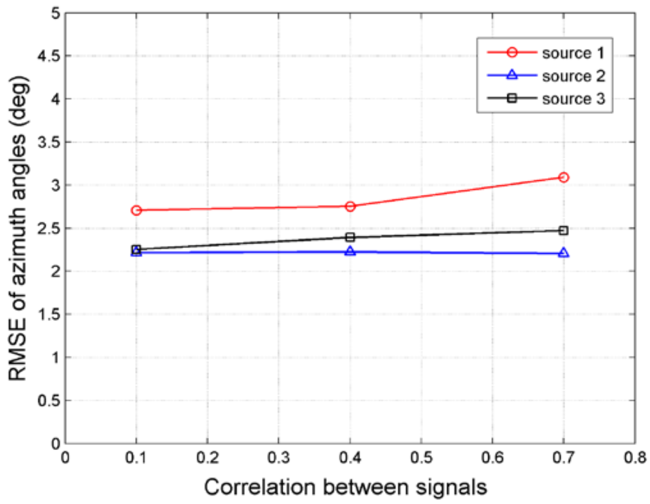

Fig. 5 RMSE versus correlation between signals

The response of the network for test data is presented in Fig. 2. Correlation between signals is 1, and distances are 7°, 15° and 35°, respectively. It can be observed that network has quite good performance for test samples. Even for the separation of 7°, it is able to distinguish two sources. Fig. 3 illustrates dependence of RMSE (Root Mean Square Error) of DOA estimates on the correlation. We can conclude that the network provides almost uniform response for all test values of correlation.

B. Detection of three correlated sources

Similar to the case with two sources, three sources are assumed to be at mutual distances of 2°, 5°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°, 100°, 110° and 120°. Initially, all sources are positioned at azimuth angles of -180°, -180°+dist and -180°+2·dist, and then moved towards 180° in steps of 1°. The correlation between signals is varied from uncorrelated case, weakly correlated up to fully coherent signals (coefficients 0, 0.5 and 1, respectively). The test set is formed assuming mutual distances of 12°, 23°, 32°, 64° and 96°. The correlation between signals is 0.1, 0.4 and 0.7. Hence, the network is trained with 10440 and tested with 4053 samples. After the training procedure is finished, the test statistics of the obtained neural models are analysed. The best neural model has 958 hidden neurons, the correlation coefficient of 0.9993 and average error of 0.7%.

In Fig. 4, response of the RBF-NN is illustrated for three mutual distances between sources and correlation coefficient of 1. The response of the network appropriately follows the movement of three sources from -180° to 180°. RMSE of DOA estimates is presented in Fig. 5 for each source separately. The second source is the most accurately detected and almost not dependent on the correlation. Estimation error of the first and the third source is slightly more expressed and rises as the correlation between signals becomes stronger. Also, it can be observed that neural model adequately separates three sources at small mutual distances such as 7°. For instance, the MUSIC algorithm has a problem to separate closely spaced sources even when the signals are uncorrelataed. In the case of correlated signals, the algorithm has problem also with more separated sources. This problem is illustrated in Figs. 6, 7, 8 and 9. Corresponding outputs of RBF-NNs are given in Table 1.

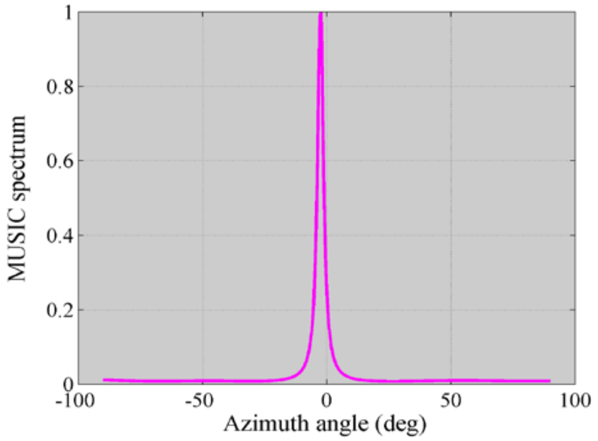

Fig. 6 MUSIC spectrum of two correlated signals (ρ=1) at azimuth positions φ_1=-10° and φ_2=5°

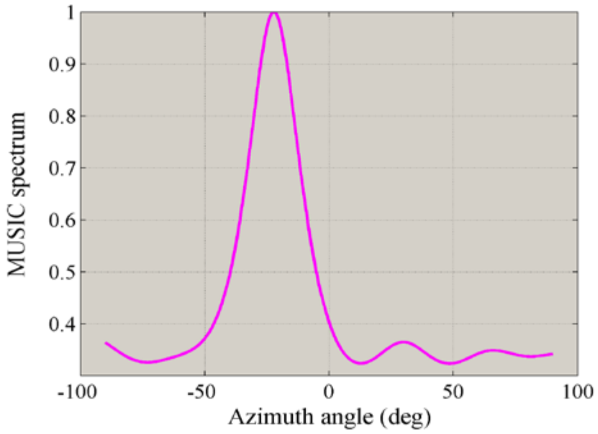

Fig. 7 MUSIC spectrum of two correlated signals (ρ=1) at azimuth positions φ_1=-40° and φ_2=-5°

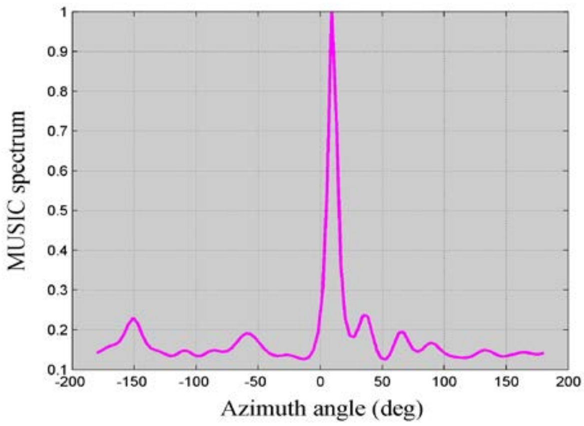

Fig. 8 MUSIC spectrum of three correlated signals (ρ=1) at azimuth positions φ_1=-2°, φ_2=10° and φ_3=22°

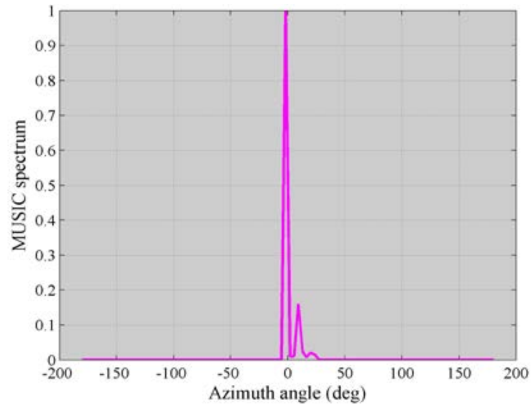

Fig. 9 MUSIC spectrum of three uncorrelated signals (ρ=0) at azimuth positions φ_1=-2°, φ_2=10° and φ_3=22°

TABLE I
DOA ESTIMATES BY RBF-NNS

Actual azimuth angles (deg)			DOA estimates (deg)		
φ_1=-10	φ_2=5		φ_{1es}=-8.48	φ_{2es}=2.25	
φ_1=-40	φ_2=-5		φ_{1es}=-41.47	φ_{2es}=-2.32	
φ_1=-2	φ_2=10	φ_3=22	φ_{1es}=-4.97	φ_{2est}=9.09	φ_{3es}=23.16
φ_1=-2	φ_2=10	φ_3=22	φ_{1es}=-3.82	φ_{2es}=9.64	φ_{3es}=23.05

V. CONCLUSION

In this paper, detection of correlated EM signals is presented. Neural networks are used as a tool to extract the information about directions of arrival from the spatial covariance matrix. It is shown that networks are able to deal with differently positioned and correlated signals. Correlated and closely spaced sources are appropriately separated and detected. The accuracy of results could be further improved using more precise data on the positions of sources in the training set.

ACKNOWLEDGEMENT

This work was supported by the project TR-32052 of the Serbian Ministry of Education, Science and Technological Development.

REFERENCES

[1] R. Schmidt, “Multiple Emitter Location and Signal Parameter Estimation,” *IEEE Trans Antennas Propagation*, vol. 34, pp. 276-280, 1986.

[2] R. Roy and T. Kailath, “ESPRIT-Estimation of Signal Parameters via Rotational Invariance Techniques,” *Trans Acoust, Speech*, Signal Processing, vol. 37, pp. 984 – 995, 1989.

[3] T. J. Shan, M. Wax, and T. Kailath, “On Spatial Smoothing for Estimation of Coherent Signals,” *Trans Acoust, Speech, Signal Processing*, vol. 33, 806 – 811, 1985.

[4] S. Haykin, *Neural Networks*, New York, IEEE Press, 1994.

[5] H. Demuth, M. Beale, “Neural Network Toolbox”, (The Mathworks, Inc., MA, USA, 1998).