Three-Wire Star-Shaped Grounding Electrode in the Vicinity of the Semi-Cylindrically Shaped Ground Inhomogeneity

Nenad N. Cvetković[1], Dragan D. Vučković[1], Miodrag S. Stojanović[1] and Aleksa S. Ristić[1]

Abstract –**The influence of the road, treated as a ground inhomogeneity, on the grounding system in its vicinity is analyzed in the paper. Using an estimation method, the road is approximated by a domain having semi-circular cross-section. The analysis includes using of quasi-stationary image theory and the Green`s function for the point source in the vicinity of the cylinder of circularly shaped cross-section, which is for the first time explicitly given in this paper. The leakage current distributions are obtained applying the Method of Moments. Afterwards, the grounding system characteristics are determined.**

Keywords – **Green's function methods, grounding, nonhomogeneous media.**

I. Introduction

Different technological systems like power facilities, telecommunication systems, or lightning protection systems include a grounding system as a necessary part. Very often, the road is located in the vicinity of such objects. If the road was treated as a ground inhomogeneity, one can expect that there is a certain influence on the grounding system characteristics placed in their vicinity.

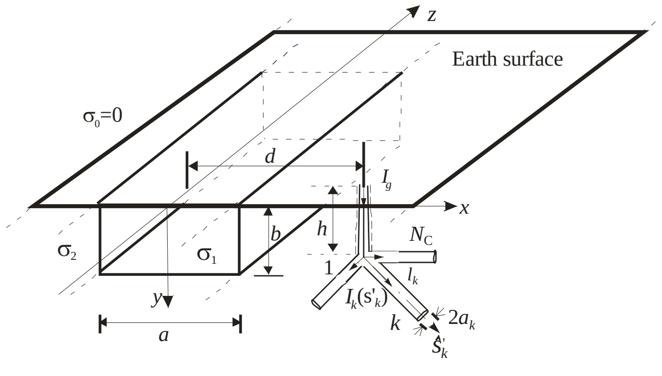

Fig. 1. The star-shaped grounding system in the vicinity of the road

In this paper, onesemi-analytical approach to the quasi-stationary analysis of this influence is proposed. The road is modeled as a homogeneous domain of rectangular cross-section, and afterwards approximated by a semi-circular cross-section cylinder using the estimation method [1]. That resulted in a need tousethe quasi-stationary Green`s function of the point source in the vicinity of such domain given in [2].

The rest of the analysis is performed applying the Method of Moments (MoM) [3] in a usual way. The geometrical parameters of analyzed grounding systems, as well as used conductivity values of the surrounding ground, and the road (the inhomogeneity) have been taken from the official publications, [4]-[5], and from [6]. The resistance and potential distribution in the vicinity of the grounding system are determined.

II. Theoretical Background

A. Description of the Problem

The wire star-shaped grounding system placed in the vicinity of the road, Fig. 1, is analyzed. It is fed by an isolated earthing conductor with current I_g.

Although the domain of the road usually consists of a few sub-domains made of different materials [6], in this case it is modeled as a homogenous cylindrical domain having rectangular ($a \times b$) cross-section of specific conductivity σ_1.

The road is surrounded by the homogeneous ground of known conductivity σ_2 where the grounding system is placed. Corresponding Cartesian coordinate system is assigned.

Star-shaped electrode is formed of N_C mutually connected wires, having length l_k and circle cross-section of radii a_k, $a_k << l_k$, $k = 1, 2, ..., N_C$. Conductors' axis and corresponding orts are labeled by s'_k, while longitudinal current distributions are denoted by $I_k(s'_k)$, $k = 1, 2, ..., N_C$.

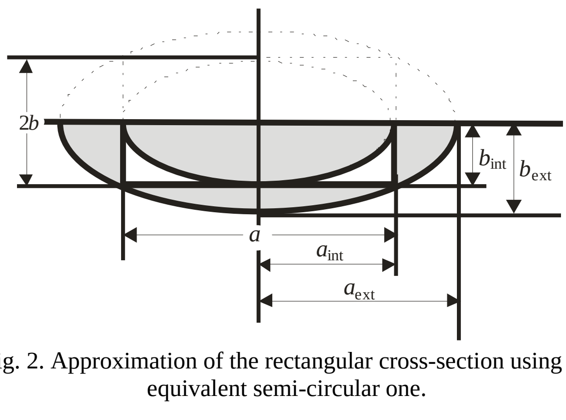

Fig. 2. Approximation of the rectangular cross-section using an equivalent semi-circular one.

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B. Approximating the Road with a Domain of Semi-Circular Cross-Section using the Estimation Method

The estimation method allows approximation of the road domain (modeled as an infinitely long domain of rectangular cross-section), by a cylinder of semi-circular cross-section having equivalent radius a_{eq}. This is carried following the approach described in [1].

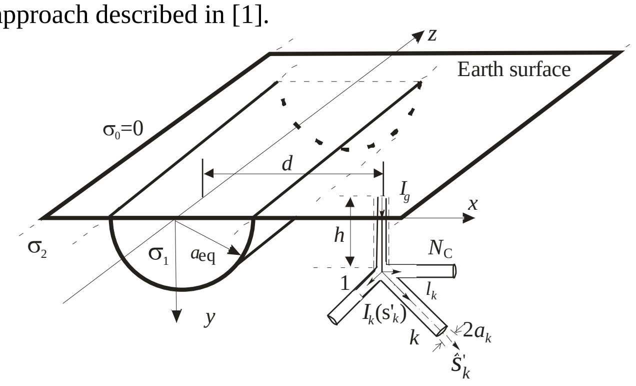

Fig. 3. The star-shaped wire grounding system in the vicinity of the infinitely long cylinder of semi-circular cross-section.

The interval that includes the value of the equivalent radius a_{eq} (Fig. 3) can be estimated using additional domains (Fig. 2) whose cross-sections are confocal ellipses of semi-axis

$$a_{\text{int}} = a/2 \text{ and } b_{\text{int}} = b \text{ for the interior ellipse,} \quad (1)$$

$$\text{i.e. } a_{\text{ext}} = \sqrt{a(a+2b)}/2 \text{ and } b_{\text{ext}} = \sqrt{2b(a+2b)}/2 \quad (2)$$

for the exterior one. The interval is defined by values

$$a_{\text{eq min}} = (a_{\text{int}} + b_{\text{int}})/2 = (a+2b)/4 \text{ and} \quad (3)$$

$$a_{\text{eq max}} = (a_{\text{ext}} + b_{\text{ext}})/2 = \sqrt{a+2b}\left(\sqrt{a}+\sqrt{2b}\right)/4 . \quad (4)$$

Finally, the approximate value of the equivalent radius is

$$a_{\text{eq}} = \frac{a_{\text{eq min}} + a_{\text{eq max}}}{2} = \frac{a+2b+\sqrt{a+2b}\left(\sqrt{a}+\sqrt{2b}\right)}{8} . \quad (5)$$

This way, the problem is reduced to the case of the grounding system in the vicinity of a cylinder of semi-circular cross-section, Fig. 3.

C. Green`s Functions

In order to obtain potential of the system from Fig. 3 the Green`s function for the point source in the vicinity of the infinitely long cylinder having circular cross-section of radii a_{eq} is applied, Fig. 4. This was carried out following the procedure for the internal point source given in [2]. The corresponding Cartesian (x, y, z) and cylindrical coordinate systems (r, θ, z) are assigned, where is

$$x = r\cos\theta, y = r\sin\theta, z = z . \quad (6)$$

The source position (point M) is defined by the field vector $\vec{R}_0 = x_0\hat{x} + y_0\hat{y} + z_0\hat{z} = r_0\hat{r} + z_0\hat{z}$ andthe field vector of the image source placed at the point M’ is

$$\vec{R}'_0 = x_0\hat{x} - y_0\hat{y} + z_0\hat{z} . \quad (7)$$

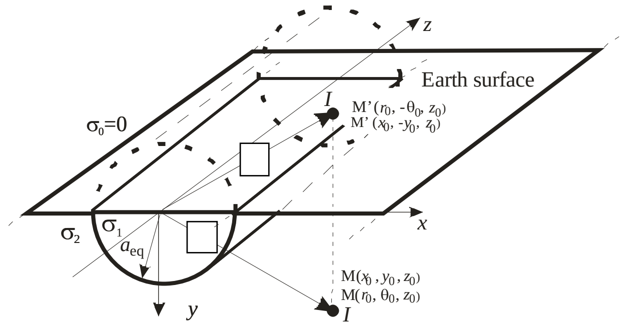

Fig. 4. The point current source near the semi-cylinder

The potential of the point source near a semi-cylinder (Fig. 4) can be expressed as

$$\varphi_1 = \frac{I}{\pi^2\sigma_2}\sum_{m=0}^{\infty}\Big\{\xi_m \cos m\theta \cos m\theta_0 \times$$
$$\times \int_0^{\infty} A_m(\lambda) I_m(\lambda r) I_m(\lambda r_0)\cos[\lambda(z-z_0)]d\lambda\Big\}, r < a_{\text{eq}}, \quad (8)$$
$$\xi_0 = 1, \xi_m = 2,\ m = 1,2,...$$

and

$$\varphi_2 = \frac{I}{4\pi\sigma_2}\left(\frac{1}{\left|\vec{R}-\vec{R}_0\right|} + \frac{1}{\left|\vec{R}-\vec{R}'_0\right|}\right) +$$
$$+ \frac{I}{\pi^2\sigma_2}\sum_{m=0}^{\infty}\Big\{\xi_m \cos m\theta \cos m\theta_0 \times \quad (9)$$
$$\times \int_0^{\infty} B_m(\lambda) I_m(\lambda r_0) K_m(\lambda r)\cos[\lambda(z-z_0)]d\lambda\Big\}, r > a_{\text{eq}},$$
$$\xi_0 = 1, \xi_m = 2,\ m = 1,2,...$$

In the previous expression, I_m and K_m denote the modified Bessel functions of the first and second kind, respectively.

The coefficients $A_m(\lambda)$ and $B_m(\lambda)$, $m = 0,1,2,...$ in expressions (8), i.e. (9), are given byfollowing relation:

$$A_m = \Delta_{A_m}/\Delta_m,\ B_m = \Delta_{B_m}/\Delta_m,\ m = 0,1,2,... \quad (10)$$

Labels Δ_A, Δ_B and Δ in (10) are defined as

$$\Delta_{A_m} = K_m(\lambda r_0) I_m(\lambda r_0)/(\lambda a_{\text{eq}}), \quad (11)$$

$$\Delta_{B_m} = I_m(\lambda a_{\text{eq}}) I_m(\lambda r_0) I'_m(\lambda a_{\text{eq}}) K_m(\lambda r_0)\left(1-\frac{\sigma_1}{\sigma_2}\right) \quad (12)$$

and

$$\Delta_m = [I_m(\lambda r_0)]^2 [(\sigma_1/\sigma_2) I'_m(\lambda a_{eq}) K_m(\lambda a_{eq}) - - I_m(\lambda a_{eq}) K'_m(\lambda a_{eq})]. \quad (13)$$

The corresponding Green`s functions for the point source from Fig. 4 can be defined as

$$G_1(\vec{R}, \vec{R}_0) = \varphi_1 / I \text{ and } G_2(\vec{R}, \vec{R}_0) = \varphi_2 / I . \quad (14)$$

where φ_1 and φ_2 are given by (8), i.e. (9).

D.Application to the Star-Shaped Grounding System

The longitudinal currents along the wires of the grounding system from Fig. 3 are $I_k(s'_k)$, $k = 1,..., N_C$. Electrical scalar potential at an arbitrary point of the grounding system's surroundings can be expressed as

$$\varphi_{1/2}(\vec{R}) = \sum_{k=1}^{N_C} \int_{l_k} G_{1/2}(\vec{r}, \vec{s}'_k) I_{leak}(s'_k) \, \mathrm{d} s'_k \quad (15)$$

where index "1" denotes the potential inside the semi-cylinder ($r < a_{eq}$) and "2" the potential in the surrounding domain ($r > a_{eq}$). Also, $\vec{s}'_k$ in (16) presents the position of the element of the *k*-thconductor $\mathrm{d} s'_k$, $k = 1,..., N_C$. The current that leaks from the wire element $\mathrm{d} s'_k$, $k = 1,..., N_C$ is labeled by $\mathrm{d} I = I_{leak}(s'_k) \mathrm{d} s'_k$, where

$$I_{leak}(s'_k) = -\partial I_k(s'_k) / \partial s'_k = -I'_k(s'_k) , \quad (16)$$

is density of the leakage current per unit length.

Based on the analysis described in the previous text, the Method of Moments is applied for characterization of the system from Fig. 3. Each conductor of the star-shaped electrode is uniformly divided into finite number of segments M_k, $k = 1,..., N_C$. Assuming that the star-shaped electrode surface is approximately equipotential, it is possible to match potential value $\varphi = U$ at the surface point in the middle of the segments on the k-th conductor, defined by field vector $\vec{R}_{ni}$, $n = 1,..., N_C$, $i = 1,..., M_n$. Previously described procedure results in a system of linear equations

$$\varphi(\vec{R} = \vec{R}_{ni}) \cong U = \left[\sum_{k=1}^{N_C} \sum_{j=1}^{M_k} \frac{I_{kj}}{\Delta_{kj}} \int_{\Delta_{kj}} G_2(\vec{R}_{ni}, \vec{s}'_k) \, \mathrm{d} s'_k \right], \quad (17)$$
$$n = 1,..., N_C, \; i = 1,..., M_n.$$

In (17), Δ_{kj} and I_{kj}, $k = 1,..., N_C$, $j = 1,..., M_k$ are lengths and leakage currents of the segments on the conductors of the star-shaped electrodes' system, respectively. The solution of the system of equations (17) are total leakage currents of the segments, I_{kj}, $k = 1,..., N_C$, $j = 1,..., M_k$. The feeding current is equal to the sum of leakage currents, i.e.

$$I_g = \sum_{k=1}^{N_C} \sum_{j=1}^{M_k} I_{mn} . \quad (18)$$

The resistance of the grounding system is now

$$R_g = U / I_g . \quad (19)$$

After determining the unknown leakage currents I_{kj}, $k = 1,..., N_C$, $j = 1,..., M_k$, the potential distribution of the system from Fig. 3, has the form

$$\varphi_{1/2}(\vec{R}) = \left[\sum_{k=1}^{N_C} \sum_{j=1}^{M_k} \frac{I_{kj}}{\Delta_{kj}} \int_{\Delta_{kj}} G_{1/2}(\vec{R}, \vec{s}'_k) \, \mathrm{d} s'_k \right]. \quad (20)$$

As in (20), indexes "1" and "2" correspond to the semi-cylinder, and the surrounding domain, respectively.

III. Numerical Results

The proposed procedure is applied on the road and grounding systems realized in practice. The values of the geometrical and electrical parameters of the road, surrounding ground, and the grounding are chosen based on the ones from [4]-[6].

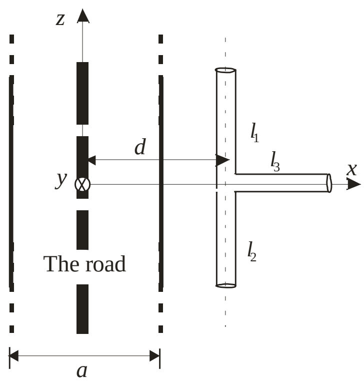

Fig.5. Three-wire grounding system

The grounding system formed of $N_C = 3$ conductors of length $l_1 = l_2 = l_3 = 5\,\mathrm{m}$, and wires' cross-sections radii $a_1 = a_2 = a_3 = 5\,\mathrm{mm}$, is analyzed in this part of the paper. Conductors are buried at depth $h = 0.8\,\mathrm{m}$, parallel to the ground surface, and along the road, Fig. 5. The dimensions of the road domain are $a = 10\,\mathrm{m}$ and $b = 4.75\,\mathrm{m}$. Using (5), we get $a_{eq} = 5.88\,\mathrm{m}$ for the equivalent radius. The system is placed at distance 3m from the boundary of the equivalent semi-cylinder, so the distance from the road axis is $d = a_{eq} + 3\,\mathrm{m} = 8.88\,\mathrm{m}$ .The described procedure is applied for $M_1 = M_2 = 25$ segments.

The normalized resistance of the grounding system from Fig. 5 versus ratio σ_2 / σ_1 is shown in Fig. 6. The normalization factor is ρ_2 / d_{nor}, where $\rho_2 = 1/\sigma_2$ is the specific resistivity of the surrounding ground and $d_{nor} = 1\,\mathrm{m}$.

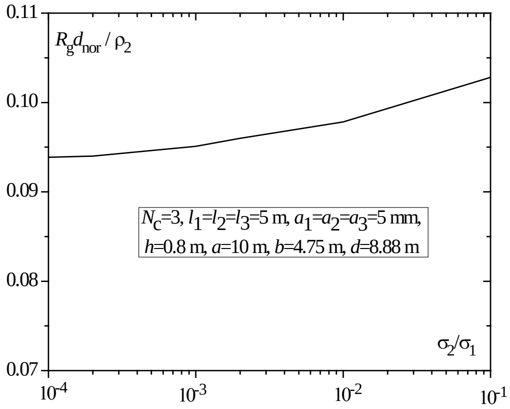

Fig.6. The normalized resistance of the grounding system from Fig. 5 versus ratio σ_2/σ_1.

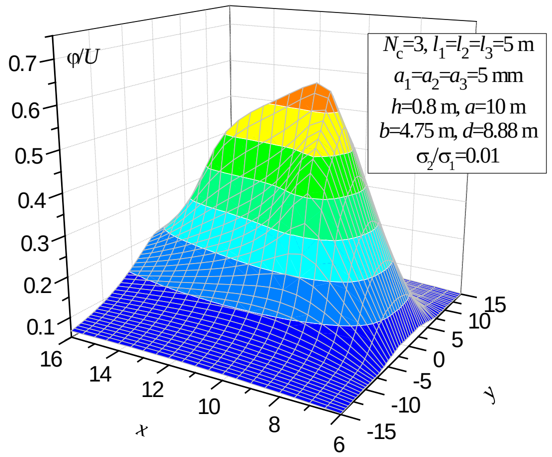

Fig.7. The normalized potential distribution on the ground surface above the electrode from Fig. 5.

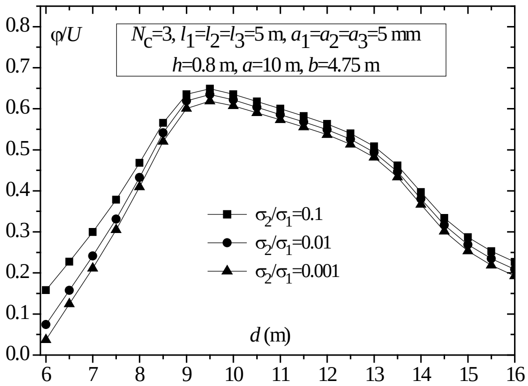

Fig.8. The normalized potential distribution of the grounding system from Fig. 5along x-axis with ratio σ_2/σ_1 as a parameter.

The normalized potential distribution on the ground surface above the ground electrode from Fig. 5 is shown in Fig. 7. The potential is normalized by the potential of the grounding system U. Values of $\sigma_1 = 0.1\,\mathrm{S/m}$ and $\sigma_2 = 0.01\,\mathrm{S/m}$ are adopted for conductivities, and the rest of the parameters are the same as in Fig. 6.

In Fig. 8, the normalized potential distribution along x-axis, i.e. for y=0 and z=0, with ratio σ_2 / σ_1 as a parameter, is presented. The rest of the parameters is the same as in Fig. 7.

The shape of the obtained functions from Figures 6-8 was expected. It is noteworthy that conductivities ratio σ_2 / σ_1 has noticeable influence on potential distribution, especially in the space between the road and the grounding system. Actually, the step-voltage is decreasing with the increase of the ratio σ_2 / σ_1.

IV. Conclusions

Asemi-analytical procedure for modeling influence of the road on the characteristics of grounding system in its vicinity is presented in this paper. It is based on the modeling of the road with a semi-cylindrical domain. The obtained results show that this influence exists for real geometrical and electrical parameters of the analyzed system, especially when the grounding system is placed close to the road, which was expected.

Proposed method considers application of the quasi-stationary Green`s function for the point source placed near the infinitely long homogenous cylinder.

The method can be applied to analysis of other types of ground inhomogeneities that can be modeled as horizontally placed cylinders or semi-cylinders (riverbeds, different kinds of ducts, etc.)

Acknowledgement

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