Modelling Self-Excitation Overvoltage in an Induction Motor With Individual Compensation of Reactive Power

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Abstract **– In this paper we have studied the problem of overvoltage appearance occuring after an induction motor with individual compensation of reactive power was disconnected from the power supply. Phenomenon of self-excitation and overvoltage appearance have been described by the appropriate system of differential equations, including important nonlinearities of machine's parameters. Mathematical model was implemented on the computer and the simulation was performed using parameters of the laboratory 1.5 kW induction motor. Experimentally obtained data are in good agreement with simulation results, thus proving that developed model can be utilized as a tool in situations when it is necessary to determine the maximum capacitance allowed for individual compensation of reactive power. Results have confirmed the opinion that for the modern design of induction motors, characterized with highly saturated magnetic core, established limits of maximum capacitance for individual compensation could be exceeded.**

Keywords – **self-excitation, overvoltage, induction motor, individual compensation, reactive power.**

I. Introduction

Individual compensation of reactive power is performed by connecting the capacitor bank directly to the terminals of a device that demands reactive power from the electrical grid. The application of this method is primarily focused on induction motors, as major consumers of reactive power. The method is very effective and inexpensive, because it does not require the use of additional equipment (contactors, fuses, units for reactive power control) which is necessary if central or group compensation is utilized. The cost of the compensation is significantly reduced in this manner, and its reliability is increased. Reactive power of the capacitor bank is usually selected to be approximately equal to the Q_0, which is the value of reactive power consumed by the induction motor at no load. This limit had been defined by the recommendations established during mid 20th century [1], and was frequently cited later in the relevant literature [2]. Limitation of the capacitor's reactive power has been primarily established in order to avoid possible problems during transient regimes. Two most dangerous phenomena described in the literature are appearance of excessive dynamic torques during re-establishment of power supply after the short interruptions, and appearance of overvoltage caused by self-excitation.

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In large systems, where great number of different induction motors operate in different time intervals, analysis of the economic feasibility of individual compensation becomes a complex problem. If compensation is performed on each of the existing machines, large amount of the installed reactive power will be out of effective use, during periods when some machines are not working. On the other hand, if individual compensation is performed only on certain machines, it will inevitably result in low cumulative power factor when uncompensated motors operate in parallel.

Recently, the method of "improved individual compensation of reactive power" appeared in the literature ([3]). It is essentially based on utilization of oversized capacitor banks for individual compensation, on a number of selected machines that are in continuous (or at least long-lasting) operation. Generated excess reactive power is spent on compensation of reactive power in machines that are not provided with their own capacitor banks. It has been shown that an appropriate choice of machines to be compensated and values of capacitors to be used, led to the cumulative effect approximately the same as if there was a regulated central compensation. In the same time, the level of investment was much lower than if regulated central compensation had been really made. This approach demands violation of recommendations for the maximum permissible reactive power of capacitor bank for individual compensation.

The experience of the authors who carried out individual compensation with oversized capacitor banks ($Q_C > Q_0$) is that the aforementioned problems almost do not exist in practice [3]. They have tried to explain such situation by the fact that magnetic core of the contemporary induction machines is far more saturated, compared to the usual design from mid 20th century. However, they have used relatively simple, graphical method, suitable for analysis of steady state conditions.

In [4], we have analysed conditions that are necessary for existence of self-excitation in an induction machine. We have also developed and experimentally verified mathematical model of voltage build-up process in a self-excited induction generator [5], considering that machine was not loaded and that there was no external voltage supply. However, the proposed model is not suitable for analysis of overvoltage that could appear as a result of disconnecting from the power grid.

In this paper, we go one step further, developing the more accurate model of the self-excitation overvoltage in an individually compensated induction motor, caused by disconnection from the power grid. The main goal of the work presented here is to help solving dilemma whether it is allowed, or not, to perform individual compensation of reactive power using capacitor bank whose reactive power exceeds the value of Q_0.

II. MATHEMATICAL MODEL

The process of self-excitation and overvoltage appearance is quite complex, and can not be fully depicted without solving mathematical model consisted of several nonlinear differential equations.

The mathematical model developed for this occasion is based on the theory of arbitrary reference frame [6]. Equivalent circuits for two orthogonal axes, *q* and *d*, are shown in Figs. 1 and 2, indicating that stationary reference frame is used. For simpler dynamic models of transient phenomena in induction machine, the most common approach is to neglect core losses, and also to consider magnetization inductivity L_m as constant. Bearing in mind that the actual goal of this model is to realistically describe the change of voltage at the terminals of the stator during the period after disconnection from the network, it is necessary to take into account the changes of mutual inductivity caused by variation of magnetization current [7]. This is done by using a nonlinear inductive element $L_m = f(I_m)$ in magnetization branches of equivalent circuits shown on Figs. 1 and 2. Also, the accuracy of voltage waveform prediction is increased by accurate modeling of iron core losses. This is performed by adding a nonlinear resistor $R_c = f(E)$ in parallel with magnetization inductivity, where E is effective value of *emf* induced in stator.

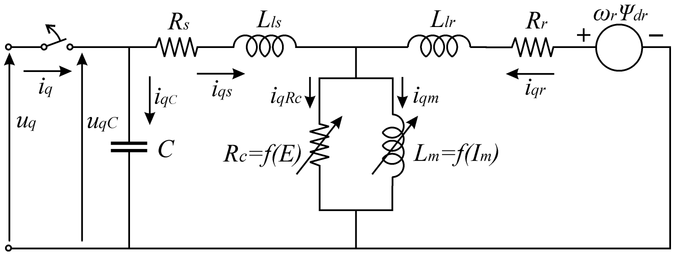

Fig. 1. Equivalent circuit for *q* axis

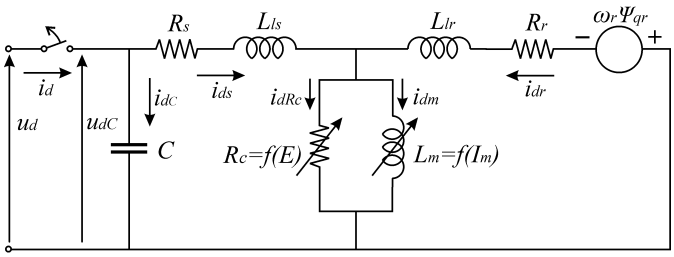

Fig. 2. Equivalent circuit for *d* axis

Regarding the circuits shown on Figs. 1 and 2, the following set of differential equations for currents can be written:

$$pi_{qs} = \frac{1}{L_{ls}}\left(u_{qC} - R_s \cdot i_{qs} - R_c \cdot i_{qRc}\right) \tag{1}$$

$$pi_{qr} = \frac{1}{L_{lr}}\left(- R_r \cdot i_{qr} - R_c \cdot i_{qRc} + \omega_r \cdot \psi_{dr}\right) \tag{2}$$

$$pi_{qRc} = \frac{1}{L_m}\left(- u_{qC} + R_s \cdot i_{qs} + L_s \cdot pi_{qs} + L_m \cdot pi_{qr}\right) \tag{3}$$

$$pi_{ds} = \frac{1}{L_{ls}}\left(u_{dC} - R_s \cdot i_{ds} - R_c \cdot i_{dRc}\right) \tag{4}$$

$$pi_{dr} = \frac{1}{L_{lr}}\left(- R_r \cdot i_{dr} - R_c \cdot i_{dRc} - \omega_r \cdot \psi_{qr}\right) \tag{5}$$

$$pi_{dRc} = \frac{1}{L_m}\left(- u_{dC} + R_s \cdot i_{ds} + L_s \cdot pi_{ds} + L_m \cdot pi_{dr}\right) \tag{6}$$

where $p = \dfrac{d}{dt}$.

One should notice that by substituting the Eqs. (1) and (2) into the Eq. (3), and also by substituting the Eqs. (4) and (5) into the Eq. (6), time derivatives of currents i_{qs}, i_{qr}, i_{ds} and i_{dr} at the right sides of Eqs. (3) and (6) can be eliminated.

The necessary flux linkages are calculated as:

$$\psi_{qr} = L_{lr} \cdot i_{qr} + L_m \cdot \left(i_{qs} + i_{qr} - i_{qRc}\right) \tag{7}$$

$$\psi_{dr} = L_{lr} \cdot i_{dr} + L_m \cdot \left(i_{ds} + i_{dr} - i_{dRc}\right) \tag{8}$$

and the electromagnetic torque is defined by:

$$T_e = \frac{3}{2} \cdot P \cdot \left(\psi_{qr} \cdot i_{dr} - \psi_{dr} \cdot i_{qr}\right) \tag{9}$$

where P represents the number of machine's pole pairs.

The electrical speed of the rotor is given as:

$$\omega_r = P \cdot \omega_m \tag{10}$$

and the mechanical speed is defined by Newton's equation:

$$p\omega_m = \frac{1}{J}\left(T_e - K_t \cdot \omega_m\right) \tag{11}$$

where K_t is constant representing friction coefficient and J is the moment of inertia.

While the machine is connected to the power supply (circuit breaker is closed), the voltage across capacitor is equal to the voltage applied to the stator, i.e. $u_{qC} = u_q$ and $u_{dC} = u_d$. After the circuit breaker is opened, the capacitor's voltage becomes an independent variable whose components along *q* and *d* axis can be calculated as:

$$u_{qC} = \frac{1}{C} \cdot \int i_{qC} \cdot dt = -\frac{1}{C} \cdot \int i_{qs} \cdot dt \tag{12}$$

$$u_{dC} = \frac{1}{C} \cdot \int i_{dC} \cdot dt = -\frac{1}{C} \cdot \int i_{ds} \cdot dt \tag{13}$$

In order to solve system of Eqs. (1)-(13), it is necessary to know actual values of parameters L_m and R_c. During each discrete time interval of the simulation, it is possible to calculate effective value of magnetization current as:

$$I_m = \frac{\sqrt{i_{qm}^2 + i_{dm}^2}}{\sqrt{2}}$$

$$= \frac{\sqrt{\left(i_{qs} + i_{qr} - i_{qRc}\right)^2 + \left(i_{ds} + i_{dr} - i_{dRc}\right)^2}}{\sqrt{2}} \tag{14}$$

and effective value of *emf* induced in stator as:

$$E = L_m \cdot \frac{\sqrt{p i_{qm}^2 + p i_{dm}^2}}{\sqrt{2}} \tag{15}$$

After that, it is easy to calculate actual values of L_m and R_c, assuming that functional dependencies $L_m = f(I_m)$ and $R_c = f(E)$ have been previously determined.

Presented system of equations can be successfully solved using *MATLAB/Simulink* solver *ode23tb*, which is designed for stiff problems.

III. Experimental Verification

The proposed mathematical model was tested using three-phase, squirrel cage, Y-connected induction machine, with rated data of $1.5\,\mathrm{kW}$, $380\,\mathrm{V}$, $50\,\mathrm{Hz}$, $3.2\mathrm{A}$, 2860 rpm. Parameters of the machine's equivalent circuit have been obtained from standard no-load and locked rotor tests, while the moment of inertia and friction coefficient have been calculated regarding the results of retardation test at no load conditions. Values are given in Table I.

TABLE I
VALUES OF THE MACHINE'S PARAMETERS

Stator resistance, R_s	4.05 $[\Omega]$
Rotor resistance, R_r	2.75 $[\Omega]$
Stator leakage inductivity, L_{ls}	0.0138 $[H]$
Rotor leakage inductivity, L_{lr}	0.0088 $[H]$
Friction coefficient, K_t	0.0017 $[Nm \cdot s \cdot rad^{-1}]$
Moment of inertia, J	0.023 $[kg \cdot m^2]$

Nonlinear function $L_m = f(I_m)$ was identified using the data obtained from no-load test that was performed for several different values of the applied voltages. It is represented as two-segment curve (see Fig. 3), defined by:

$$L_m = \begin{cases} 0.722 & \mathrm{I} \\ 0.808 - 0.35 I_m + 0.073 I_m^2 - 0.0071 I_m^3 + 2.6 \cdot 10^{-4} I_m^4 & \mathrm{II} \end{cases} \tag{16}$$

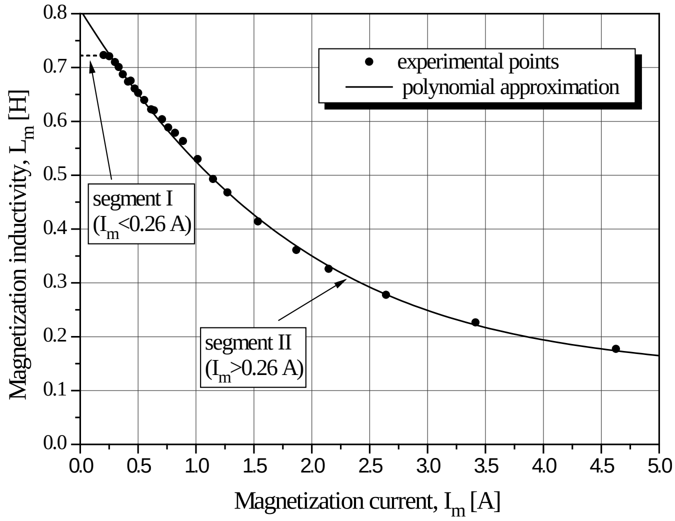

Fig. 3. Magnetization inductivity versus magnetization current

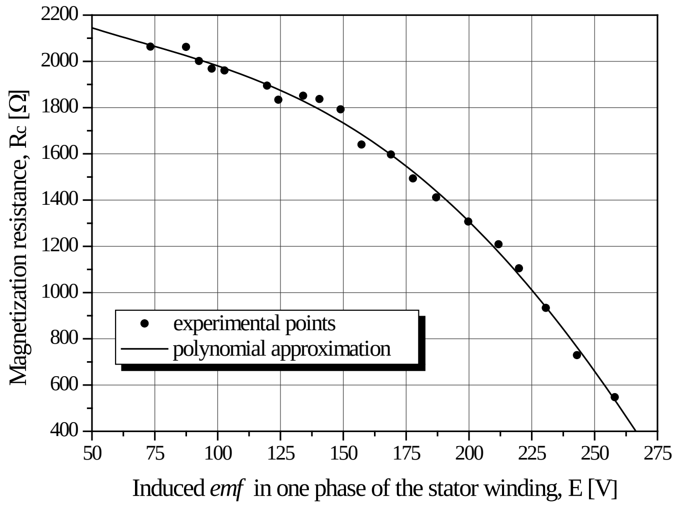

Fig. 4. Magnetization resistance versus induced *emf*

Value of variable resistor $R_c = f(E)$, placed in the magnetization branch, is defined using the 4th degree polynomial approximation (see Fig. 4) defined as:

$$R_c = 2379 - 6.7E + 0.055E^2 - 0.00031E^3 + 3.7 \cdot 10^{-7} E^4 \tag{17}$$

Model was experimentally tested using the symmetrical three-phase capacitor bank with rated capacitance of $C = 35\mu F$ per phase, connected directly to the stator's terminals. The test machine demands reactive power of $Q_0 = 1210 VAr$ when it operates at no load and with rated voltage applied on the stator. It is clear that the power of the used capacitor bank, $Q_C = 3 \cdot \omega \cdot C \cdot U_f^2 = 1588 VAr$, exceeded the limit defined in the literature, because the ratio of $Q_C / Q_0 = 1.31$ was obtained. Machine was at no load operation, and the waveform of stator's voltage was recorded before, and shortly after disconnection from the power supply. Recorded waveform is shown in Fig. 5.

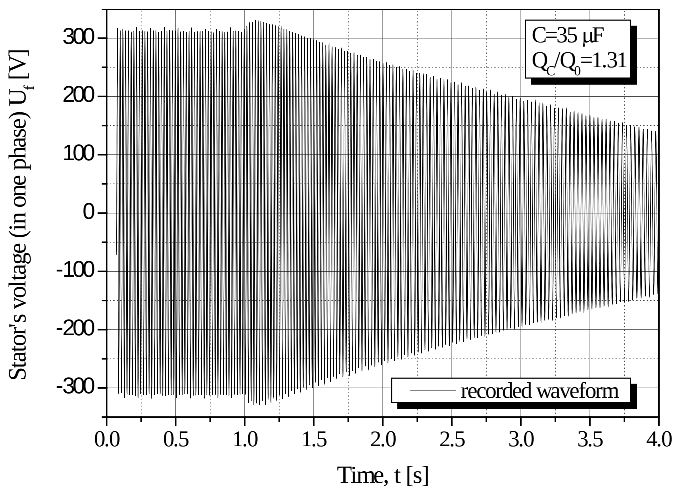

Fig. 5. Recorded waveform of stator's voltage after disconnection

The developed mathematical model was used for computer simulation of stator's voltage behavior after machine was disconnected from the grid. Calculated waveform is presented in Fig. 6, and it is in good agreement with experimental result.

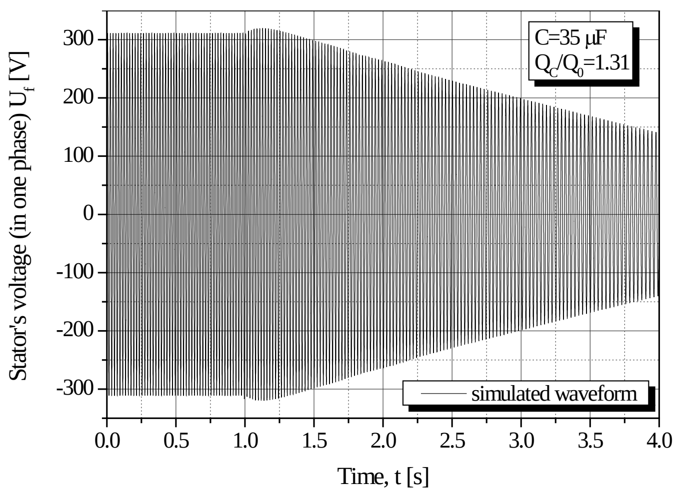

Fig. 6. Simulated waveform of stator's voltage after disconnection

Observing the results shown in Figs. 5 and 6, one can see that the maximum value of the self-excitation overvoltage is only about 7 % higher than the stator's rated voltage, although the capacitor bank with $Q_C / Q_0 = 1.31$ was used. This result should not be generalized, because the overvoltage may vary from one induction machine to another. However, if the motor parameters are known, and if the main nonlinearities describing magnetic circuit have been successfully identified, it is possible to make better choice of capacitors for individual compensation, using this model. It is very important to accurately represent functional dependence of mutual inductivity between stator and rotor (magnetization inductivity) versus magnetizing current, otherwise the obtained results will not be realistic. Considering the fact that magnetic circuits of modern induction machines usually have heavily saturated magnetization characteristics, results justify the view that nowadays individual compensation can be performed using capacitor bank whose power exceeds value of Q_0, as it has been suggested in [3].

IV. Conclusion

Developed model gives realistic image of self-excitation overvoltage occurring at the stator of an induction machine with individual compensation of reactive power. Simulation results are in good agreement with experimentally recorded values. Therefore, the model can be exploited as a useful additional tool during the selection of the capacitors for individual compensation of reactive power. Although the model was tested on the example of the small induction motor with rated power of 1.5 kW, it has a universal character and can be also used for prediction of overvoltages in large induction motors.

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