Wave Digital Approach in Characterization of T-junction Discontinuity in Microstrip Stub-line Structures

Biljana P. Stoģiĺ

Abstract – **The purpose of this paper is to show that the one-dimensional wave digital approach can be used in analysis of stub-line structures. A stub-line structure, divided into uniform sections, can be efficiently modeled by wave digital network. Frequency response is obtained by direct analysis of formed block-based network. Also, a complete understanding of microstrip circuits requires characterization of T-junction discontinuities included in the circuits. Two different approaches to the modeling of the equivalent network of T-junction discontinuity are presented in this paper. Open T-resonator circuit, proving the response accuracy of the new technique, is given.**

Keywords – **Wave digital approach, wave digital networks, microstrip circuits, stub-line structures, T-resonator circuit.**

I. Introduction

The modeling of the planar structures by wave digital elements is based on well known theory of wave digital filters (WDFs). A detailed review of WDF theory is given in references [1-4]. A large variety of WDF-based techniques has been developed for a wide range of applications [2]. Recently, great effort has been placed on modeling of microwave planar structures are modeled by one-dimensional (1D) [5-8], [10-14], and by two-dimensional (2D) [5], [9] wave digital elements (WDEs). Standard WDEs, such as delay, adder, multiplier and adaptor, are used in design of wave digital networks (WDNs). The reader is refered to [7], [10-13] for a thorough introduction to WDNs.

Microstrip lines are widely used in microwave and RF circuits due to its superior compatibility to the circuit environment for easy integration and manufacturing. In practice, there does not essentially exist just one type of microstrip structures, but a whole variety of quite distinct subclasses (stepped-impedance filters, linearly tapered nonuniform transmission lines, stub-line structures, elliptic filters), each of which can again be divided into varies families, etc. This reflects the richness of WDNs, and the most appropriate one have to be chosen for structure at hand.

In the previously published paper [9], microstrip structure with stub is analyzed by use of 2D wave digital approach. In the paper [14], the authors have had mainly deal with 1D wave digital approach. In fact, in view of their geometry, stub-line structures are also natural candidates for 1D approach. A way for connecting segments in the structure with stubs is also developed and also represented in the Section II in order to address both readers with and without background in 1D wave digital approach.

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Microstrip circuits inevitably incorporate transmission line discontinuities of one type or another. A complete understanding of microstrip circuits requires characterization of various discontinuities included in the circuit. Two different approaches to the modeling of the equivalent network of T-junction discontinuity are described in the Section III.

In the Section IV, simulation validation of the proposed modeling approach is provided by means of an open T-resonator circuit.

The main contribution of this paper is developed wave digital network model for modeling the equivalent circuit of T-junction discontinuity. The equivalent circuit elements can be determined by closed-form expressions or by carrying out EM simulations. Here, it is shown how WDN model can be derived starting with multiport capacitor in parallel branch [9].

II. Model of the Structure with Stub

In the paper [14], one-dimensional wave digital approach is shown to be suitable for the blocked-based analysis of microstrip stub-line structures.

The formed Simulink model of WDN given in [14] is depicted in Fig. 1. The blocks **TLine_1**, **Stub_2** and **TLine_3** represent uniform transmission line (UTL) segments. The blocks **ADP-S** and **ADP-L** represent two-port series adaptors, and block **ADP_T1S2T3** three-port parallel adaptor with port 2 being dependent.

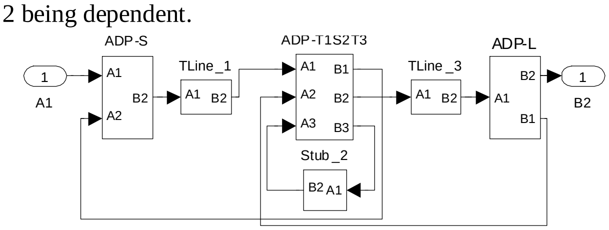

Fig. 1. Simulink model of the structure

III. Modeling of the T-junction Discontinuity

Here, only discontinuities associated with the T-pattern resonator, such as open end and T-junction, are considered. Since discontinuity dimensions are usually much smaller that the wavelength in a microstrip, they can be modeled by lumped-element equivalent ciruits.

A T-junction discontinuity is very offten used in the microstrip circuits, such as microstrip stubs, power dividers, etc [15].

An open-end discontinuity occurs frequently in a number of circuits such as resonators, matching stubs, filters, and microstrip antennas. A closed form expressions for calculating the excess length of transmission line [16] are used here.

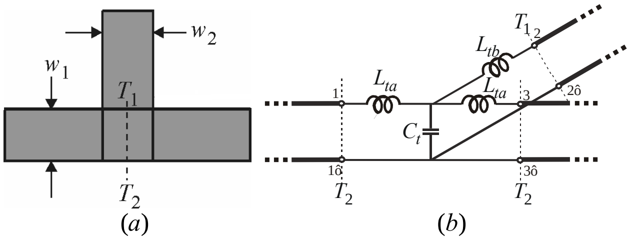

Fig. 2. (*a*) Symmetric T-junction, and
(*b*) equivalent network of the discontinuity

For a symmetric T-junction structure depicted in Fig. 2*a*, the equivalent network is shown in Fig. 2*b*. It consists of the inductances L_{ta} and L_{tb} in the series branches, and the parallel capacitance C_s. The inductances and capacitance can be obtained by closed form expressions given in the papers [3-4]. Here, two different ways for modeling the equivalent circuit of the T-junction discontinuity are given. The modeling procedures are based on:

- decreasing line lengths, and
- a wave digital element.

A. Modeling by Decreasing Line Lengths

The simplest modeling approach of the equivalent discontinuity network involves changing line lengths. In order to absorbe discontinuity effects a new line lengts are counted. Corrections for T-junction effects are done as follows: the physical length of the series line which length is l_1 and width w_1 is decreased by $w_2/2$ value, i.e. it is decreased by one half of width of the stub line. The physical length of the stub line with length l_2 and width w_2 is decreased by $w_1/2$ value, i.e. it is decreased by one half of width of the series line [17].

The WDN of a T-resonator circuit with discontinuities is obtained in a few steps. First, new lengths of the structure segments are calculated. Then, the procedure described in the section II can obviously also be applied to the obtained structure. Finally, WDN of a nonuniform planar microstrip structure is the same as that one shown in Fig. 1. A number of sections in UTL segments differs from the previous one because the segments lengths are being changed.

This approach of modeling tee-junction discontinuity is very acceptable because there are not new blocks here. In the case of a microstrip stub-line structure the proposed method reduces drastically the computation time while giving acceptable accuracy.

B. Wave Digital Element - Direct Realization of the LC Elements of the Equivalent Discontinuity Network

In the other modeling approaches of the step discontinuity, new blocks have to be included in the WDN. A WDN depicted in Fig. 3 consists of three types of building blocks (*UE*, two-port adaptors and *WDE_Tjunction* element).

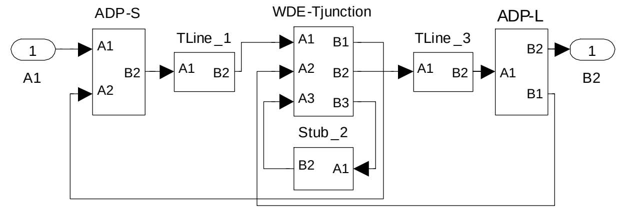

Fig. 3. Simulink model of the structure with discontinuities being modeled by block

As has been shown previously, T-junction equivalent network is approximated by inductors in the series branches and capacitor in the parallel branch. A starting point in realization of **WDE_Tjunction** element is the four-port network containing capacitor in parallel branch, Fig. 4.

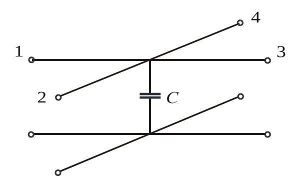

Fig. 4. A four-port C analog network

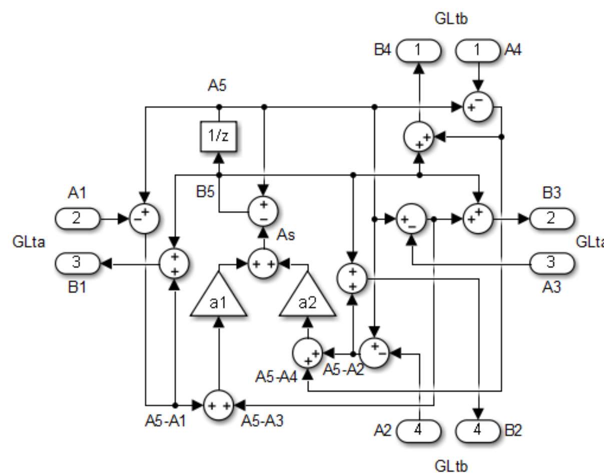

Fig. 5. Symmetrical four-port wave digital network of capacitor

The five-port parallel adaptor with dependent port five is used for modeling capacitor in parallel branch. In this way the four-port wave digital network of the capacitor in parallel branch (WC_Cen), shown in Fig. 5, is obtained [1-3]. The WC_Cen is symmetrical because of $L_1 = L_3$ and $L_2 = L_4$ for homogenous structures. This network corresponds to centrally placed segments. For this wave digital network, the equations for wave variables are

$$A_5 = z^{-1} \cdot B_5, \quad (3)$$

$$A_S = \alpha_1(2A_5 - A_1 - A_3) + \alpha_2(2A_5 - A_2 - A_4), \quad (4)$$

$$B_5 = A_5 - A_S, \quad (5)$$

$$B_k = B_5 + A_5 - A_k, \quad k = 1,2,\cdots,4 \quad (6)$$

where A_k and B_k are incident and reflected wave variables.

The multiplier coefficients can be expressed as follows

$$\alpha_1 = \frac{2 \cdot GL_{ta}}{2 \cdot GL_{ta} + 2 \cdot GL_{tb} + GC} \quad (7)$$

and

$$\check{U}_2 = \frac{2 \cdot GL_{tb}}{2 \cdot GL_{ta} + 2 \cdot GL_{tb} + GC}, \quad (8)$$

where GL_{ta} and GL_{tb} are port conductances.

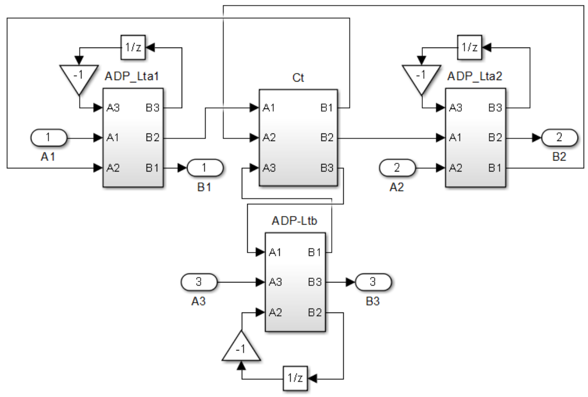

Fig. 6. *Simulink* model of **WDE_Tjunction** block

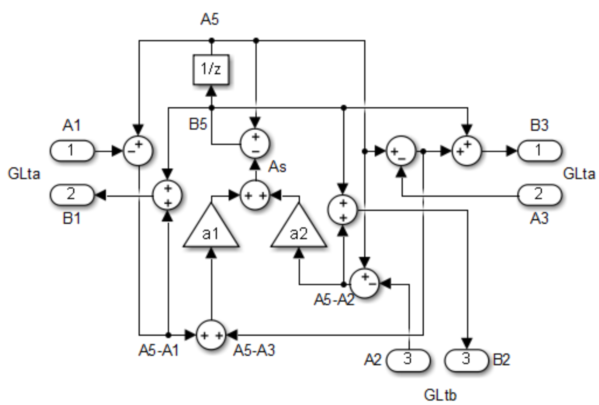

Fig. 7. *Simulink* WDN model of **WCt** block

The wave digital element of T-junction (**WDE_Tjunction** block) is shown in Fig. 6. It consists of one WCt element and three three-port series adaptors with wave digital elements of inductors (a single delay and a multiplier with coefficient -1) placed on their dependent ports. WDN of WCt element is given in Fig. 7. The equations for wave variables in *z*-domain are

$$B_k = \frac{(1+z^{-1})[\alpha_1(A_1+A_3)+\alpha_2 A_2)]}{1-z^{-1}[1-2\alpha_1-\alpha_2]} - A_k, \quad k=1,2,3, \quad (9)$$

and in *t*-domain are

$$\begin{aligned} b_k(n) &= \alpha_1[a_1(n)+a_3(n)+a_1(n-1)+a_3(n-1)]+ \\ &+\alpha_2[a_2(n)+a_2(n-1)]-a_k(n)+[1-2\alpha_1-2\alpha_2)]\cdot \quad (10) \\ &\cdot[a_k(n-1)+b_k(n-1)] \end{aligned}$$

where $n=0,1,2,\ldots$, $a_k(-1)=b_k(-1)=0$ and $k=1,2,3$.

IV. T-Resonator Circuit Example

A microstrip structure with a single stub, depicted in Fig. *2a*, is used for verification of the proposed method. The structure is fabricated on CuFlon substrate with dielectric constant $\varepsilon_r = 2.17$ and width $h = 0.508\,mm$.

TABLE I
Transmission Line Parameters without Modeled Discontinuity

nv	d [mm]	w [mm]	Zc [Ohm]	Tv [ps]
1	30.0000	4.7100	21.9255	140.4001
2	30.0000	15.7600	7.5216	144.2831
3	30.0000	4.7100	21.9255	140.4001

TABLE II
Transmission Line Parameters with Modeled Discontinuity

nv	d [mm]	w [mm]	Zc [Ohm]	Tv [ps]
1	22.1200	4.7100	21.9255	103.5217
2	27.9196	15.7600	7.5216	134.2776
3	22.1200	4.7100	21.9255	103.5217

T-resonator circuit is approximated by connection of 3 transmission lines with parameters given in Tables I and II. In this case, T-junction discontinuity is modeled by decreasing lengths of the line in the junction. The effect of the open stub is compensated by increasing length of the segment UTL2 [16].

A minimal number of sections for the given error, can be found as described in the papers [8] and [9] where multiple factor $q \geq 1$ is used. In the case where discontinuity is being modeled, for given error $n\_er = 0.01\%$, a total minimal number of sections in *WDN* is $n_t = \sum_{k=1}^{3} n_k = 122$. For segments UTL1 and UTL3, a number of sections n_k is 37, and for segment UTL2 is 48. A total delay for the digital model of the structure is $T_t = n_t \cdot T_{\min} / q = 341.3209\,ps$ where $q = 37$ is a multiple factor and $T_{\min} = \min\{T_1, T_2, T_3\} = 103.5217\,ps$ is a minimum delay. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{3} T_k = 341.3418\,ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = n_t / T_t = 357.4130\,GHz$. In this case, a relative error of delay is $er = -0.0061\%$. Adaptor coefficients are $\alpha_S = -\alpha_L = 0.3903$ and $\alpha_1 = \alpha_2 = 0.4069$.

The S_{21} parameters of the structure at the frequencies from $300\,MHz$ to $6\,GHz$ are shown in Fig. 8. The transmission parameter was measured by a network analyser. A very good agreement between the simulated results was also obtained for many other structures of different shapes with T-junctions, which are not presented here. The measured result shows a small difference of the resonance frequency in comparison with the simulated result. This difference is caused by the microstrip T-junction and the open-end, which are not described accurate enough by the models. This may be due to the closed form expressions which are used.

According to these findings, a simple 1D wave digital approach can produce results that are similar to the much more sophisticated methods. The simulation showed that the discontinuities of the microstrip T-resonator structure had a great effect on the resonance frequencies.

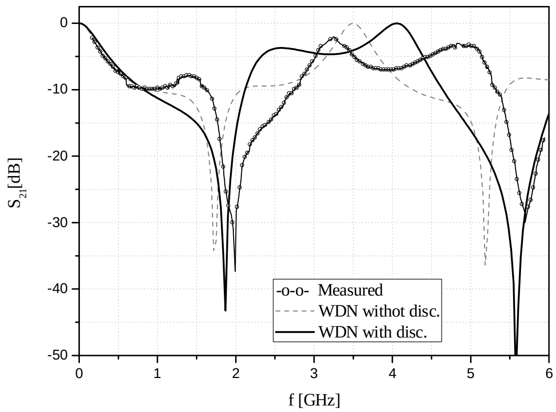

Fig. 8. The measured and simulated transmision parameters of the open T-resonator circuit

V. Conclusion

The primary purposes of this paper are twofold.

The first objective is to give an original and general method to characterize the behaviour of microstrip stub-line structures in both the frequency and time domains. A very simple block-diagram method of analysis of the WDN is used here. WDN is formed directly in the Simulink toolbox of the MATLAB environment. Frequency response is obtained by direct analysis of formed block-based wave digital network.

The second objective, but not the least, is the study of discontinuities associated with the T-pattern resonator. A simplified approach has been established to study the characterization of the microstrip discontinuities (open-end stubs and tee-junction). Two different ways of modeling T-junction discontinuity are given. Also, WDN model of the equivalent multiport circuit of T-junction is developed.

The broadband accuracy of the suggested procedure is validated by two examples realized in the microstrip line technique, such as an open T-resonator circuit. The results of the analysis obtained by the WDN have shown a very good agreement with those obtained by other programs mentioned above. The results of one T-resonator are compared with measurements performed by Vector Network Analyzer in the frequency range $300\,MHz$ - $6\,GHz$.

The proposed approach can be used by microwave engineers because of the associated computational accuracy. It can be used for analyzing the transmission lines of various nonuniform shapes present in practice.

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