Software Model for Optimization of Infill Balise Location in ETCS, Level 1 in Railway Transport

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***Abstract* – A drawback of railway ETCS, L1 is that a change of the signal aspect of a signal cannot be transmitted dynamically in the on board system. For this reason infill balises are used. Their location is a matter of discussion. A mathematical model is demonstrated here, which allows to optimize the location of infill balises, as the time gained from unnecessary halting and accelerating must be maximal, under certain limit conditions.**

Keywords – **ETCS L1, infill, braking curves calculation.**

I. Introduction

In many countries worldwide, including beyond the borders of the EU, the European standardized system for automatic train protection ETCS, Level 1, is implemented. With this system of the so-called balises, located along the railway, information about the signal aspects of the signals is transmitted to the locomotive. Usually, the signal aspect shows two velocities – the one at which the current signal should be passed and the one at which the next signal should be passed. When the signal aspect of the signal changes to more permissive after the locomotive has already passed the previous signal, there is no way for the locomotive equipment to update its information because the previous signal balise has already been passed. Thus, the locomotive is compelled to move at a lower velocity than that actually prescribed, and in some cases even halt, if the aspect before its updating has been "stop." This leads to unnecessary braking and accelerations, resulting in longer travelling time and power loss. In order to avoid such losses, the so-called infill balise is installed before some signals. The infill balise location is disputable and depends on the velocities at which the train passes the previous signal, the distance to it, and differs for the various railway administrations. In order to determine the most appropriate location of the infill balise, a software product is developed for automatic calculation of the infill balise location based on various input data. It is based on a number of analytical calculation methods from the area of mathematics, physics and safety systems in the field of railway transport. Various additional data are also calculated, which are useful for designing the section in which the infill balise is to be located. The product has a graphic interface facilitating the work with it and data visualization.

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II. Principles of Building the Mathematical Model

As in every mathematical model, some approximations are accepted here as well, sufficiently close to the real-life situations, allowing the use of standard mathematical and physical functions.

The basic adopted approximation is that braking and accelerating curves are parabolas whose vertex M is on the ordinate axis (Fig. 1). This type of parabola is the basis of the approximations of the given model and is preset with the following dependences:

- parabola equation : $y(x) = ax^2 + c$; (1)

roots: x_1, x_2 as $x_1 = -x_2$

parametre a – always negative $(a < 0)$

parametre c – always positive $(c > 0)$

vertex M with coordinates $M(0,c)$;

discriminant $D = b^2 - 4ac$ => $D = -4ac$ always $D > 0$

- Vieta's formulae

$$x_1 + x_2 = -\frac{b}{a}; \qquad x_1 x_2 = \frac{c}{a}$$

If the linear relocation l is taken into account, the function will appear as follows:

$$y_1(x) = a(x - l)^2 + c.$$

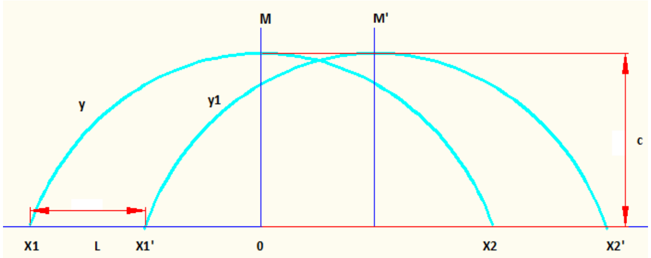

Fig. 1. Graphical view of equations

III. Modeling the Braking and Accelerating Behaviour

A. Braking curve

The braking curve is approximated as part of a parabola and calculated as a velocity function of the way– $V = f(s) = as^2 + c$, as V indicates the instantaneous velocity in a point of the way S. For easier perception of mathematical calculations the above formula is used – $f(x) = ax^2 + c$. The initial point of the coordinate system $O(0,0)$

coincides with the location of the so-called warning signal (*Пс*). The target point or the point where the train must stop is the entry signal (*Вх*) – point x_2 – the greater positive root in the equation. The value of the coefficient c is identical with the velocity at which the train enters at the warning signal. The position of the warning signal, entry signal and entry velocity is determined based on traction estimates which are not subject of this study and are accepted as preset. Due to the specificity of the subject matter, the measurement units are respectively preset as S – way [m], V – velocity [km/h], t – time [s].

In calculating the curve, two main problems are solved:

- to calculate the quadratic function of the parabola for preset location of the warning and entry signal and entry velocity at the warning signal;
- to calculate the time at each point of the way - $t = f(s)$ for preset location of the warning and entry signal and entry velocity at the warning signal,.

After the respective transformations and derivations, the analytical expressions for the sought curves are obtained. The graph of the time function is of logarithmic type and is shown in Fig.2. The graph of the velocity function is shown in Fig.3.

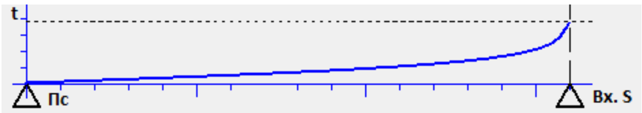

Fig. 2. Deceleration time function

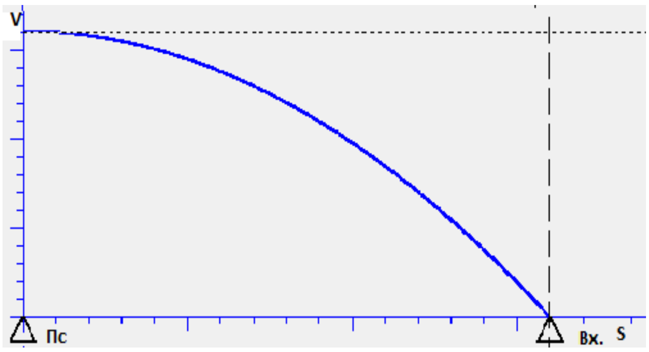

Fig.3. Deceleration velocity function

B. Acceleration curve

The acceleration curve is approximated as a part of a parabola (Fig.3 quadrant II) and is calculated as a velocity function of the way – $V = f(s) = as^2 + c$ as by V we indicate the instantaneous velocity in a point of the way S. For easier perceiving of the mathematical calculations, we will use the symbols generally accepted in mathematics – $f(x) = ax^2 + c$. We perceive as an initial point of the coordinate system $O(0,0)$ as the location of the warning signal. Due to the fact that the acceleration curve (unlike the part of the primary parabola located in quadrant II) is situated in quadrant I, and its initial point is located after the warning signal, in the curve equation a horizontal relocation is added relative to the primary parabola (Fig.1). Therefore, the equations of the acceleration curve are accordingly transformed: – $V = f(s) = a(s - l)2 + c$ and $f(x) = a(x - l)^2 + c$. The parametre l is always greater than 0 and greater than the distance between the warning and entry signal, i.e., $(l > x_2)$ (Fig.1).

The maximum velocity of the acceleration curve is determined by the specific situation and is accepted as preset. Due to the specificity of the subject matter, the measurement units are preset accordingly: S – way [m], V – velocity [km/h], t – time [s]. Upon calculation of the acceleration curve two main problems are solved:

- to calculate the parabola's parametres for preset location of the warning and entry signal, maximum acceleration velocity, starting acceleration point of $0\ km/h$ and preset maximum acceleration;
- to calculate the instant in each single point – $t = f(s)$ for preset location of the warning and entry signal, the maximum acceleration velocity, starting acceleration point of $0\ km/h$ and preset maximum acceleration.

After the respective transformations and derivations, the analytical expression for the sought curves and drawn graphs (Fig.4) are obtained. The graph of the time function is of logarithmic type.

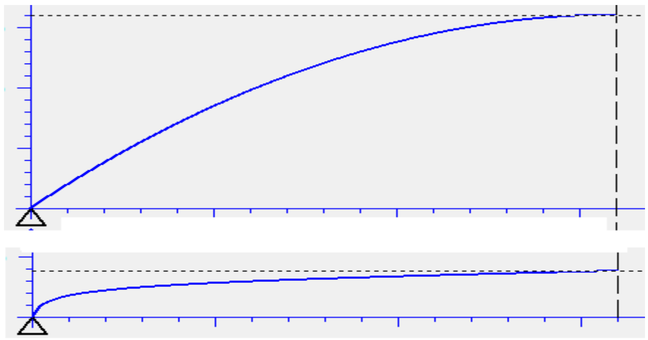

Fig.4. Acceleration, velocity and time functions

IV. Determining the Infill Balise Location

A. Movement with constant acceleration and with constant deceleration

Here we will provide some basic formulae and graphs for movement with constant deceleration (Fig.5) and with constant acceleration (Fig.6), reworked for specific purposes, which are used in further calculations. The following symbols are used in the figures:

- initial velocity – u ;
- final velocity – v ;
- travelled way – ΔS (Δx) ;

- acceleration – a ;
- time for travelling a way with definite initial and final velocity – Δt
- average velocity $-v_{avg} = \dfrac{u+v}{2}$

where:

$u < v$ – with constant acceleration $a = const > 0$; $u > v$ with constant deceleration $a = cost < 0$;

$u = v - a = 0$;

<u>Main equation</u>:

(2)

<u>Travelled way</u>:

$$\Delta x = V_{avg} \times \Delta t \qquad (3.a)$$

$$\Delta x = u \times \Delta t + \frac{1}{2} a \Delta t^2 \qquad (3.b)$$

$$\Delta x = \frac{v^2 - u^2}{2a} \qquad (3.c)$$

<u>Acceleration</u>:

$$a = \frac{v^2 - u^2}{2 \times \Delta x} \qquad (4)$$

<u>Time</u>:

$$\Delta t = \frac{\Delta x}{V_{avg}} = \frac{2 \times \Delta x}{u + v} \qquad (5)$$

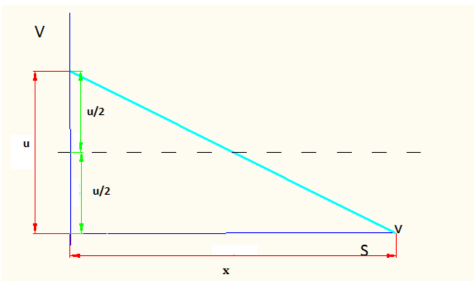

Fig.5. Average velocity calculation

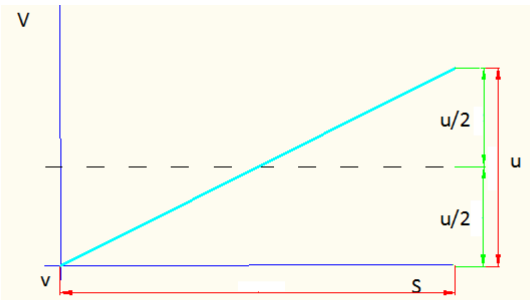

Fig. 6. Average velocity calculation

B. Determining the infill balise location

In railway transport the function of train velocity of the way – $V = f(s)$ is considered as basis for initial calculations. Time is a determining factor in calculating the dependences between the separate elements of the law of motion: travelled distance, velocity, acceleration. That is why in creating the mathematical model of great significance is to find the dependences of the way, velocity and acceleration on time. Upon preset law (equation) $V = f(s)$ are calculated the functions (equations) $S = s(t)$, $V = v(t)$, $A = a(t)$, where S – way, V – velocity, A – acceleration, as velocity is the first derivative from $s(t)$, and the acceleration is the second derivative from $s(t)$, i.e.,

$V = s'(t)$, $\quad A = v'(t)$, $\quad A = s''$

The location of the **infill balise** is determined by two main factors, which are related in inversely proportional dependence. Both factors are based on the time t_1, for which the train travels the distance from the warning signal to the **infill balise and the time t_2, for which the train travels the distance from the infill balise to the entry signal. Significant impact is exerted by the human factor, and yet it is very difficult or impossible to take this into account mathematically, and therefore, it is neglected in calculations, save in some very specific cases. The first factor is the probability for the aspect at the entry signal to change for the time t_1 . The closer** to the entry signal an **infill balise is, the greater** the probability for the change to be read, and to respond to it appropriately, will be. The second factor is the gained time and power upon timely detecting of the aspect's change. **The closer** to the warning signal an **infill balise is located, the greater the** gain of time and energy will be. Several methods **adopted in global practice** exist for calculation of the **infill balise** position**.**

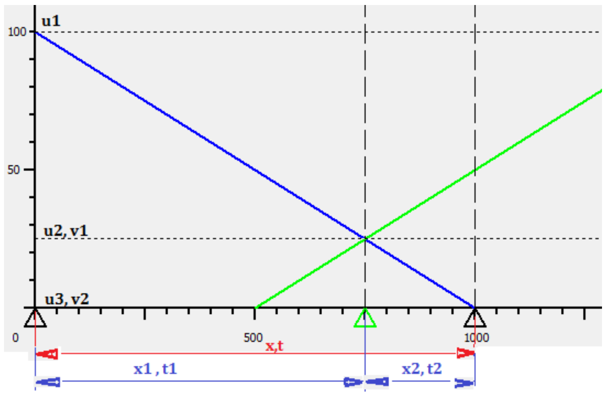

Fig.7. Theoretical probability calculation

The most widely spread method (that may as well be defined as classical) is based on the movement with constant deceleration (acceleration) (Fig.7). According to it, the probability for the aspect of the entry signal to change when the train is between the warning signal and the **infill balise and the probability** for the aspect of the entry signal to change when the train is between the **infill balise and the entry signal must be equal. This means that** $t_1 = t_2 = \frac{1}{2} t$.

Upon detailed development of the kinematic formulae it is proved that, irrespective of the entry velocity at the warning signal and the distance between the warning signal and the entry signal the ratio $\frac{x_1}{x_2} = 3$ is preserved. The pros of the method are: easy mathematical interpretation, constant $\frac{x_1}{x_2}$, the movement is with constant deceleration.

In real-life conditions (the movement is parabolic) far greater error occurs. The probability for the aspect of the entry signal to change when the train is between the warning signal and the **infill balise and the** probability for the aspect of the entry signal to change is constant $\frac{t_1}{t} = \frac{t_2}{t} = \frac{1}{2}$.

The second method is based on the same theoretical postulates as the one above, but it uses parabolic interpretation in calculating. The mathematical justification is very complicated and is very seldom used in real calculations.

The third method (Fig.8) is based on the assumption that the most optimal position of the infill balise is at $t_1 = t_2$, i.e., the time for which the train traverses curve $v_1 - v_2$ equals the time for which it traverses curve $v_2 - v_3$. The method is comparatively new. It has a complex mathematical interpretation.

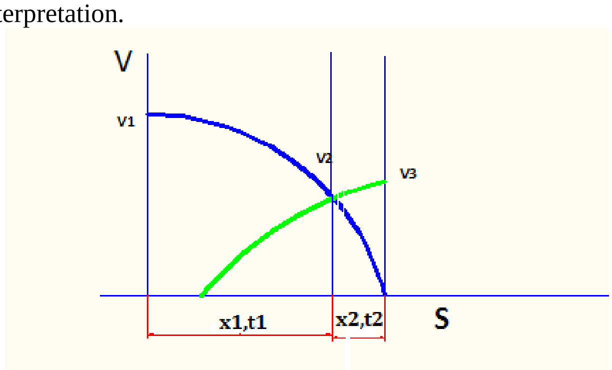

Fig.8. Real probability calculation

A **graphical – analytical** method that is very flexible has been elaborated and proposed. In this method movement is approximated as a parabola (variable acceleration). There are four variable parametres in place for determining the input parametres and an abundant "toolbox" for calculation of various output parametres which are tremendously useful to the designer in considering a specific situation. The relevant software is developed, with the help of which every change in the input parametres is visualized immediately and enables the designer to assess the optimal position of the infill balise in compliance with the requirements of the terms of reference and the allowed tolerances, both for input and for output data. This method also provides a possibility for analysis of the already existing situation and its optimization with minimum changes and expenses. As is widely known, the economic factor becomes increasingly significant in contemporary developments, which is why special attention should be given to reducing the consumption of time and power and to increasing the track carrying capacity of a particular section, and also to reducing one-off investments in facilities on account of design optimization.

Graphical interface is developed for visualization of the method. Upon correct selection of the user data a screen appears, similar to that in Fig. 9.

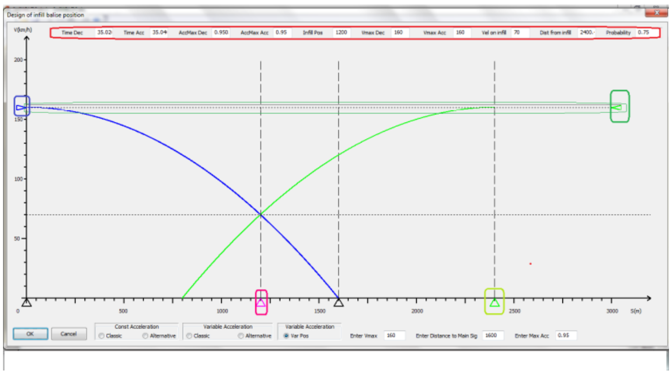

Fig.9. Flexibility calculation of infill balise position

On the screen are displayed the braking curve and the acceleration curve. The position of infill balise Δ is calculated via the classical parabola method. The parabolas are calculated and drawn according to the initial input data. In the upper part of the screen (red) fields with some output data are shown.

V. Conclusion

As a result of the performed analyses for description of the movement of trains between the warning and entry colour light signal a selection is made of the most appropriate interpretation of this movement. Mathematical and software model is developed and a method is proposed for determining the location of the infill balise, depending on the position of the infill balise Δ, the position at the end of the acceleration Δ, the entry velocity at the warning signal (0 ÷ 210), the maximum acceleration velocity (0 ÷ 210). With the created graphic interface the input data may be smoothly changed, as they are calculated forthwith and appear in the output data fields.

References

[1] Vishy Shanker, The Ideal Positioning of Infill Balises, IRSE News, ISSUE 172, Nov.2011

[2] David Glendinning, Graeme Stainlay, ETCS Revealed – The Railcorp Expereince, IRSE Technical Meeting “All about ATP”, Sydney, March, 2008.

[3] Tractive effort, acceleration and braking, The Mathematical Association, 2004