Comparative Analysis of the MSD and MSDM Watermarking Algorithms Based on the Schur Decomposition

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Abstract –**The first part of the paper describes the SDM watermarking algorithm, based on the Schur decomposition, and the MDB algorithm for detection and removal of impulse noise. The SDM algorithm has been modified in the part for watermark extraction. In the second part of the paper, proposed algorithm is tested, for resistance on superimposed impulse noise and filtering with the MDB algorithm. Testing results are presented in tables and in the graphs.**

Keywords – **Schur decomposition, watermarking, impulse noise, noise detection, filtering.**

I. INTRODUCTION

The widespread use of computer networks leads to intensive exchange of multimedia data (pictures, video, audio,...). However, copyright protection and proof of ownership is a big problem in distribution of digital images. In order to solve these problems, a principle is used of inserting visible or invisible information in the image with aim to prove ownership of the author, i.e. distributor. Inserting of hidden information in the digital image with the aim to prove ownership rights is called a digital watermark [1], [2]. The basic characteristics of digital watermark are insensitivity, robustness, capacity, non-inverse and ability to provide positive proof of ownership [3]. For inserting a watermark in the image, many transformations could be used such as DCT [3], [4] (*Discrete Cosine Transform*), DWT [5], [6] (*Discrete Wavelet Transform*), SVD (*Singular Value Decomposition*) transformation [2], [7], and Schur Decomposition. Insertion of the watermark by applying Schur decomposition can be performed for inserting the watermark into unitary matrix [8] (SD watermarking algorithm) and inserting the watermark in the upper triangular matrix [9] (SDM watermarking algorithm).

In this paper a modification of SDM algorithm for inserting a watermark has been done [9] with aim to increase efficiency in extracting the watermark. In addition, an analysis of the efficiency of the algorithm in conditions with existence of impulse noise has been done. On watermarked image, impulse noise, salt and pepper, are superimposed. From watermarked image with superimposed impulse noise a watermark has been extracted. Algorithm for detecting and removing impulse noise is applied on watermarked image with superimposed noise. From filtered image a watermark has been extracted. Based on the obtained results which are presented in tables and graphs, a comparative analyze was made with results obtained in the paper [10]. As a measure of quality, MSE is applied (Mean-Square Error).

The paper is organized in the following way: In section II Schur decomposition was described and MSDM algorithm for inserting and extracking the watermark, based on it. In section III are presented the obtained results and analyze of results. Conclusion is given in section IV.

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II. ALGORITHM

A. Shur decomposition

For apply Schur decomposition, image $A_{M\times N}$ is divided on blocs, $H_{Mb\times Nb}$. Schur decomposition is performed over each blocs:

$$A = U \times D \times U', \quad (1)$$

where $U_{Mb\times Nb}$ is unitary matrix and $D_{Mb\times Nb}$ is upper triangular matrix. In the paper [10] MSD watermarking algorithm has been presented, that is performed by inserting the watermark into a unitary matrix *U*. In the paper [9] SDM watermarking algorithm is presented, in which the insertion of one bit of the watermark is performed in the diagonal of the upper triangular matrix *D*. In this paper the efficiency of the algorithm in terms of SDM superimposed impulse noise has analyzed. In addition to this, over an image in which watermark is inserted and superimposed impulse noise, a DB algorithm for removing impulse noise has been applied, which was proposed in the paper [11] and which is modified from authors of this paper in order to increase the efficiency of detecting the infected pixels (MDB algorithm) [12]. The following part of the paper describes the SDM algorithm from [9] and its modification (MSDM).

B. SDM algorithm

SDM watermarking algorithm for inserting watermark is realized trough the following steps:

Input: original image $A_{M\times N}$, binary watermark $w_{\mathrm{Mz\times Nz}}$, bloc dimension $M_b\times N_b$.

Output: image with the watermark A_w.

Step 1: The original matrix *A* divided on $X\times Y$ blocs $H_{Mb\times Nb}$, where $X = \lceil M / M_b \rceil$ and $Y = \lceil N / N_b \rceil$.

Step 2: Schur decomposition is applied to all blocs *H*:

$$H_{i,j} = U_{i,j} \times D_{i,j} \times U_{i,j}^{T}, \qquad (2)$$

where U is unitary matrix, D is upper triangular matrix and $1 \le i \le \lceil M / M_b \rceil$ and $1 \le j \le \lceil N / N_b \rceil$.

Step 3: Insertion of one bit of watermark, $bw_{i,j}$ in $D_{i,j}$ bloc of matrix $H_{i,j}$:

$$D'_{i,j} = D_{i,j} \cdot (I + \alpha \cdot I \cdot bw_{i,j}), \qquad (3)$$

where I is unit matrix and $D'_{i,j}$ is upper triangular matrix with embedded watermark.

Step 4: Reconstruction of the bloc with embedded watermark:

$$H'_{i,j} = U_{i,j} \times D'_{i,j} \times U_{i,j}^{T}, \qquad (4)$$

Step 5: Watermarked image A_w is obtained from blocs H'.

SDM watermaking algorithm for extracting the watermark is realized through the following steps:

Input: Watermarked image A_w, bloc dimensions $M_b \times N_b$.
Output: Reconstructed binary watermark $w'_{Mz \times Nz}$.

Step 1: The watermarked matrix A_w is divided on $X \times Y$ blocs $H'_{Mb \times Nb}$, where $X = \lceil M / M_b \rceil$ and $Y = \lceil N / N_b \rceil$.

Step 2: Schur decomposition is applied to all blocs H':

$$H'_{i,j} = U'_{i,j} \times D'_{i,j} \times (U'_{i,j})^{T}, \qquad (5)$$

where U is unitary matrix, D' is upper triangular matrix and $1 \le i \le \lceil M / M_b \rceil$ and $1 \le j \le \lceil N / N_b \rceil$.

Step 3: Extraction one bit of watermark, bw' from the matrix D':

$$I_M \cdot w'_{i,j} = D_{i,j}^{-1} \cdot (D'_{i,j} - D_{i,j}) / \alpha \quad \Rightarrow$$

$$bw'_{i,j} = \frac{diag(D'_{i,j} - D_{i,j}) \cdot diag(D'_{i,j} - D_{i,j})^{T}}{\sqrt{diag(D_{i,j}) \cdot diag(D_{i,j})^{T}}}. \qquad (6)$$

Step 4: Watermark w' is obtained from extracted bits bw'.

C. MSDM algorithm

In order to increase the quality of extracted watermark, authors of this paper are modified the SDM algorithm from [10] (MSDM algorithm). A modification of the following steps of SDM algorithm was done:

a) in the part for inserting the watermark:

Step 3: Diagonal of block of upper triangular matrix D has analyzed. The block is not appropriate, for insertion bit of the watermark, if the diagonal has two or more elements with the value 0.

b) in the part for extracting the watermark:

Step 3: Extraction one bit of watermark, bw' from the matrix D':

IF (real($D'_{(1,1)}$) - real($D_{m(1,1)}$)) > 0.01 | (real($D'_{(1,1)}$) -...
...real($D_{m(1,1)}$)) < -0.01
$bw'_{i,j}$=1;
ELSE
$bw'_{i,j}$=0;
END.

III. EXPERIMENTAL RESULTS AND ANALYZE

A. Experiment

For the purpose of testing the watermarking algorithm based on Schur decomposition (MSDM watermarking algorithm) the following experiment was conducted:

Step 1: Original image A is divided into blocs $M_b \times N_b$. Using algorithm based on Schur decomposition (MSDM watermarking algorithm), in blocs is inserted binary watermark W dimensions $M_w \times N_w$ with coefficient of inserting α. Watermaked image A_w is obtained from blocs with inserted watermark.

Step 2: In the watermarked image A_w impulse noise were superimposed (salt and pepper) with different percent of p.

Step 3: From the watermaked image with superimposed impulse noise, A_{ws} watermark has been extracted W_e

Step 4: Over an image A_{ws} MDB algorithm for detection and elimination of impulse noise has been applied.

Step 5: From the filtered image A_w^* a watermark has been extracted W'_e.

As a measure of quality of filtered image and extracted watermark, the mean squared error is applied MSE:

$$MSE = \frac{\sum_{ij} \left(x_{ij} - y_{ij} \right)^2}{M \times N} \qquad (7)$$

where x_{ij} - is pixel element of original image, y_{ij} - is pixel element of reconstructed image, $M \times N$-image dimensions.

The values of the coefficient of inserted watermark are α={0.001, 0.025, 0.05, 0.075, 0.1}. Impulse noise was varied from p=10-70%.

B. Base

Images (dimensions 512x512) presented on Fig. 1 presents the image's base for the experiment: a) Lena, b) Girl, c) Baboon, d) Barbara, e) Boat and f) Peppers. Image, dimensions (128×128), presented in the Fig. 2, were used as watermark.

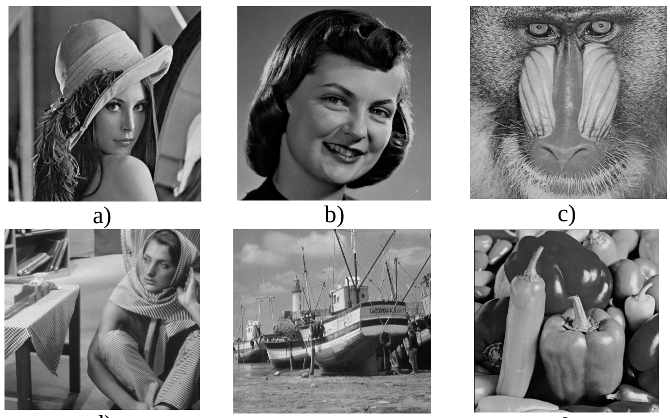

Fig. 1 Images used in the paper: a) Lena, b) Girl, c) Baboon, d) Barbara, e) Boat and f) Peppers.

Fig. 2. Watermark.

C. Results

The Table I presents the results of MSE for image Lena, after watermarking with MSD algorithm (MSE_{x1}), and results after applying the MDB algorithm for removing impulse noise (MSE_{xr1}). Also the Table I, presents results of MSE for Lena, after watermarking with MSDM algorithm (MSE_{x2}) and results after applying the MDB algorithm (MSE_{xr2}). The table II presents the results for the MSE for extracted watermark. Results for MSE are represented with: a) MSE_{ws1} (with superimposed impulse noise), b) MSE_{wr1} (after filtering impulse noise with MDB algorithm) for watermark inserting with MSD algorithm. Results for MSE are represented with: a) MSE_{ws2} (with superimposed impulse noise), b) MSE_{wr2} (after filtering impulse noise with MDB algorithm) for watermark inserting with MSDM algorithm. Fig. 3 shows diagrams for MSE for Lena: a) after watermarking with SDM and MSDM algorithms (MSE_x) and b) after filtering with MDB algorithm (MSE_{xr}) also shows MSE for extracted watermark from Lena c) after superimposed impulse noise and d) after applying MDB algorithm. Fig. 4 shows the appearance of extracted watermark (MSD algorithm): a) without attacks (Fig. 4.a), b) after superimposed impulse noise (p=10%) (Fig. 4.b) and c) after filtering (Fig. 4.c). Also Fig. 4 shows the appearance of extracted watermark (MSDM algorithm): a) without attacks (Fig. 4.d), b) after superimposed impulse noise (p=10%) (Fig. 4.e) and c) after filtering (Fig. 4.f). Fig. 5 shows the appearance of test images watermarked with MSDM algorithm, with the inserting coefficient α=0.05 and superimposed noise of *p*=30% (Figs. 4.a, 4.b and 4.c), and image appearance after applying the MDB algorithm for filtering (Figs. 4.c, 4.d and 4.e).

TABLE I
Mean Square Error for image Lena

Coef. Insert (α)	Perc. Noise (*p*)	MSE_{x1}	MSE_{xr1}	MSE_{x2}	MSE_{xr2}
0.01	0	13.1	100.2	13.1	88.2
	10		104.8		93.6
	30		116.6		107.4
	50		137.1		129.9
	70		186.7		181.6
0.05	0	42.8	129.9	43.1	112.2
	10		131.9		117.3
	30		138.6		130.4
	50		154.1		152.4
	70		198.7		203.4
0.1	0	136.6	223.3	137.2	187.4
	10		217.8		191.9
	30		209.3		203.7
	50		209.1		224.8
	70		238.2		274.1

TABLE II
MEAN SQUARE ERROR FOR THE WATERMARK

Coef. Inserting (α)	Perc. Noise (*p*)	MSE_{ws1} (10^{-3})	MSE_{wr1} (10^{-3})	MSE_{ws2}	MSE_{wr2}
0.01	0	5	7.7	7693	7921
	10	300	110	9436	9396
	30	470	230	9829	9685
	50	500	320	9806	9609
	70	520	390	9585	9218
0.05	0	0.18	0.67	7694	7921
	10	270	20	9436	9404
	30	430	68	9829	9699
	50	470	130	9809	9632
	70	510	230	9586	9273
0.1	0	0.12	0.18	7694	7921
	10	240	4.5	9438	9411
	30	390	28	9829	9706
	50	450	80	9809	9651
	70	490	180	9586	9336

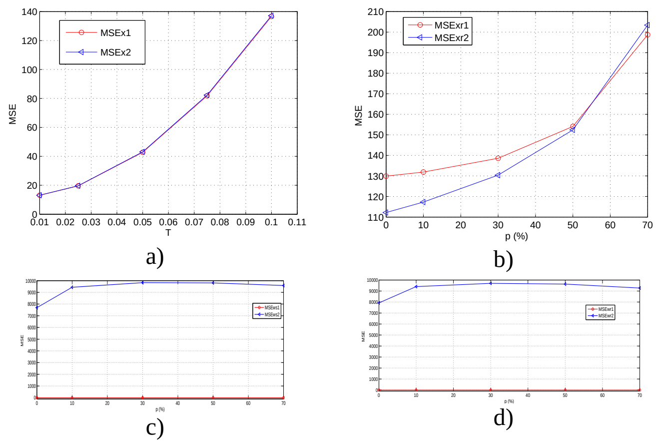

Fig. 3. MSE for: a) Lena after watermarking, b) Lena after watermarking and filtering with MDB algorithm, c) extracted watermark from Lena after superimopsed impulse noise, d) extracted watermark from Lena after filtering with MDB algorithm

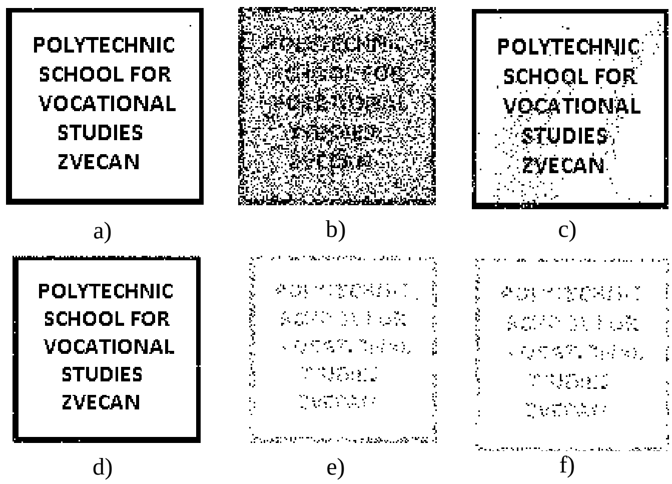

Fig. 4. The extracted watermark (MSD algorithm): a) without attacks, b) after superimposing impulse noise (*p*=10%), c) after filtering with MDB algorithm; and watermark extracted (MSDM algorithm): d) without attacks, e) after superimposing impulse noise (*p*=10%), f) after filtering with MDB algorithm.

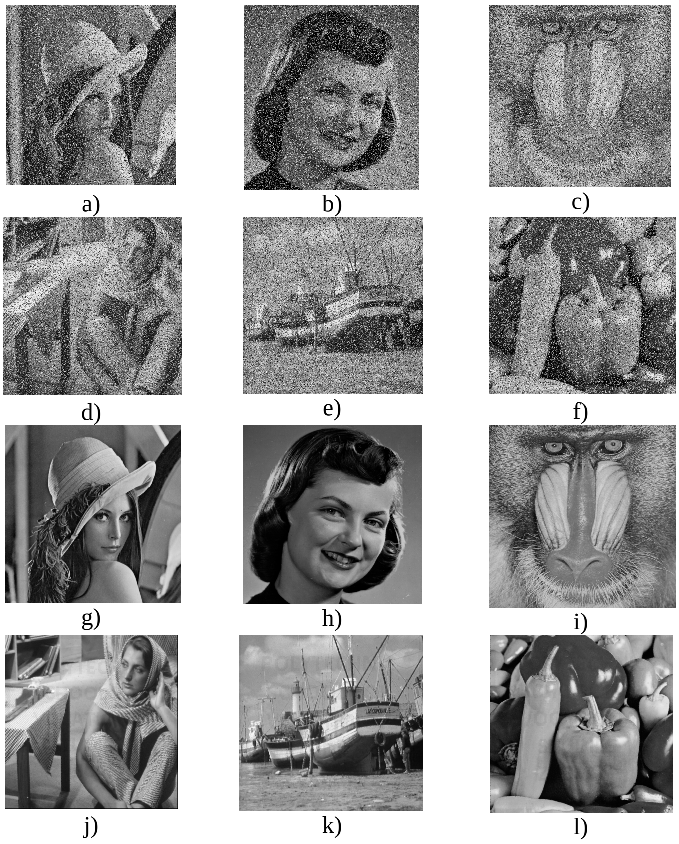

Fig 5. The watermarked test images, with inserting coefficient α=0.05 and superimposed noise *p*=30%: a) Lena, b) Girl, c) Baboon, d) Barbara, e) Boat, f) Peppers, and images after applying the MDB algorithm: g) Lena, h) Girl, i) Baboon, j) Barbara, k) Boat and Peppers.

C. Analyse

Based on the results shown in Tables I and II and in Figs 1-5 can be concluded that:

a) Applied the MSDM watermarking algorithm for inserting watermark in the image, does not lead to visual degradation.

b) The watermark, which is inserted into the image with MSDM algorithm, after extraction, visually is satisfactory quality.

c) Visual characteristics of watermarked image, inserted with MSDM algorithm, are possible to improve for very high percentage of superimposed impulse noise, even up to 70%.

d) After superimposing of 10% impulse noise to watermarked image, extracted watermark, applying MSDM algorithm, visually is not satisfactory quality.

e) MSE of extracted watermark (*p*=10%) after filtering with MDB algorithm, for MSD algorithm applied in the paper [11] is MSE=0.27, while for MSDM algorithm, presented in this paper, is MSE=9436, which present statistically extremely high percentage of errors.

f) MSE of extracted watermark (*p*=30%) after filtering with the MDB algorithm is MSE=0.068 (MSD algorithm) which is for $14x10^4$ times less in relation to MSE of watermark extracted by applying the MSDM algorithm.

IV. CONCLUSION

The paper analysed the MSDM watermarking algorithm and comparing with MSD algorithm. Watermarked images are filtered with the MDB algorithm for detection and removing impulse noise. The analysis was done for the coefficient of insertion α={0.01,0.025,0.05,0.075,0.1} with variation of superimposed impulse noise *p*=10-70%. Measure of success of algorithms is demonstrated with quality measure MSE.

Detailed comparative analysis of the MSE parameters, and visual appearance of test images and extracted watermark, indicates that the MSD algorithm [11] is suitable for inserting a watermark and shows satisfactory resistance for superimposed impulse noise. Watermark extracted, with SDM algorithm [10] modified in this paper (MSDM algorithm), is visually satisfactory quality.

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