Calculation of Capacitance of the Rectangular Coaxial Lines with Offset Inner Conductor by Strong FEM

Vladimir V. Petrovic[1] and Žaklina J. Mančić[2]

Abstract – **In this paper, capacitance per unit length of rectangular coaxial transmission lines with offset nonzero-thickness inner conductor, having an isotropic and anisotropic dielectric, using strong FEM formulation is calculated. The results were compared with the results obtained by the weak FEM and commercial software FEMM, which uses node-based first-order basis function. Based on that, appropriate conclusions are made.**

Keywords – **Quasi-static analysis, Finite element method, Strong FEM formulation, lines with rectangular cross section, offset inner conductor, isotropic and anisotropic dielectric, capacitance per unit length.**

I. Introduction

Problem of capacitance per unit length of square or rectangular lines calculation, especially lines with offset inner conductor is topical in theory and practice. The paper [1] gives a review of the literature, dealing with this task and it performs the calculation of capacitance of the rectangular coax line with offset inner conductor by using the weak FEM formulation [2]. This paper deals with calculation of capacitance per unit length of square and rectangular coaxial lines filled with isotropic and anisotropic dielectric by using strong FEM formulation [3-6]. The results are compared with those obtained by weak FEM [1] and by commercial software FEMM [6]. FEM is a very suitable method for the analysis of closed polygonal structures and it can be simply used for analysis of geometries with anisotropic dielectrics, unlike the methods that use Green's function (e.g., MoM or EEM) for which an additional complicated step of anisotropic Green's function determination is needed [7]. Besides classifying FEM into strong and weak formulation, this method can be classified as a node-based [1,6,8,9] and non node-based (with hierarchical basis functions) [2-5, 10-12]. Node-based FEM can be found much more often than non node-based FEM. However, weak FEM formulation is usually presented in the literature, while strong formulation can rarely be found. In weak FEM formulation, only function's continuity condition is exactly satisfied, whereas in strong FEM formulation, boundary conditions for the both function and its first derivative are satisfied exactly [2-5,10-12]. In this paper are obtained for the third order basis functions ($n = 3$).

II. Brief Description of the Strong FEM Formulation

FEM approach in this paper is based on hierarchical strong basis functions of higher (arbitrary) order that are constructed by using mutual multiplication of 1D strong basis functions [13]. Consider a two-dimensional domain, uniform with respect to *z*-axis, Fig. 1, filled with linear inhomogeneous dielectric without free charges, in which the distribution of electrostatic potential, $V(x,y)$, is the unknown function. Let the problem be of the closed type: on one part of the domain boundary (C_1), boundary conditions of the first kind (given V), and on the rest of the boundary (C_2), boundary conditions of the second kind (given $\partial V/\partial n$), are imposed (Fig.1). (Boundary condition of the second kind here is equivalent to given $D_n = -\varepsilon\,\partial V/\partial n$.) Differential equation for $V(x,y)$ can be defined with:

$$\mathrm{div}_S(\varepsilon\,\mathrm{grad}_S V) = 0\,, \tag{1}$$

In previous equation div_S and grad_S denote surface divergence and gradient, respectively. Calculation domain is divided into M sub-domains (elements) in FEM solution of Eq. (1).

Exact solution $V(x,y)$ is expressed as a linear combination of basis functions with unknown coefficients, $V \approx f = \sum_{j=1}^{N} a_j f_j$.

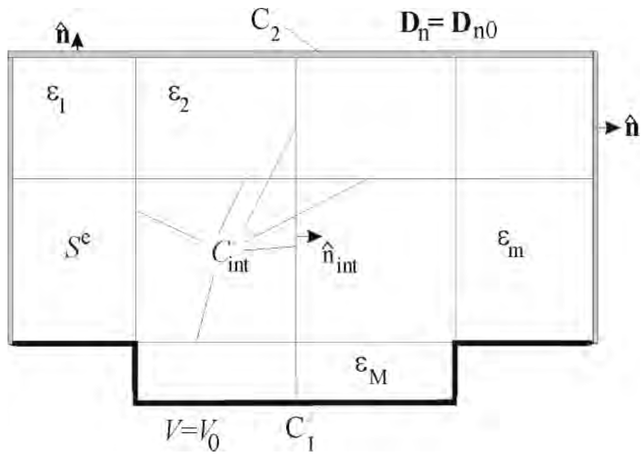

Fig. 1. Two-dimensional calculation domain divided into elements.

The system of linear algebraic equations for unknown coefficients is obtained by applying the weak Galerkin formulation [14, 15], and it is defined with:

$$[K_{ij}][a_j] = [G_i]\,,\ i, j = 1,\ldots,N\,, \tag{2}$$

where

$$K_{ij} = \int_S \varepsilon\left(\mathrm{grad}\, f_i\right)\left(\mathrm{grad}\, f_j\right)\mathrm{d}S\ , G_i = \int_{C_2} f_i\, D_{n0}\,\mathrm{d}l\ . \tag{3}$$

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In previous equation with D_{n0} is denoted a normal component of vector $\mathbf{D}$ on the contour C_2, whereas i and j represent global serial numbers of basis functions. Furthermore, S represents the union of all the element's surfaces, defined with $S = \bigcup_{e=1}^{M} S^e$. Next, rectangular elements of arbitrary order are utilized for strong formulation. Strong basis functions automatically satisfy continuity of potential V (C^0 continuity) and continuity of D_n (generalized C^1 continuity) on interelement boundaries (C_{int} in Fig..1). Complete set of strong basis functions for 2-D problems in homogeneous (isotropic or anisotropic) media is presented in [13]. Instead of ε, for anisotropic dielectrics it should be used $\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & \varepsilon_y \end{bmatrix}$ in equation (3) .

III. NUMERICAL EXAMPLES

I. Square coaxial line with offset inner conductor

For a square coaxial line with offset inner conductor, Fig. 2, for $b/a=4$, results for normalized capacitance per unit length, C'/ε, are presented in Fig. 3. When the inner conductor is moved from the center and positioned closer to the outer conductor, the normalized capacitance increases. The results of C'/ε in the case when $b/a=4$ are compared with the corresponding results obtained by FEMM [6] and results obtained by weak FEM [1]. The results are shown in Fig. 3 and an excellent agreement can be observed. In this case, it is not possible to exploit symmetry for the problem solution. In all the cases the mesh that consists of 288 rectangular elements is used for strong and weak FEM. This resulted in 1152 unknowns for strong FEM and 2448 unknowns for weak FEM formulation. In order to obtain results of the similar accuracy by using FEMM software, the number of nodes (which is equal to the number of unknowns) was between 3980 and 4130 while the number of triangular mesh elements was between 7592 and 7830.

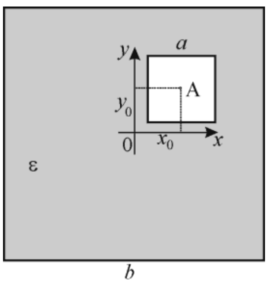

Fig. 2. Square coaxial line with offset inner conductor. Coordinate origin is in the center of the outer conductor.

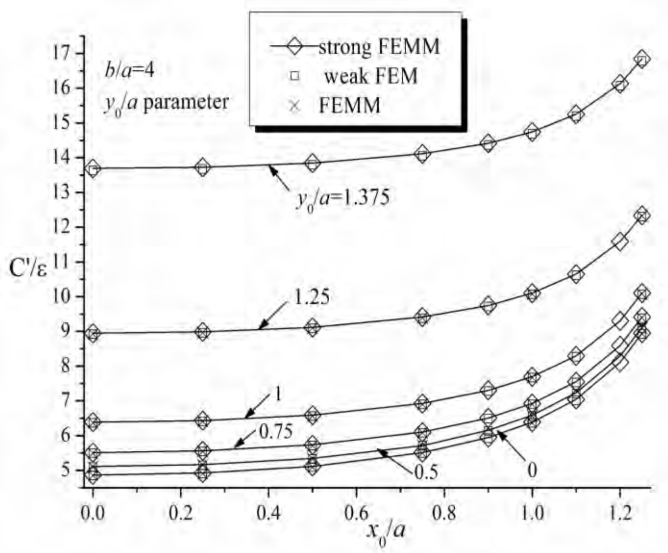

Fig. 3. Ratio C'/ε depending on x_0/a, where y_0/a is parameter, $b/a=4$ and dielectric is isotropic.

II. Rectangular coaxial line with offset inner conductor

For rectangular coaxial line, Fig. 4, C'/ε dependance of x_0/a is shown in Fig. 5.

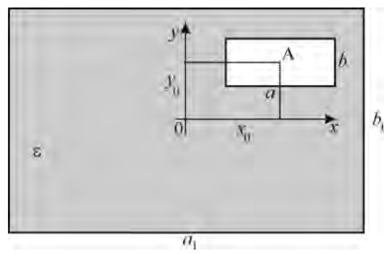

Fig. 4. Rectangular coaxial line with offset inner conductor

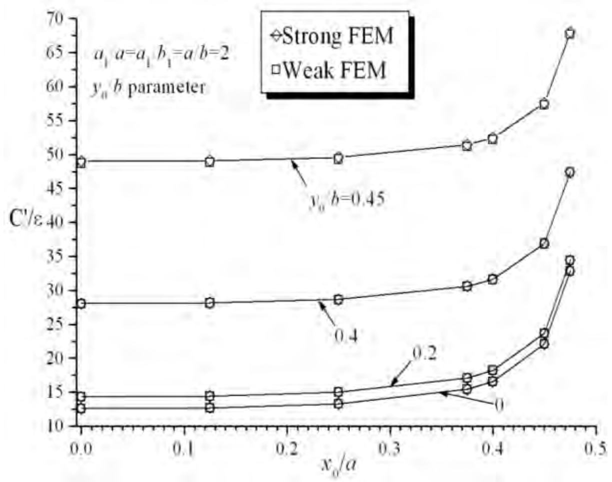

Fig. 5. Ratio C'/ε depending on x_0/a, where y_0/b is a parameter, $a/b=2$ and dielectric is isotropic, Fig. 4.

III. Rectangular coaxial line with offset inner conductor and multilayered dielectric

Fig. 6 shows the structure with layered isotropic dielectric in which the inner conductor was moved in direction *t*.

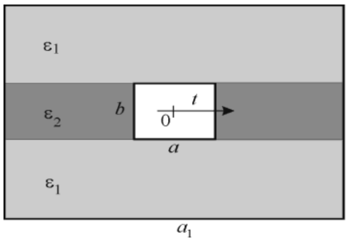

Fig. 6. Rectangular coaxial line with offset inner conductor and multilayer isotropic dielectric

In Fig. 7 dependence of the normalized effective permittivity $\varepsilon_e / \varepsilon_1$ on $\varepsilon_1/\varepsilon_2$ for two different values of t/b for a square coaxial line from Fig.1 is shown, where $a_1/a = a_1/b_1 = a/b = 2$.

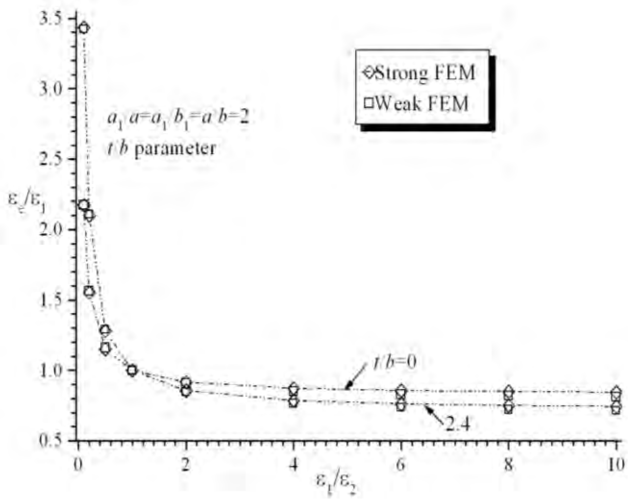

Fig. 7. Normalized effective permittivity $\varepsilon_e / \varepsilon_1$ of a rectangular coaxial line with offset inner conductor and multilayered isotropic dielectric, Fig.6, for two different values of ratio t/b.

IV. Square coaxial line with offset inner conductor and anisotropic dielectric

For a square coaxial line with offset inner conductor, Fig. 1, for $b/a = 4$, filled with anisotropic dielectric Sapphire, where $\bar{\bar{\varepsilon}} = [\varepsilon_x \; \varepsilon_y]$, results for relative permittivity ε_{re}, are presented in Fig. 8, for the following cases: a) $\varepsilon_x = 9.4$, $\varepsilon_y = 11.6$ and b) $\varepsilon_y = 9.4$, $\varepsilon_x = 11.6$. The required number of unknowns for strong FEM formulation is 1152 and for weak FEM formulation is 2448, whereas the number of rectangular elements is 288. On the other hand, FEMM requires the number of unknowns between 3964 and 4088, whereas the number of triangular elements is between 7559 and 7804. From Fig. 8 both effects of the proximity and anisotropy can be noticed, as described in detail in [2, 4, 5]. Moreover, an excellent agreement with FEMM results can be noticed, which proves that the strong FEM can be successfully applied for an accurate and efficient calculation of rectangular coaxial line with offset anisotropic dielectric.

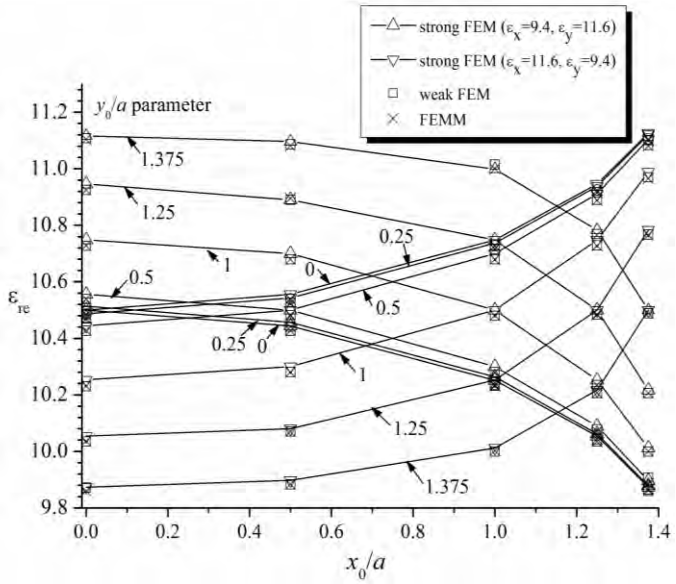

Fig. 8. Effective relative permittivity ε_{re}, of a rectangular coaxial line with offset inner conductor and anisotropic dielectric Sapphire, Fig. 2, for different ratios y_0/a.

CONCLUSION

Based on numerical examples shown in section III it can be concluded that the strong FEM formulation of the higher order and hierarchical basis functions can successfully be applied for accurate and efficient analysis of transmission lines with offset inner conductor of finite thickness in the case of isotropic and anisotropic dielectrics. Excellent agreement of obtained results and those obtained by weak FEM and commercial software FEMM has been observed. The advantage of strong FEM formulation compared to weak FEM is approximately one half of the number of unknowns. The advantage of both strong and weak FEM, is more than 25 times smaller number of required finite elements with respect to FEMM.

ACKNOWLEDGEMENT

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