Chromatic Polynomial Heuristics for Connectivity Prediction in Wireless Sensor Networks

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Abstract – **Connectivity related problems in wireless sensor networks are an active area of research. A common requirement in connectivity management is the utilization of a robust and scalable unsupervised learning method. In this paper an isomorphism of a specially defined hypergraph is used to derive a clustering method. The proposed method builds chordal graph and estimates their chromatic polynomial.**

Keywords – **Hypergraph, Isomorphism, Wireless sensor Networks.**

I. INTRODUCTION

One of the simplest and most basic of tasks in a Wireless Sensor Network (WSN) is the procedure of neighbour discovery. It allows a given node to choose upon all of the nodes that it could possibly reach, through its neighbours. Reachability, in turn, is directly related to routing, which is a NP-problem in general. Since, the network's time dynamic state is not changing completely at random, then a node discovery procedure should use an unsupervised clustering method. Currently, there are a lot of available clustering tools in the literature [1]. The most commonly used clustering algorithms are k-Nearest Neighbour and K-Means. Both can scale well with appropriate approximations, but unfortunately, they perform badly in higher dimensions as the distance between data points becomes uninformative. One way to go about this problem is to try to use appropriate projections in lower dimensions and then, use the modes of the projections to make inferences about the clusters [2]. An alternative to this is to represent the data set as vertexes of a hyper-graph and then, to try to find such a hyper-edge that maximizes a criteria for a subset of vertexes to belong to a cluster. An example of this is given in [3]. In general, clustering problems are well defined in the scope of hyper-graph theory [4]. A common problem of most clustering algorithms, which utilize some kind of distance such as an Euclidean norm, is that the number of clusters and the termination conditions are hard to be determined. Hence, it is possible to have empty clusters or clusters that have a number of clusters within themselves. Moreover, these algorithms are very sensitive to the scaling of the data that will artificially make certain clusters more or less separable.

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Furthermore, in the case of connection evaluation the clusters are also time dependent. Hence, the system can't spend much resources on cluster estimation, if they are going to loose their relevance in a short period of time. Hence, the model needs some time dependent predicting power of the network structure.

The most common approach to these issues is to use biologically inspired methods like Genetic Algorithms (GA) [5], Evolutionary Algorithms (EA) [6], Ant Colony (AC) [7] or others like in [8]. Probably the most used algorithm is Particle Swarm Optimisation (PSO) [9]–[12] and its many variants. In this paper we take a different approach. We define the problem in a hyper-graph theory framework similar to that in [13], which then after a number of appropriate transformations are made will result in a heuristic cost function for the cluster of close neighbours with respect to a single node.

The rest of the paper is organized as follows: Section II derives the heuristic cost function; Section III explains the data set obtained from the simulation and the respective results; Section IV presents some concluding remarks.

II. INDEX INVARIANCE

Let us define the hyper graph $G=(\{V\},\{E\})$ with vertex set $\{V\}$ and hyper-edge set $\{E\}$, where the degrees of the hyper-edges $d\leq|\{V\}|$ are arbitrary. An example of such a graph is given in Fig. 1. For each vertex V_i there exists a node in the network. We assume that two sensors are neighbors and constitute a hyper-edge E^2 of degree two, if they are pairwise in each other's range of coverage.

When the vertex V_i makes a first time connection to a set of vertexes $\{V\}$, it makes no a priori difference between the labels of the vertexes as long as they are different. For that reason, let us define the notion of index invariance.

A hyper-graph $S_\sigma=(\{V\},\{E\})$ with vertex set $\{V_\sigma\}=\{V_i,V_j,V_k\ldots\}$ and hyper-edge set $\{E\}$ is index invariant, if the incidence matrix $[I]$ of the graph is upper triangular and the elements of the vector of unique indexes $\sigma=\{i,j,k\ldots\}$ can be rearranged in an arbitrary way $\{\sigma\}$, without this causing change in the graph S_σ and this is given in Eq. (1).

$$S_\sigma, I_S^{\triangle}, \vec{\sigma} \equiv S_\sigma, I_S^{\triangle}, \{\sigma\} \quad (1)$$

An example of such hyper-graph Sσ is given in Fig 1.

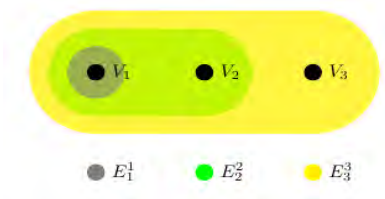

Fig. 1. An example of an index invariant hypergraph.

Theorem 2.1: An Index Invariant Graph (IIG) S_σ and its dual graph S_σ^D are isomorphic. An alternative statement is that the transpose matrix $[I]^T$ of the matrix $[I]$ induces such a graph S_σ^D, that is isomorphic to S_σ.

We prove the theorem by induction on the number of vertexes n. The theorem is true for n = 1, 2, 3, hence, assume it is true for all graphs with no more that n vertexes. For the induction step, we add one vertex V_{n+1} and one hyper-edge E_{n+1} to $I(n)$ which results in $I(n+1)$ in Eq. (2).

$$I_S^{\triangle}(n+1) = \left\| \begin{array}{c|ccc|c} & E_1 & \dots & E_n & E_{n+1} \\ V_1 & & & & 1 \\ \vdots & & I_S^{\triangle}(n) & & 1 \\ V_n & & & & 1 \\ \hline V_{n+1} & 0 & 0 & 0 & 1 \end{array} \right\| \qquad (2)$$

We transpose Eq. (2). Since $I(n)$ is upper triangular, then its transposed matrix $I^T(n)$ is lower triangular in Eq. (3).

$$\left[I_S^{\triangle}(n+1)\right]^T = \left\| \begin{array}{c|ccc|c} & E_1 & \dots & E_{n-1} & E_n & E_{n+1} \\ V_1 & & & & & 0 \\ \vdots & & \left[I_S^{\triangle}(n)\right]^T & & & 0 \\ V_{n-1} & & & & & 0 \\ V_n & & & & & 0 \\ \hline V_{n+1} & 1 & 1 & 1 & 1 & 1 \end{array} \right\| \qquad (3)$$

We assumed by the induction hypothesis, that all matrices $I^T(n)$ with a size smaller or equal to n can be transformed into upper triangular $I^T(n)=I(n)$. Next, we note that any reordering in the columns and rows of $I^T(n+1)$ in Eq. (3), does not change row V_{n+1} and column E_{n+1}, neither the structure of the graph. We move the rows of $I^T(n+1)$ with one position downwards, so that V_1 goes in place of V_2, V_2 goes in place of V_3 and likewise for each row $V_{i-1}=V_i$ other than V_{n+1}. Next, V_n goes in place of V_1 which gives Eq. (4).

$$\left[I_S^{\triangle}(n+1)\right]^T = \left\| \begin{array}{c|ccc|c} & E_1 & \dots & E_{n-1} & E_n & E_{n+1} \\ V_n & & & & & 0 \\ V_1 & & & & & 0 \\ \vdots & & I_S^{\triangle}(n) & & & 0 \\ V_{n-1} & & & & & 0 \\ \hline V_{n+1} & 1 & 1 & 1 & 1 & 1 \end{array} \right\| \qquad (4)$$

In Eq. (4) we swap row V_n and V_{n+1}, which yields Eq. (5).

$$\left[I_S^{\triangle}(n+1)\right]^T = \left\| \begin{array}{c|ccc|c} & E_1 & \dots & E_{n-1} & E_n & E_{n+1} \\ V_{n+1} & 1 & 1 & 1 & 1 & 1 \\ V_1 & 1 & 1 & 1 & 1 & 0 \\ \vdots & & I_S^{\triangle}(n-1) & & & 0 \\ V_{n-1} & & & & & 0 \\ \hline V_n & 0 & 0 & 0 & 1 & 0 \end{array} \right\| \qquad (5)$$

We move columns E_i with one rightwards, so that E_1 goes in place of E_2, E_2 goes in place of E_3 and every column other than E_{n+1} with one rightwards $E_{i-1}=E_i$. Next, E_n goes in place of E_1. Finally, we swap columns E_n and E_{n+1} which gives Eq.(6).

$$\left[I_S^{\triangle}(n+1)\right]^T = \left\| \begin{array}{c|ccc|c} & E_{n+1} & E_1 & \dots & E_{n-1} & E_n \\ V_{n+1} & 1 & 1 & 1 & 1 & 1 \\ V_1 & 0 & 1 & 1 & 1 & 1 \\ \vdots & & I_S^{\triangle}(n-1) & & & 1 \\ V_{n-1} & & & & & 1 \\ \hline V_n & 0 & 0 & 0 & 0 & 1 \end{array} \right\| \qquad (6)$$

Eq. (6) proves the theorem, since the matrix is upper triangular.

We can substitute in Eq. (1) to obtain Eq. (7)

$$S_\sigma, I_S^{\triangle}, \vec{\sigma} \equiv S_\sigma, I_S^{\triangle}, \{\sigma\} \equiv S_\sigma^D, \left[I_S^{\triangle}\right]^T, \{\sigma\} \qquad (7)$$

A direct consequence of Eq. (7) is that for each IIG there exists exactly one dual, while the two of them are isomorphic to one another. Because the ordering of the labels of the vertexes of S_σ and the ordering of the labels of the hyper-edges of the dual S_σ^D are the same and are of no consequence, then the two graphs are indistinguishable from one another $S_\sigma=S_\sigma^D$.

Let us examine the IIG $S_\sigma(n)$ and one of its sub hypergraph $S_\sigma(k), \leq n$, as it is done in Fig. 2. There exist $|S_\sigma(k)| = \prod_{i=0}^{k}(n-i)$ number of sub graphs, that are indistinguishable by Eq. (7). Therefore, we consider them as one and the same graph leading to Eq. (8).

$$\left\{S_\sigma(k)_i\right\}_{i=1}^{\prod_{i=0}^{k}(n-i)} \equiv \left\{S_\sigma(k)\right\} \qquad (8)$$

We note at this point that the number of sub IIGs of size k coincides with the chromatic polynomial $C_p = |S_\sigma(k)|$ of the complete graph G_k with k vertexes and n colors. Let us observe Fig. 2 further in order to prove that the IIG is a special case of a complete graph.

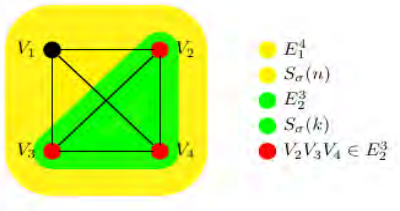

Fig. 2. An example of a sub IIG.

A vertex that is not in $S_\sigma(k)$ must have a label that is different than the labels of each vertex in $S_\sigma(k)$. That is true for all $S_\sigma(k)$ and all k. If we build a 2-uniform graph K_n having the same vertex set as $S_\sigma(k)$ where two vertexes of K_n are neighbors if they have different labels, then we can see that there is an edge of degree 2 between each two vertexes of K_n. This means that K_n is a complete graph and an IIG is a special case of a complete graph.

Therefore, Eq. (8) allows us to evaluate the index invariance with the chromatic polynomial $C_p(S_2, \lambda)$ [14] of the 2-section graph (S)2 .

$$C_p((S)_2, \lambda) = \prod_{i \in \{\sigma((S)_2)\}} [\lambda - N(V_i)]^{|\sigma_i|}, \qquad (9)$$

where $N(V_i)$ is the number of neighbors of the simplicial vertex V_i in the simplicial decomposition σ and $|\sigma|$ is the total number of vertexes having that number of neighbors in the decomposition.

Because the metric grows fast, we can normalize each polynomial with its minimum value and further take the logarithm. Let n be the number of vertexes in the tree, then we write Eq.(10).

$$M_{etric} = \log\left[\frac{\prod_{i \in \{\sigma((B_g)_2)\}} [\lambda - N(V_i)]^{|\sigma_i|}}{\prod_{i=0}^{|n|-1}(\lambda - i)}\right] \qquad (10)$$

Another consequence of Eq. (8) is that if two IIGs have a common vertex, then they form an entire IIG, meaning that two independent IIGs do not intersect. The inverse is also true. If two IIGs do not intersect, then they are independent.

III. Simulation and Results

For the purposes of this study we view the network from the perspective of a single initial node. In Fig. 3 it is shown as follows: all nodes are marked as crosses and their coverage range is given in blue; the initial node and its coverage range in green; with red - all neighbouring relations; in black – cover trees rooted at the initial node.

On each iteration, the coverage of each sensor changes, which changes the structure of the graph, which in turn changes the access capabilities and resources. On the other hand, in each time interval requests enter the system. At each time new nodes can become connected to the initial node and in the same time some nodes might get disconnected as the range of each node changes. Naturally, if a node has no resources to accept packets or the path between the nodes gets disconnected packets will be dropped.

After we accumulate data on the network structure as it changes in time, we can then compute each structure's corresponding heuristics. On Fig. 4 we plot the metric in an increasing order and the packet loss rate that is measured for that structure.

The results show, that with the increase of the heuristics, the probability of packet loss also rises. For instance, two hyper-edges with indexes e1 = 12 and e1 = 19, have heuristics H1 = 0.56 and H2 = 1.58 and their respective packet success probability is P1 = 0.43 and P2 = 0.33.

It must be also noted, that the metric is time slice specific, while th change in packet losses can be subject to differential processes.

In conclusion, the sensors should negotiate a maximum acceptable value for the heuristics and participate only in such hyper-edges that are bellow that value in order to minimise their packet loss in the network.

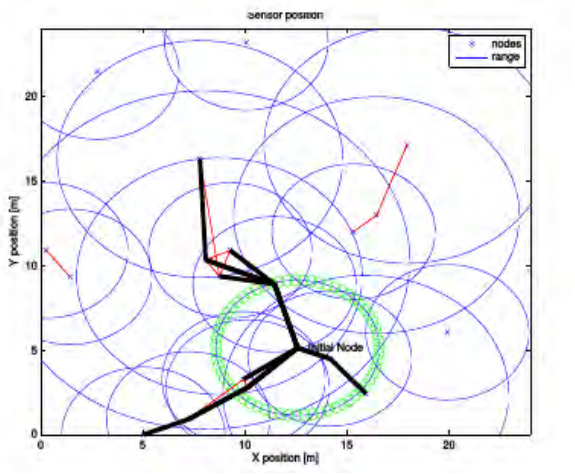

Fig. 3. A State of a WSN.

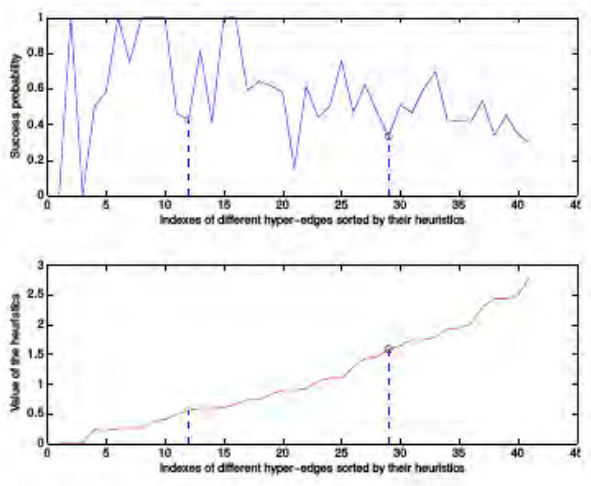

Fig. 4. An example of a sub IIG.

IV. Conclusion and Future Work

In this paper a method for evaluating the connectivity in a wireless sensor network is proposed. In the future we need to compare our results with different algorithms that tackle the same problem and also we need to increase the simulation complexity with respect to the sensor behaviour and environment.

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