Outage Performance of Satellite-Terrestrial Multiuser Networks with Fixed Gain AF Relay

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Abstract – **We investigate the outage performance of satellite-terrestrial multiuser network with single fixed gain amplify-and-forward relay. Driven by opportunistic scheduling strategy, new expression for the cumulative distribution function of the highest end-to-end signal-to-noise ratio (SNR) is derived. Based on this result, the outage probability of the system over shadowed Rician fading satellite-relay link and Rayleigh fading relay-user channels, is evaluated. Compact numerical results, analyzing the impact of number of service users, shadowing effects and the average SNR values, are also given together with independently performed simulations.**

Keywords – **hybrid satellite-terrestrial network, multiusers, opportunistic sheduling, outage probability, relaying technology**

I. INTRODUCTION

Deployment of satellite systems over challenging wireless environments is a great solution in providing high date rates, extended coverage area and variety of services for terrestrial users [1]. Disaster recovery is one of the important issues that interpreted basic utilization of geostationary satellites for broadband communications. Due to the fact that the line-of-sight (LoS) links between the satellite and terrestrial destination points are often blocked by bad weather conditions or surrounding obstacles, the usage of relaying technology is required. Among the different cooperative strategies, amplify-and-forward (AF) is the most effective relaying strategy according to its low complexity, sufficient performance gain and low implementation cost [2].

The main concept of relaying technology refers to the scenario with point-to-point link configuration having a single source and a single destination terminal. In order to establish communication between the source and many remote or scattered multiusers, point-to-multipoint dual-hop AF configuration can be utilized [3-4]. In point-to-multipoint multiuser applications, selecting the mobile user with the strongest channel i.e. oportunistic scheduling strategy can provide larger performance gains.

In open technical literature, there are numerous published papers of end-to-end performance of hybrid satellite-terrestrial relay systems over different fading or shadowed fading environments. Capitalizing on the analysis over shadowed fading links, various performance evaluations of dual-hop AF systems with a satellite are proposed. In [5] focusing on the AF relaying over shadowed Rician fading paths, the derived analytical results are given in the form of infinite series. In [6], the hybrid land mobile satellite (LMS) system with satellite-relay shadowed Rician link and a relay-to-destination free space optical link was analysed.

Also, the hybrid satellite-terrestrial networks with single user scenario have been obtained in [7-10]. A LMS system with the source-to-relay path under shadowed Rician and the relay-destination link under Nakagami-m fading conditions was investigated in [7] and [8]. In [7], the outage probability performance of hybrid networks was provided. The error probability evaluations of the similar hybrid systems were proposed in [9]–[10]. Since future development of satellite communications implies servicing a number of terrestrial users, the multiuser relaying techniques were also considered in related works. The performance analysis of hybrid multiuser system has been reported in [11]. The analytical expressions for evaluating ergodic capacity and the outage probability (in the form of infinite series) of multiuser system applying channel-state-information based AF relay, were presented.

In this paper, we propose the outage probability performance evaluation of hybrid satellite terrestrial multi-user network with fixed gain relay. The analysis is performed under the assumption that satellite-relay channel can be modeled by shadowed Rician distribution and relay-to mobile users paths as Rayleigh fading channels. Novel analytical expression is derived in closed-form for integer values of shadowing parameter m, which describes LoS component of satellite-relay link. From practical point of view, this result can be used in many real scenarios obtaining required engineering accuracy.

II. SYSTEM MODEL

Dual-hop communication between satellite and multiple users is performed via fixed gain AF relay, as shown in Fig. 1. The satellite broadcasts the signal to the relay in the first transmission phase, the relay amplifies the signal and in the second phase relay resends the signal to N users. The satellite transmits the modulated signal $s(t)$ over the link between the satellite and relay with channel complex fading coefficient h_1.

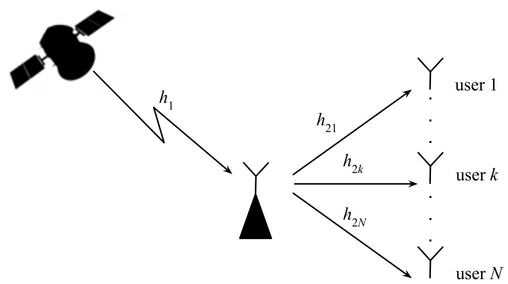

Fig. 1. System model for hybrid satellite-terrestrial multiuser system

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The received signal at the relay is given by

$$y_R = \sqrt{E_s} h_1 s(t) + n_R , \tag{1}$$

where E_s denotes the source average energy and n_R is additive white Gaussian noise (AWGN) with variance σ_R^2. The signal at the relay is multiplied by a gain G. The relay transmits amplified signal to the k-th user over channel with complex fading coefficient h_{2k}. The received signal at the k-th user is expressed as

$$y_k = \sqrt{E_R} h_{2k} G\left(\sqrt{E_s} h_1 s(t) + n_R\right) + n_{Dk} \tag{2}$$

where E_s denotes the relay output average energy and n_{Dk} is AWGN at the k-th user with variance σ_{Dk}^2.

The fixed gain relay is implemented in the system so the gain depends on the statistical channel state information of the satellite-relay link in a way

$$G^2 = \frac{E_R}{C\sigma_R^2} = \frac{E_R}{\sigma_R^2 + \mathrm{E}\left[E_s|h_1|^2\right]} = \frac{E_R}{\sigma_R^2(1+\bar{\gamma}_1)}, \tag{3}$$

where C is a constant determined by the relay gain and $\bar{\gamma}_1$ is average signal-to-noise ratio (SNR) of the satellite-relay link (E[.] denotes expectation).

The instantaneous SNR at the k-th user can be determined as

$$\gamma_{eqk} = \frac{E_s|h_1|^2 E_R|h_{2k}|^2 G^2}{E_R|h_{2k}|^2 G^2\sigma_R^2 + \sigma_D^2} . \tag{4}$$

With the introduction of instantaneous SNR of the satellite-relay and relay-user link as $\gamma_1 = E_s|h_1|^2 / \sigma_R^2$ and $\gamma_{2k} = E_R|h_{2k}|^2 / \sigma_D^2$ respectively, and substituting (3) in (4), the equivalent SNR at k-th user has the form [4]

$$\gamma_{eqk} = \frac{\gamma_1\gamma_{2k}}{\gamma_{2k} + C} . \tag{5}$$

From (3), the constant C is $C = 1 + \bar{\gamma}_1$.

III. Channel model

We assume that a hybrid satellite-terrestrial system, from the satellite to the users, consists of two links. First, LoS link between satellite and relay experiences Rice fading where the LoS amplitude fluctuation is modeled by Nakagami-*m* distribution. The relay-user links are assumed to follow Rayleigh fading.

A. Satellite-relay channel

The shadowed Rician distribution was proposed in [12] for modeling the signal amplitude over LMS channel and the probability density function (PDF) of channel gain has a form

$$p_{|h_1|^2}(x) = \alpha \exp(-\beta x)\, {}_1F_1(m;1;\delta x) . \tag{6}$$

where ${}_1F_1(.;.;.)$ denotes the confluent hypergeometric function [13, (9.210.1)], and parameters α, β and δ can be calculated as

$$\alpha = \frac{1}{2b}\left(\frac{2bm}{2bm+\Omega}\right)^m, \quad \beta = \frac{1}{2b}, \quad \delta = \frac{\Omega}{2b(2bm+\Omega)}, \tag{7}$$

wherein $2b$ is the average power of the multipath components, Ω is average power of the LoS component and m is Nakagami-*m* parameter.

The PDF of the instantaneous SNR, $\gamma_1 = E_s|h_1|^2 / \sigma_R^2$, is [11]

$$p_{\gamma_1}(x) = \frac{\alpha}{E_s/\sigma_R^2}\exp\left(-\beta\frac{x}{E_s/\sigma_R^2}\right){}_1F_1\left(m,1,\delta\frac{x}{E_s/\sigma_R^2}\right). \tag{8}$$

Average SNR in this case has the following form

$$\bar{\gamma}_1 = \frac{\mathrm{E}\left[|h_1|^2\right]E_s}{\sigma_R^2} = (2b+\Omega)\frac{E_s}{\sigma_R^2} . \tag{9}$$

For integer values of the fading parameter m the function ${}_1F_1(.;.;.)$ using [14, (07.20.03.0009.01)] and [14, (05.02.06.0005.01)] can be presented in a simple mathematical form

$${}_1F_1(m;1;z) = e^z \sum_{k=0}^{m-1}\binom{m-1}{m-1-k}\frac{1}{k!}z^k . \tag{10}$$

Substituting (10) into (8) the PDF can be written as

$$p_{\gamma_1}(\gamma_1) = \frac{\alpha}{E_s/\sigma_R^2} e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}\gamma_1} \sum_{k=0}^{m-1}\frac{1}{k!}\binom{m-1}{m-1-k}\left(\frac{\delta}{E_s/\sigma_R^2}\right)^k \gamma_1^k . \tag{11}$$

The expression of the PDF given by (11) only applies to integer values of m, and it can be used to determine the upper and lower bounds of system performance for any value of parameter m.

Based on (11), the cumulative density function (CDF) can be determined using [13, (3.351.1)]

$$F(\gamma) = \alpha\sum_{k=0}^{m-1}\frac{1}{k!}\binom{m-1}{m-1-k}\frac{\delta^k}{(\beta-\delta)^{k+1}} \times\left(\Gamma(k+1) - \Gamma\left(k+1, \gamma\frac{\beta-\delta}{E_s/\sigma_R^2}\right)\right), \tag{12}$$

where $\Gamma(.)$ is the gamma function [13, (8.310.1)] and $\Gamma(.,.)$ is the incomplete gamma function [13, (8.350.2)].

B. Relay-users channels

Multiple users are linked to the relay over Rayleigh fading channels. The corresponding PDF and CDF of the instantaneous SNR over Rayleigh fading channel between relay and k-th user has a form

$$p_{\gamma_{2k}}(\gamma_2) = \frac{1}{\bar{\gamma}_2}\exp\left(-\frac{\gamma_2}{\bar{\gamma}_2}\right) \text{ and } F_{\gamma_{2k}}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}_2}\right) \tag{13}$$

respectively. We assume that all average SNRs are equal to $\bar{\gamma}_2 = E_R|h_{2k}|^2 / \sigma_D^2$.

Opportunistic multiuser scheme is adopted and the relay selects the user which channel has the highest instantaneous SNR, i.e. $\gamma_2 = \max_{1\le k\le N}\gamma_{2k}$. Then, the CDF of selected end-to-end SNR is

$$F_{\gamma_2}(\gamma)=\prod_{k=1}^{N}F_{\gamma_{2k}}(\gamma). \qquad (14)$$

In the case of accepting an opportunistic protocol in relay-multiuser communication, the equivalent SNR can be defined as in (5), wherein γ_{2k} is replaced by γ_2, i.e. $\gamma_{eq}=\gamma_1\gamma_2/(\gamma_2+C)$.

IV. Outage performance evaluation

Outage probability is one of the performance measures that can be useful in system design. The outage performance can be obtained as the probability that the end-to-end SNR falls below a threshold.

The outage probability for hybrid system with fixed gain AF relay can be evaluated in the following way

$$\begin{aligned}F_{\gamma eq}(\gamma_{th})=&\int_0^{\gamma_{th}}P_r\left(\gamma_2>\frac{C\gamma_{th}}{\gamma_1-\gamma_{th}}|\gamma_1\right)p_{\gamma_1}(\gamma_1)\mathrm{d}\gamma_1\\&+\int_{\gamma_{th}}^{\infty}P_r\left(\gamma_2<\frac{C\gamma_{th}}{\gamma_1-\gamma_{th}}|\gamma_1\right)p_{\gamma_1}(\gamma_1)\mathrm{d}\gamma_1.\end{aligned} \qquad (16)$$

The closed-form solution for the first integral, namely for

$$I_1=\int_0^{\gamma_{th}}P_r\left(\gamma_2>\frac{C\gamma_{th}}{\gamma_1-\gamma_{th}}|\gamma_1\right)p_{\gamma_1}(\gamma_1)\mathrm{d}\gamma_1=\int_0^{\gamma_{th}}p_{\gamma_1}(\gamma_1)\mathrm{d}\gamma_1, \qquad (17)$$

is given by (12).

Substituting (11) and (14) in the second integral of (16), under the assumption of equal average SNRs over all relay-users channels, we get

$$\begin{aligned}I_2=&\int_{\gamma_{th}}^{\infty}\left(1-e^{-\frac{C\gamma_{th}}{\bar{\gamma}_2(\gamma_1-\gamma_{th})}}\right)^N p_{\gamma_1}(\gamma_1)\mathrm{d}\gamma_1=\frac{\alpha}{E_s/\sigma_R^2}\sum_{k=0}^{m-1}\binom{m-1}{m-1-k}\\&\times\frac{1}{k!}\left(\frac{\delta}{E_s/\sigma_R^2}\right)^k\int_{\gamma_{th}}^{\infty}\left(1-e^{-\frac{C\gamma_{th}}{\bar{\gamma}_2(\gamma_1-\gamma_{th})}}\right)^N e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}\gamma_1}\gamma_1^k\mathrm{d}\gamma_1.\end{aligned} \qquad (18)$$

Applying binomial expansion and introducing a change of variables, $x=\gamma_1-\gamma_{th}$ in (18), the integral I_2 becomes

$$\begin{aligned}I_2=&\frac{\alpha}{E_s/\sigma_R^2}e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}\gamma_{th}}\sum_{k=0}^{m-1}\sum_{l=0}^{N}\sum_{p=0}^{k}\frac{1}{k!}\binom{m-1}{m-1-k}\binom{N}{l}\binom{k}{p}\\&\times(-1)^l\left(\frac{\delta}{E_s/\sigma_R^2}\right)^k\gamma_{th}^{k-p}\int_{\gamma_{th}}^{\infty}e^{-\frac{Cl\gamma_{th}}{\bar{\gamma}_2 x}}e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}x}x^p\mathrm{d}x.\end{aligned} \qquad (19)$$

For l=0, I_2 can be solved in closed-form as

$$\begin{aligned}I_2^{l=0}=&\frac{\alpha}{E_s/\sigma_R^2}e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}\gamma_{th}}\sum_{k=0}^{m-1}\sum_{p=0}^{k}\frac{1}{k!}\binom{m-1}{m-1-k}\binom{k}{p}\\&\left(\frac{\delta}{E_s/\sigma_R^2}\right)^k\gamma_{th}^{k-p}\left(\frac{\beta-\delta}{E_s/\sigma_R^2}\right)^{-1-p}\Gamma(p+1),\end{aligned} \qquad (20)$$

while for $l\neq0$ the closed-form solution of I_2 has a following form

$$\begin{aligned}I_2^{l\neq0}=&\frac{2\alpha}{E_s/\sigma_R^2}e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}\gamma_{th}}\sum_{k=0}^{m-1}\sum_{l=1}^{N}\sum_{p=0}^{k}\frac{1}{k!}\binom{m-1}{m-1-k}\\&\binom{N}{l}\binom{k}{p}(-1)^l\left(\frac{\delta}{E_s/\sigma_R^2}\right)^k\left(\frac{Cl\gamma_{th}E_s/\sigma_R^2}{\bar{\gamma}_2(\beta-\delta)}\right)^{\frac{1+p}{2}}\\&\gamma_{th}^{k-p}K_{-1-p}\left(2\sqrt{\frac{Cl\gamma_{th}(\beta-\delta)}{\bar{\gamma}_2E_s/\sigma_R^2}}\right)\end{aligned} \qquad (21)$$

with $K_n(.)$ denoting the modified Bessel function of the second kind [13, (8.407.1)]

Finally, the expression for outage probability of the hybrid satellite-terrestrial multiuser system with fixed gain AF relay can be evaluated as

$$\begin{aligned}P_{out}(\gamma_{th})=&\alpha\sum_{k=0}^{m-1}\frac{1}{k!}\binom{m-1}{m-1-k}\frac{\delta^k}{(\beta-\delta)^{k+1}}\\&\times\left(\Gamma(k+1)-\Gamma\left(k+1,\gamma_{th}\frac{\beta-\delta}{E_s/\sigma_R^2}\right)\right)\\&+\frac{\alpha}{E_s/\sigma_R^2}e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}\gamma_{th}}\sum_{k=0}^{m-1}\sum_{p=0}^{k}\frac{1}{k!}\binom{m-1}{m-1-k}\binom{k}{p}\\&\times\left(\frac{\delta}{E_s/\sigma_R^2}\right)^k\gamma_{th}^{k-p}\left(\frac{\beta-\delta}{E_s/\sigma_R^2}\right)^{-1-p}\Gamma(p+1)\\&+\frac{2\alpha}{E_s/\sigma_R^2}e^{-\frac{\beta-\delta}{E_s/\sigma_R^2}\gamma_{th}}\sum_{k=0}^{m-1}\sum_{l=1}^{N}\sum_{p=0}^{k}\frac{1}{k!}\binom{m-1}{m-1-k}\\&\times\binom{N}{l}\binom{k}{p}(-1)^l\left(\frac{\delta}{E_s/\sigma_R^2}\right)^k\left(\frac{Cl\gamma_{th}E_s/\sigma_R^2}{\bar{\gamma}_2(\beta-\delta)}\right)^{\frac{1+p}{2}}\\&\times\gamma_{th}^{k-p}K_{-1-p}\left(2\sqrt{\frac{Cl\gamma_{th}(\beta-\delta)}{\bar{\gamma}_2E_s/\sigma_R^2}}\right).\end{aligned} \qquad (22)$$

Novel expression (22) is derived in closed-form solution. Regardless it looks quite robust, numerical computation of (22) in many of softwares like *Matlab* or *Mathematica* is easy, because it contains simple and well-tractable mathematical functions.

V. Numerical results

In this section we present outage probability numerical and simulation results for hybrid satellite-terrestrial multiuser system. Numerical results are obtained based on the derived analytical results (22) and accompanied with independent simulation results.

In order to analyze the impact of fading and shadowing effects on outage probability we assume the heavy and average shadowing conditions over the sattelite link. For heavy shadowing condition the parameters are set as {m, b, Ω}={1, 0.063, 0.0007} and for average shadowing as {m, b, Ω}={5, 0.251, 0.279} [15].

Outage probability for different values of E_s/σ_R^2 as a function of $\bar{\gamma}_2$, is shown in Fig. 2. Relying on obtained results we can conclude that for a certain value of E_s/σ_R^2 in the low range of $\bar{\gamma}_2$, increasing of $\bar{\gamma}_2$ leads to outage probability decreasing. However, the outage floor appears at high values of $\bar{\gamma}_2$. We can also notice that this floor is only defined by the value of E_s/σ_R^2 and is totally independent of the number of users.

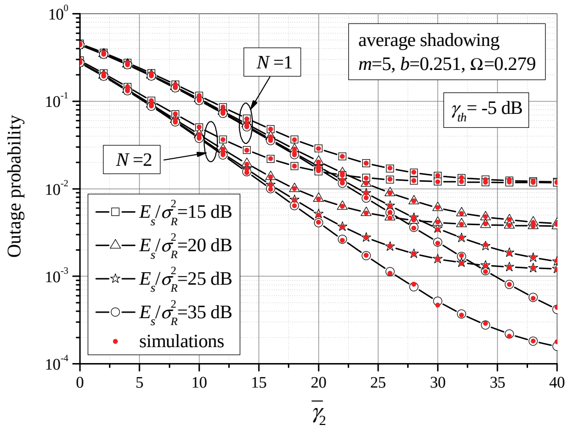

Fig. 2. Outage probability versus average relay-user SNR for various E_s/σ_R^2 and users number

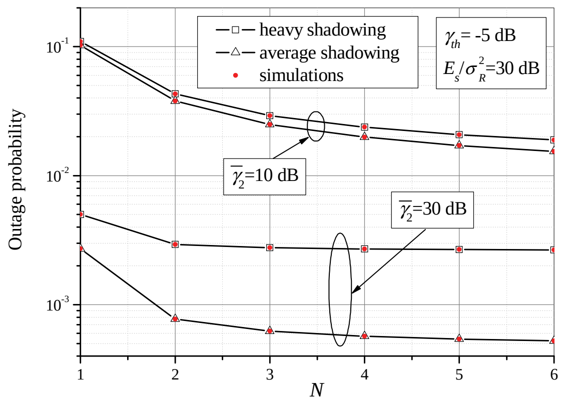

Fig. 3. Outage probability versus number of users

Outage probability versus the number of users in the hybrid multiuser system is shown in Fig. 3. Outage performance is presented for heavy and average shadowing condition over satellite link and for different average SNR of relay-user channels. With the increase of number of users, the performance gain is decreased. This gain is the greatest when we increase the number of users from one to two, when we select the user with the best relay-user channel. By increasing the average relay-user SNR, the effect of shadowing on outage probability is more pronounced. For example, for N = 4 and $\bar{\gamma}_2$ = 10 dB, the outage probability in heavy shadowing conditions is 0.02374, while in averge shadowing conditions is the same order of magnitude, 0.01987. For $\bar{\gamma}_2$ = 30 dB, changes in shadowing conditions cause the change in probability in almost of an order of magnitude (from 5.69392×10^{-4} to 2.71×10^{-3}, for the same change of conditions).

VI. Conclusion

In this paper, outage performance for hybrid satellite-terrestrial multiuser network with the fixed gain AF relay is analyzed. Analytical expression for outage probability is derived for integer fading parameter value of shadowed LoS component over satellite channels. The obtained results are in excellent agreement with the simulation results which were performed independently. We showed the effect of shadowing over the satellite-relay link, number of users, and average SNR values on the overall outage system performance.

Acknowledgement

This work was supported by Ministry of science and technology development of Republic of Serbia TR-32052.

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