Research and signal analysis with different classes of wavelets

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Abstract – **The theory of elementary waves is a powerful alternative to the classic Fourier analysis and it assures a more flexible technique for processing signals. Wavelet analysis allows the examination of data while having the capabilities to compress, filter and analyze information. In this paper signal analysis using different wavelets is made. The obtained results are compared and appropriate conclusions are made.**

Keywords – **Signal analysis, Wavelets, Wavelet Toolbox, Wavelet Transformation**

I. INTRODUCTION

In the recent years a special kind of signals called elementary waves (wavelet functions) is actively used in modern theory and practice. The wavelets are particularly effective in performing spectrum analysis and signal compression. They are one of the newest tools for decomposition of functions or continuous time signals in frequency components and the study of each frequency component with a resolution corresponding to its scale. Basically they are considered as an alternative to the Fourier transform. The wavelet transformation is one of the most popular of the time-frequency-transformations.

Wavelet analysis has begun to play a serious role in a broad range of applications, including data and image compression, signal processing, solution of partial differential equations, statistics, etc.

Wavelet analysis [1], [2], [3], [4], [6] has significant advantages that stem from the very properties of wavelet functions. The wavelet transformation allows the components of a non-stationary signal to be analyzed. The wavelets also allow filters to be constructed for stationary and non-stationary signals.

In the present paper signal analysis using wavelets is made. Haar, Daubechies (db2, db4, db7), Coiflets (coif3, coif4), Symlets (sym4, sym5, sym7) and discrete Meyer (dmey) wavelets are applied. The values of the retained energy of the considered signal are compared and appropriate conclusions are made.

II. WAVELETS AND WAVELET TRANSFORMATION

Wavelets are a class of a functions used to localize a given function in both space and scaling. A family of wavelets can be constructed from a function $\psi(t) \epsilon L_2(R)$. It denote the vector space of measurable functions that are absolutely and square integrable:

$$\int_{-\infty}^{\infty} |\psi(t)|\, dt \quad < \infty \qquad (1)$$

$$\int_{-\infty}^{\infty} |\psi(t)|^2\, dt \quad < \infty \qquad (2)$$

Sometimes $\psi(t)$ is known as a "mother wavelet" which is confined in a finite interval. "Daughter wavelets" $\psi_{a,b}(t)$ are then formed by translation $\boldsymbol{b}$ and contraction $\boldsymbol{a}$.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right),\ a, b \in\ R,\ a \neq 0 \qquad (3)$$

where $\boldsymbol{a}$ is positive and defines the scale and $\boldsymbol{b}$ is any real number and defines the shift. The pair ***(a, b)*** defines a point in the right half plane $\boldsymbol{R}_+ \times \boldsymbol{R}$.

There are a variety of wavelets. In this paper some of the most popular wavelets are used.

The **Haar** wavelet is the simplest possible wavelet. The mother wavelet function can be described as

$$\psi(t) = \begin{cases} 1,\ 0 \le t < 1/2 \\ -1, 1/2 \le t < 1 \\ 0, otherwise \end{cases} \qquad (4)$$

Its scaling function $\phi(t)$ is given in Eq.5.

$$\phi(t) = \begin{cases} 0,\ 0 \le t < 1 \\ 1,\ otherwise \end{cases} \qquad (5)$$

The **dbN** wavelets are the Daubechies' extremal phase wavelets (N refers to the number of vanishing moments).

The symlet wavelets (**symN** wavelets) are also known as Daubechies' least-asymmetric wavelets. The symlets are more symmetric than the extremal phase wavelets. In symN, *N* is the number of vanishing moments.

Coiflet scaling functions also exhibit vanishing moments. In **coifN**, N is the number of vanishing moments for both the wavelet and scaling functions.

The Meyer wavelet (**dmey**) is an orthogonal wavelet.

The wavelet transformation is often compared with the Fourier transformation, in which signals are represented as a sum of sinusoids. The main difference is that wavelets are localized in both time and frequency whereas the standard Fourier transformation is only localized in frequency.

Wavelet transforms are classified into discrete and continuous wavelet transforms.

A continuous wavelet transform (CWT) is used to divide a continuous-time function into wavelets. The CWT of a function $\boldsymbol{f(t) \in L_2 (R)}$ is defined as

$$CWT_f(a,b) = \int_{-\infty}^{\infty} \psi_{a,b}(t).f(t).dt \qquad (6)$$

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III. Signal compression using wavelets

In this section compression of a signal representing the sum of two sinusoids (Fig. 1) is made. Haar, Daubechies, Coiflets, Symlets and discrete Meyer wavelets are applied. Graphical user interface in Matlab is used for this purpose. It provides point-and-click control of software applications, eliminating the need to learn a language or type commands in order to run the application. The obtained results by using Haar wavelets are given on Figs. 1-3.

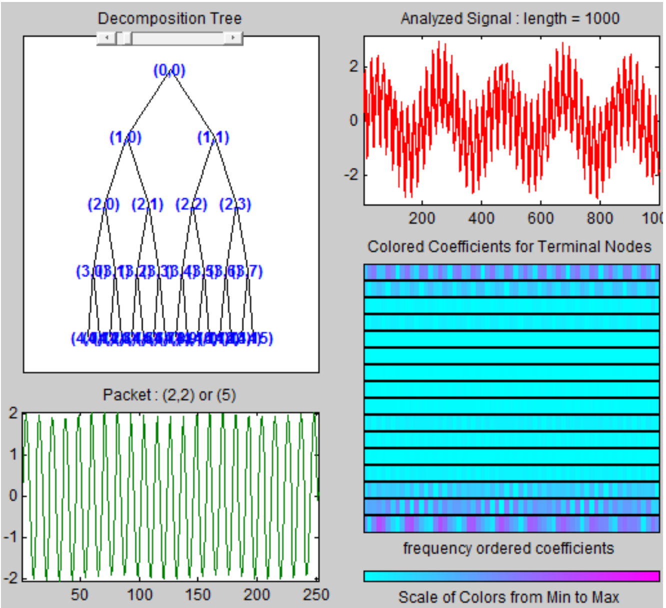

Fig. 1. Decomposition of the signal (Haar wavelets)

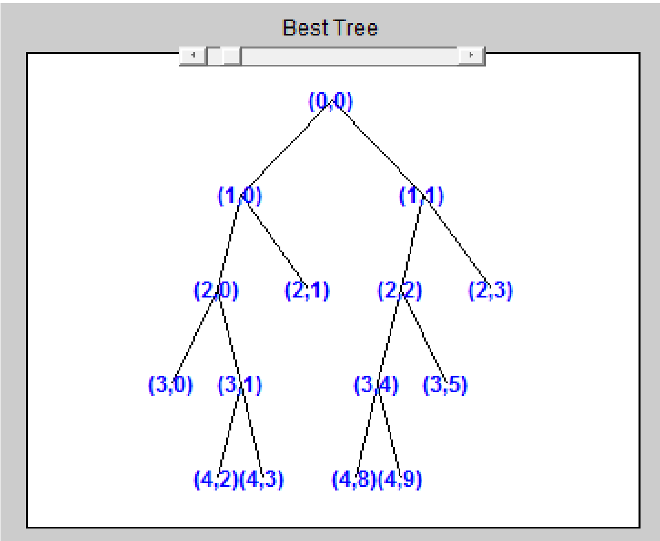

Fig. 2. Best Tree (Haar wavelets)

The original signal and compressed signal using Daubechies wavelets db2, bd4 and db7 are given on Figs.4, 5 and 6.

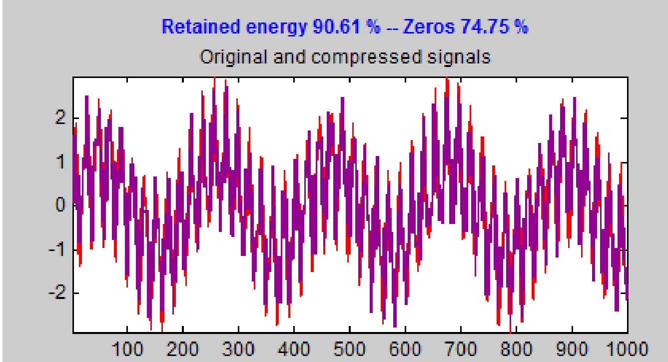

Fig. 3. Original and compressed signals using Haar wavelets

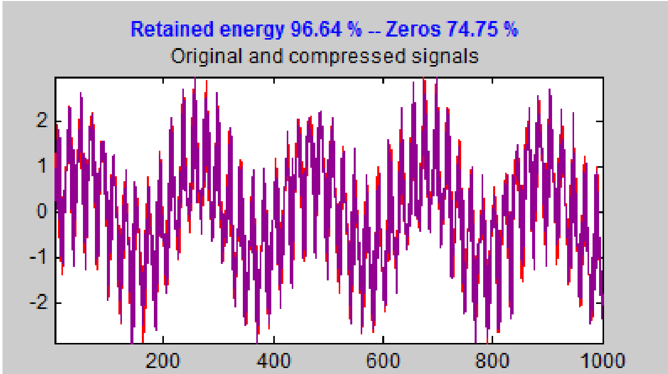

Fig. 4. Original and compressed signals using db2

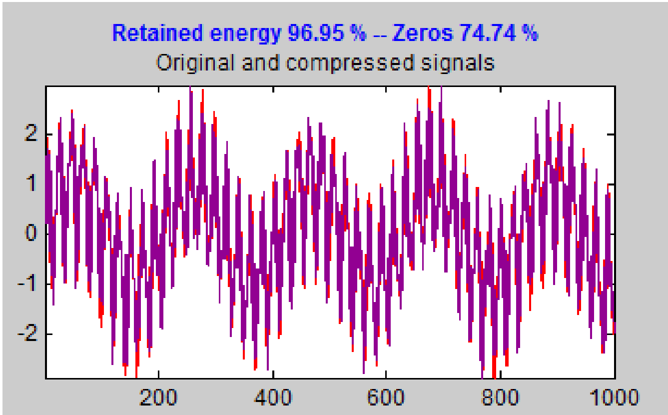

Fig. 5. Original and compressed signals using db4

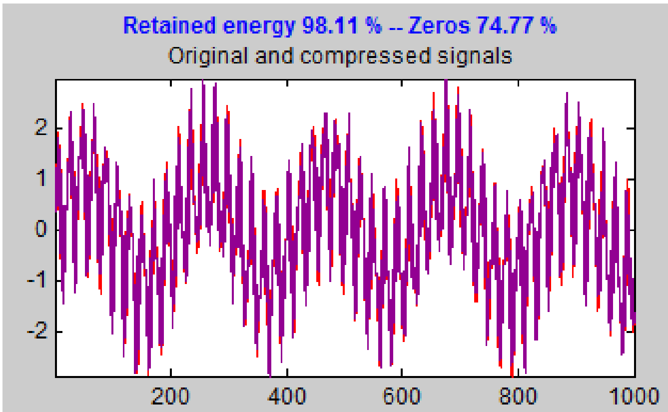

Fig. 6. Original and compressed signals using db7

The obtained results by using Coiflet wavelets are given on Figs.7 and 8.

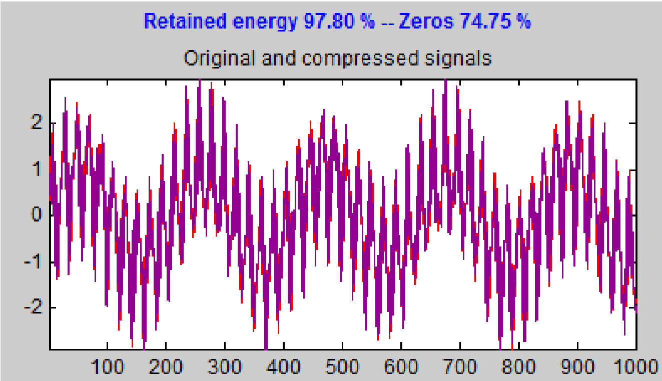

Fig. 7. Original and compressed signals using coif3

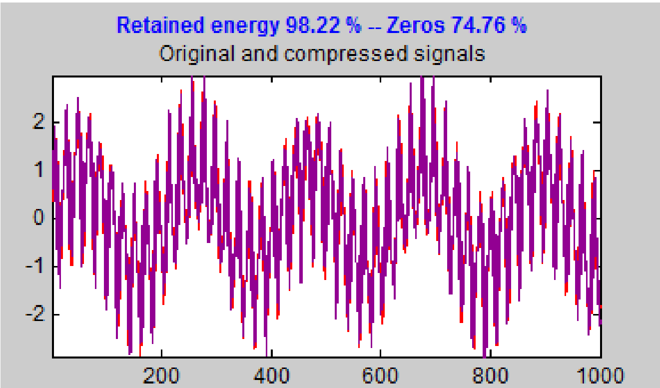

Fig. 8. Original and compressed signals using coif4

The results by using Symlet wavelets are given on Figs.9-11.

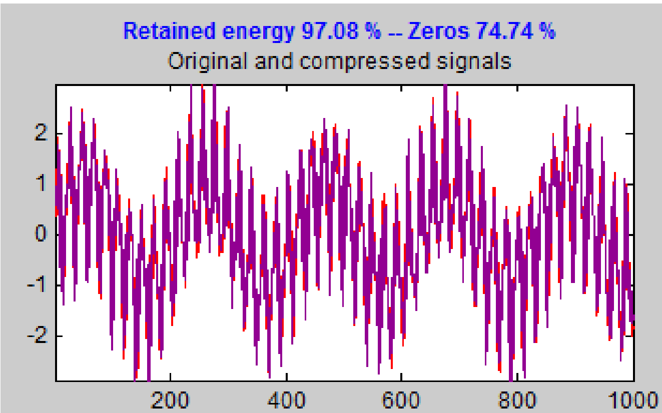

Fig. 9. Original and compressed signals using sym4

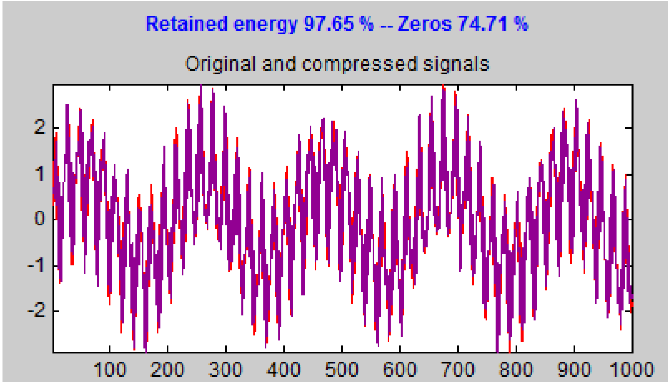

Fig. 10. Original and compressed signals using sym5

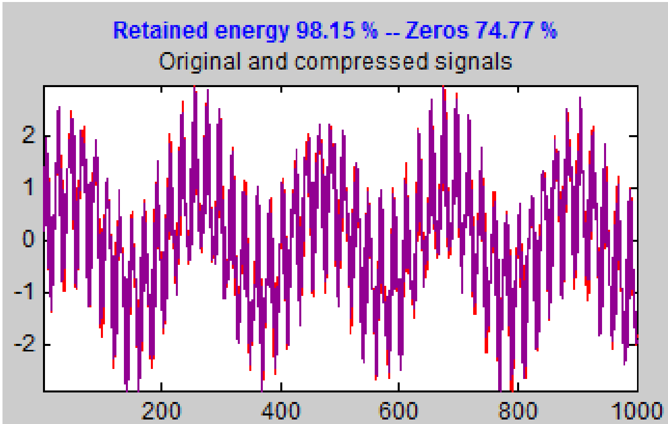

Fig. 11. Original and compressed signals using sym7

The obtained results by using discrete Meyer wavelets are given on Figs.12, 13 and 14.

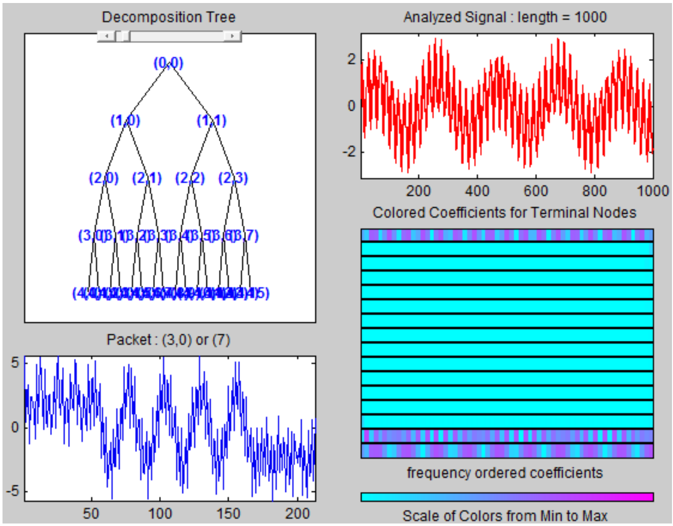

Fig. 12. Decomposition of the signal (dmey)

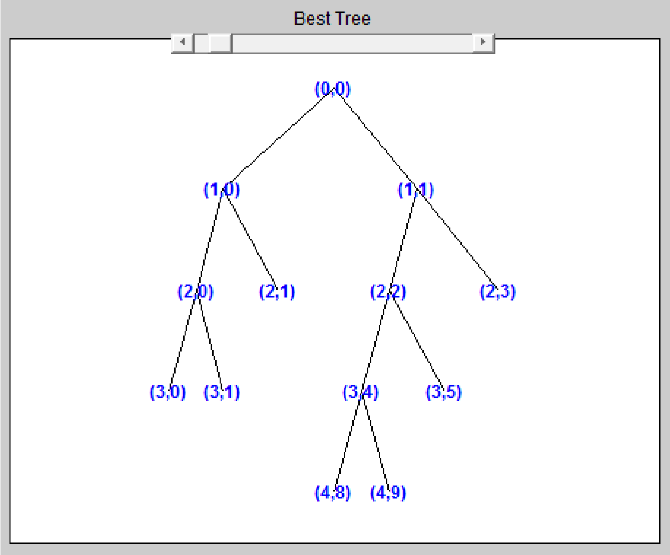

Fig. 13. Best Tree (dmey)

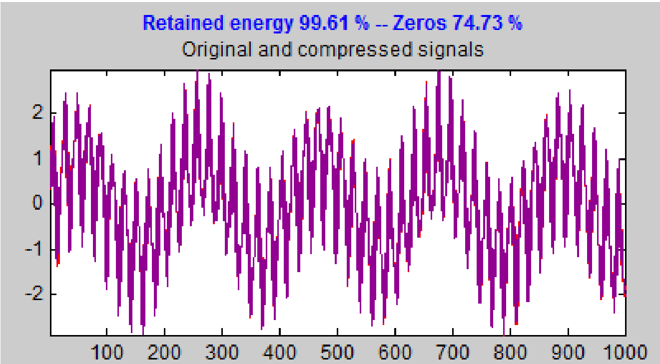

Fig. 14. Original and compressed signals using dmey

The obtained values of the retained energy of the signal using different wavelets are compared in Table I.

Table I

Retained energy

Wavelets	Global threshold	Number of zeros %	Retained energy %
Haar	1.002	74.75	90.61
db2	1.101	74.75	96.64
db4	0.9614	74.74	96.95
db7	0.7381	74.77	98.11
Coif3	0.7835	74.75	97.80
Coif4	0.6691	74.76	98.22
Sym4	0.9257	74.74	97.08
Sym5	0.847	74.71	97.65
sym7	0.7186	74.77	98.15
dmey	0.5301	74.73	99.61

IV. CONCLUSION

From the analysis of the so considered signal with different types of wavelet functions can be seen that the compression with the same parameters gives different results. When setting a threshold by which to achieve the same number of zeros of the wavelet coefficients (74.7%), it is preferable to use a discrete Meyer function. The saved energy using discrete Meyer function is 99.61%. The most accurate results are obtained using a Haar function. In Fig. 3 are visible relatively large differences between the original and the compressed signal.

The advantage of the discrete Meyer wavelet over the others can be seen in figure 14, which shows excellent overlaying of the compressed signal on the original.

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