

Optimization of the parameter of 2P Keys interpolation kernel according to the criterion of similarity to sinc function in the spectral domain

Nataša Savić¹, Zoran Milivojević¹, Zoran Veličković¹, Darko Brodić²

Abstract – This paper provides the optimization of the parameters of two-parameter Keys kernel. Optimization was performed in the spectral domain. The optimization criterion is the condition that the amplitude characteristic is a good approximation of the amplitude characteristic of the ideal interpolation kernel of the form $\sin x/x$. The second part of the paper shows an experimentally determined optimal value of the kernel parameters for an audio signal.

Keywords – Convolution, Interpolation kernel, Optimal value parameter.

I. INTRODUCTION

The measurement results are mainly expressed on a discrete set of points. Based on this data intermediate values are estimated. This process is called interpolation. The cubic convolution interpolation is usually used for working in real time [1-8]. It is suitable because it uses cubic interpolation kernels and presents a compromise between the speed of performance and the numerical precision. Parametric kernels are a very significant class of cubic interpolation kernels. They are suitable because by making a choice of parameter values it is possible to control the efficacy of the use of kernels for solving various problems. A greater number of algorithms were developed to be used for optimizing kernel parameters. Optimization is performed in the time and spectral domain. [4] suggests the application of parametric kernel for image reconstruction, and there is an algorithm for determining optimum parameter values, α_{opt} . The kernel was later termed Keys one-parameter (1P) kernel. The optimum value of Keys (1P) kernel in application to image processing is $\alpha_{opt}=-0.5$ [5] provides an algorithm for determining α_{opt} for

¹Nataša Savić is with the College of Applied Technical Sciences Niš, Aleksandra Medvedeva 20, Niš 18000, Serbia, E-mail: natasa.savic@vtsnis.edu.rs.

¹Zoran Milivojević is with College of Applied Technical Sciences Niš, Aleksandra Medvedeva 20, Niš 18000, Serbia, E-mail: zoran.milivojevic@vtsnis.edu.rs

¹Zoran Veličković is with the College of Applied Technical Sciences Niš, Aleksandra Medvedeva 20, Niš 18000, Serbia, E-mail: zoran.velickovic@vtsnis.edu.rs.

²Darko Brodić is with the University of Belgrade, Technical Faculty in Bor, Serbia, E-mail: dbrodic@tf.bor.ac.rs

Keys 1P kernel in the spectral domain through the application of Taylor expansion. The paper [6] shows how to determine α_{opt} based on the slope of the amplitude characteristic on the border of the low-pass and high pass range. The paper demonstrates [8] that by using the expansion of the amplitude characteristic in the Taylor series based on the criterion of reducing the wiggles of the amplitude characteristic in the low-pass range we can determine the optimum parameters of the Keys 2P kernel $\alpha_{opt}=-44/81$ i $\beta_{opt}=7/81$.

This paper also presents the optimization of the parameters of 2P Keys kernel. The optimization was carried out in the spectral domain. The criterion of the optimization is the similarity of the amplitude characteristics. Optimum parameter values were obtained by minimizing the deviations of the spectral characteristic in the low-pass and high pass range compared to the spectral characteristic of the ideal interpolation kernel in the form $\sin x/x$. By the application of 2P Keys kernel an interpolation of an audio signal was performed (tones G₁-G₇ piano August Forster) and the optimum values of the parameters α_{opt} and β_{opt} were obtained. Thereafter, a statistical analysis of the results was performed. The results are presented in graphs and tables.

The paper is structured as follows: section II shows an analytical form of Keys 2P interpolation kernels. Optimization of the parameters of Keys 2P kernel based on the criterion of similarity between the amplitude characteristic to the characteristic of the sinc function is presented in the section III. The experimental results and the analysis of the results are shown in section IV. Section V contains the conclusion.

II. KEYS TWO PARAMETER INTERPOLATION KERNEL

This section first describes the analytical form of 2P Keys kernel suggested in [7].

The two-parameter kernel in the form:

$$r(x) = \begin{cases} (\alpha + \beta + 2)|x|^3 - (\alpha + \beta + 3)|x|^2 + 1; & 0 < |x| \leq 1, \\ \alpha|x|^3 - (5\alpha - \beta)|x|^2 + (8\alpha - 3\beta)|x| - (4\alpha - 2\beta); & 1 < |x| \leq 2, \\ \beta|x|^3 - 8\beta|x|^2 + 21\beta|x| - 18\beta; & 2 < |x| \leq 3, \\ 0; & 3 > |x| \end{cases} \quad (1)$$

where α and β are the kernel parameters is well-known in the literature as Keys 2P kernel. For $\beta=0$ we obtain 1P Keys kernel (2) and it can be written in the form of the sum of the components:

$$r(x) = r_0(x) + \alpha r_1(x) + \beta r_2(x). \quad (2)$$

where:

$$r_0(x) = \begin{cases} 2|x|^3 - 3|x|^2 + 1 & , |x| \leq 1 \\ 0 & , |x| > 1 \end{cases} \quad (3)$$

$$r_1(x) = \begin{cases} |x|^3 - |x|^2 & , |x| \leq 1 \\ |x|^3 - 5|x|^2 + 8|x| - 4 & , 1 < |x| \leq 2 \\ 0 & , |x| > 2 \end{cases} \quad (4)$$

and

$$r_2(x) = \begin{cases} |x|^3 - |x|^2, & 0 < |x| \leq 1 \\ |x|^2 - 3|x| + 2, & 1 < |x| \leq 2 \\ |x|^3 - 8|x|^2 + 21|x| - 18, & 2 < |x| \leq 3 \\ 0; & 3 < |x| \end{cases} \quad (5)$$

B. Amplitude characteristic of the kernel

By applying the Fourier transformation (FT) on the kernel $r(x)$ the amplitude characteristic of the kernel is obtained:

$$H(f) = FT(r(x)) = H_0(f) + \alpha H_1(f) + \beta H_2(f) = H(f, \alpha, \beta), \quad (6)$$

where:

$$H_0(f) = FT(r_0(x)) = \int_{-\infty}^{\infty} r_0(x) e^{-2\pi x f i} dx, \quad (7)$$

$$H_1(f) = FT(r_1(x)) = \int_{-\infty}^{\infty} r_1(x) e^{-2\pi x f i} dx, \quad (8)$$

and

$$H_2(f) = FT(r_2(x)) = \int_{-\infty}^{\infty} r_2(x) e^{-2\pi x f i} dx, \quad (9)$$

of the spectral component of the kernel. By using (3), (4) and (5) in (7), (8) and (9) respectively after the application of partial integration, spectral components of the kernel can be written as follows:

$$H_0(f) = \frac{6 \sin^2(\pi f) - 3\pi f \sin(2\pi f)}{2\pi^4 f^4}, \quad (10)$$

$$H_1(f) = \frac{3 \sin^2(2\pi f) - 4\pi f \sin(2\pi f) - \sin(4\pi f)}{2\pi^4 f^4}, \quad (11)$$

and,

$$H_2(f) = \frac{1 - 2 \sin^2(\pi f)}{2\pi^4 f^4} \cdot (6 \sin^2(2\pi f) - 6 \sin^2(\pi f) - 6\pi f \sin(2\pi f) - 2\pi f \sin(4\pi f) + 2\pi^2 f^2) \quad (12)$$

By using (10), (11) and (12) in (6) we get the analytical form for amplitude characteristic of Keys 2P kernel which depends on the α and β parameters. The choice of optimum parameters α_{opt} and β_{opt} is described in the following section.

III. OPTIMIZATION OF KERNEL PARAMETERS

In [4] the optimization of parameter of 1P Keys kernel was performed in the time domain. The algorithm for determining the optimal parameter by minimizing the interpolation error is presented. [5] shows the optimization of 1P Keys kernel in the spectral domain. The optimum parameters of Keys 2P kernel are determined in [8] based on the criterion of the decrease of wiggles of the amplitude characteristic in the low-pass range.

A. Total mean square error

This paper presents the optimization of the 2P Keys kernel parameters in the spectral domain. The main idea is that the spectral characteristic of the kernel should be a good approximation of the ideal rectangular function $H_B(f)$ which has the value 1 in the interval [0-0.5] and value 0 in the interval [0.5-1], i.e. characteristics of the kernel in the form $\sin x/x$. Total mean square error is:

$$E_T = 2 \left(\int_0^{0.5} |1 - H(f)|^2 df + \int_{0.5}^1 |H(f)|^2 df \right), \quad (13)$$

After dividing the segment [0-1] into M subsegments, the discrete form of the total mean square error (MSET) is:

$$MSE = \frac{1}{M} \sum_{k=0}^{\frac{M-1}{2}} |1 - H(f_k)|^2 + \frac{1}{M} \sum_{k=\frac{M-1}{2}+1}^M |0 - H(f_k)|^2. \quad (14)$$

In consideration to (7) the mean square error depends on the parameters α and β . Optimum value of the parameters was obtained based on the position of the minimum value MSE (α , β).

B. The algorithm for the estimation of parameters

The algorithm for the estimation of optimum parameters α_{opt} and β_{opt} of Keys 2P kernel in the spectral domain is shown in the fig.1. The input parameters are the spectral components H_0 , H_1 and H_2 (equations 10, 11 and 12), the range of α parameter (α_{min} , α_{max}), iterative step $\Delta\alpha$, the range of β parameter (β_{min} , β_{max}), iterative step $\Delta\beta$, the range of normalized frequency f (f_{min} , f_{max}), iterative step Δf . The output values are α_{opt} , β_{opt} and the minimum value MSET_{min}.

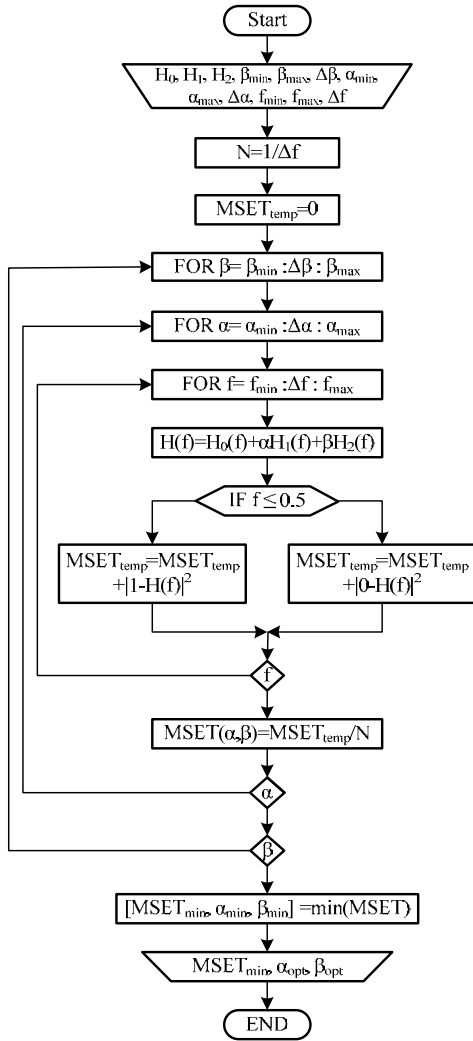


Fig. 1. Algorithm for the estimation of optimum parameters.

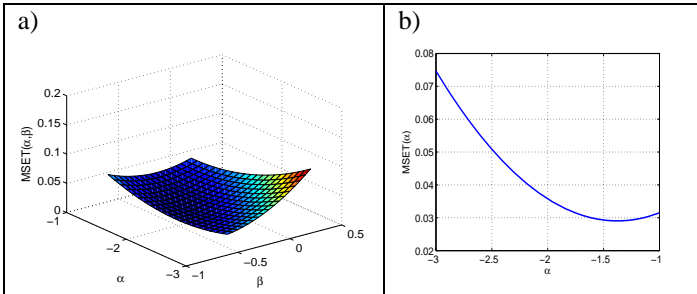


Fig. 2. a) Total mean square error between the amplitude characteristic of 2P Keys kernel, H , and the ideal H_b spectral characteristic depending on parameters α, β b) Total mean square error between amplitude characteristic 1P Keys kernel, H , and the ideal H_b spectral characteristic depending on α parameters.

TABLE II
PARAMETERS OF THE FUNCTION OF GAUSSIAN DISTRIBUTION

f_s [Hz]	α_{opt}		β_{opt}	
	σ^2	μ	σ^2	μ
44100	0.0120	-0.6929	$5.38 \cdot 10^{-4}$	0.0146
22050	0.0165	-0.7214	0.0012	0.0255
8000	0.1662	-0.6571	0.5330	0.5144

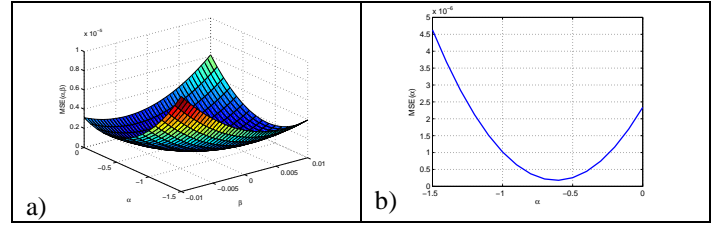


Fig. 3. a) MSET (α, β) and b) MSET (α) for tone G_3 on the following sample frequency $f_s=44.1$ kHz

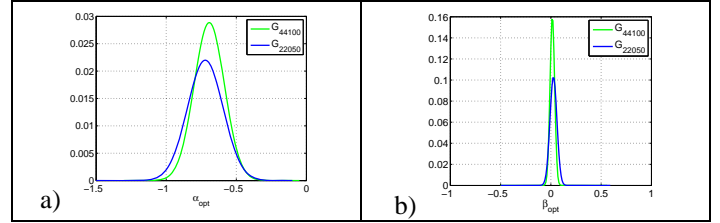


Fig. 4. Function of Gaussian distribution of optimum values a) of α parameters b) of β parameters for sample frequencies $f_s=44.1$ kHz and $f_s=22.05$ kHz.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

A. Experiment

The optimum values of the parameters α and β in the case of interpolation of the audio signal were obtained experimentally. The mean square error was determined in the process of interpolation $i+1$ sample based on $\{i-2, i, i+2, i+4\}$. This process was repeated for the whole signal starting from $i=2$ to $i=M-2$ where M is the length of the array. A statistics analysis of the results was performed by using the Gaussian distribution.

B. The basis

The basis is made of audio signals obtained by recording of G tones (G_1-G_7) performed on the August Forster piano using the sample frequencies $f_s=\{44.1, 22.05, 8\}$ kHz.

C. Results

The experiments obtained optimum values of parameters α and β Keys 2P kernel by the use of algorithm presented in figure1. for the input parameters ($\alpha_{min}=-3, \alpha_{max}=-1, \Delta\alpha=0.1, \beta_{min}=-0.6, \beta_{max}=0.2, \Delta\beta=0.05$ and $f_{min}=0, f_{max}=0.2, \Delta f=0.01$). By the application of fig.1. for the aforementioned input parameters optimum values of the parameter $\alpha_{opt}=-1.9$ and $\beta_{opt}=-0.4$ were obtained and $MSET=0.0198$. Fig.2 a) shows the mean square error of 2P Keys kernel depending on the parameters α and β , b) shows the mean square error of 1P Keys kernel depending on the parameters α (result from [8]). Minimum values of MSE in the interpolation of the tones G_1-G_7 for sample frequencies $f_s=\{44.1, 22.05, 8\}$ kHz are presented in the table 1. Fig. 3 a) shows the total mean square error (MST) for tone G_3 with the application of 2P Keys kernel, and b) shows the MST for tone G_3 with the application

of 1P Keys kernel. The parameters of the function of Gaussian distribution a) σ^2 (variance) and b) μ (mean value) were obtained based on the data from table 1 and shown in table 2. The functions of Gaussian distribution of optimum values of the parameters α and β for sample frequencies $f_s=44.1$ kHz and $f_s=22.05$ kHz are shown in fig. 4. A comparative analysis was performed with the results of the application of 2P Keys kernel where the optimum parameters were $\alpha_{opt}=-44/81$ and $\beta_{opt}=7/81$ based on the criterion of the decrease of wiggles of amplitude characteristic [8].

TABLE I
EXPERIMENTAL VALUES OF THE PARAMETERS

Tone	f_s [Hz]	α_{opt}	β_{opt}	MSE
G ₁	44100	-0.5500	0	$2.2906 \cdot 10^{-8}$
	22050	-0.6500	0.0010	$8.1374 \cdot 10^{-7}$
	8000	-0.7000	0.0110	$3.8772 \cdot 10^{-4}$
G ₂	44100	-0.5500	0.0010	$7.8752 \cdot 10^{-8}$
	22050	-0.7000	0.0060	$5.0339 \cdot 10^{-6}$
	8000	-0.8000	0.0500	$9.3589 \cdot 10^{-5}$
G ₃	44100	-0.6000	0.0005	$1.7162 \cdot 10^{-7}$
	22050	-0.6000	0.0020	$1.3603 \cdot 10^{-5}$
	8000	-0.8000	0.0500	$4.6006 \cdot 10^{-5}$
G ₄	44100	-0.7000	0.0050	$8.5367 \cdot 10^{-7}$
	22050	-0.7000	0.0145	$7.8177 \cdot 10^{-5}$
	8000	-1.0000	0.1500	$1.6737 \cdot 10^{-4}$
G ₅	44100	-0.8000	0.0075	$7.3630 \cdot 10^{-7}$
	22050	-0.7000	0.0170	$3.9116 \cdot 10^{-5}$
	8000	-1.1000	0.4000	$2.8594 \cdot 10^{-5}$
G ₆	44100	-0.8000	0.0245	$4.0872 \cdot 10^{-6}$
	22050	-0.7000	0.0380	$1.4387 \cdot 10^{-4}$
	8000	-0.2000	0.9500	$4.3180 \cdot 10^{-4}$
G ₇	44100	-0.8000	0.0635	$1.9657 \cdot 10^{-6}$
	22050	-1	0.1000	$8.3373 \cdot 10^{-5}$
	8000	0	1.9900	0.0119

A. Result analysis

Based on the experimental results shown in table 1, tbl. 2 and the optimum values of the parameters $\alpha_{opt}=-1.9$ and $\beta_{opt}=-0.4$ obtained by the application of the algorithm and optimum parameters $\alpha_{opt}=-44/81$ and $\beta_{opt}=7/81$ from [8] it can be concluded that:

- in the case of $f_s=44100$ Hz the range of the optimum values of the parameter are α , $\alpha_{opt} \in [-0.8 \div -0.55]$ and the mean value is $\overline{\alpha_{opt}} = -0.6929$, while $\beta_{opt} \in [0 \div 0.0635]$ and $\overline{\beta_{opt}} = 0.0146$;
- $\alpha_{opt} \in [-1.0 \div -0.6]$, mean value $\overline{\alpha_{opt}} = -0.7214$ and $\overline{\beta_{opt}} = 0.0255$ for $\beta_{opt} [0.001 \div 0.1]$ for $f_s=22050$ Hz.
- in case when $f_s=8000$ Hz, $\alpha_{opt} \in [-1.1 \div 0]$ and $\beta_{opt} \in [0.01 \div 1.99]$, while are $\overline{\alpha_{opt}} = -0.6571$ and $\overline{\beta_{opt}} = 0.5144$.
- the error of the estimation of the α parameter is smallest at sample frequency 22050Hz

$$\Delta_{\alpha} = |\alpha_{opt} - \overline{\alpha_{opt}}| = |-1.9 - (-0.7214)| = 1.1786,$$

$$\text{while for } \alpha_{opt}=-44/81 \cdot \Delta_{\alpha T} = |-0.5432 - (-0.7214)| = 0.1782$$

e) the error of the estimation of the β parameter is smallest at sample frequency 22050Hz

$$\Delta_{\beta} = |\beta_{opt} - \overline{\beta_{opt}}| = |-0.4 - 0.0255| = 0.4255,$$

$$\text{for the } \beta_{opt}=7/81 \text{ this error is } \Delta_{\beta T} = |0.08642 - 0.0255| = 0.0609$$

V. CONCLUSION

This paper shows an algorithm for the optimization of the parameters of Keys 2P kernel in the spectral domain. The optimization of the parameters was performed so that the amplitude characteristic of the kernel is a good approximation to the ideal amplitude characteristic. By applying the algorithm suggested in this paper the following was obtained $\alpha_{opt}=-1.9$ and $\beta_{opt}=-0.4$. An analysis of the interpolation of an audio signal (tones G₁-G₇ piano August Forster) was performed and optimum values of the parameters for each signal for the sample frequencies $f_s=\{44.1, 22.05, 8\}$ kHz were obtained. Optimum values of the parameters α and β are in the range $\alpha_{opt} \in [-1.0 \div -0.6]$ and $\beta_{opt} \in [0 \div 1.99]$. The smallest error in the estimation of the parameters is for sample frequencies $f_s=22.05$ kHz. Considering the results from [8], by comparing the errors of parameter estimation it is concluded that the kernel with the parameters obtained by the method of decreasing the wiggles of the amplitude characteristic is more precise.

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