Mathematical model of thermoelectric Peltier module

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Abstract – **Thermoelectric Peltier modules (TEM) are devices that convert electrical power in a temperature gradient. Usually in their catalogue data presents some transducer characteristics and maximum parameters, but they are insufficient to create highly efficient thermoelectric systems. The purpose of this article is to offer a relatively easy method for modelling of TEM, and the results are presented in tabular and graphic form by Matlab.**

Keywords – **Thermoelectric elements, Peltier cooler, Modelling**

I. INTRODUCTION

Thermoelectric Peltier modules (TEM) are devices which convert electrical power in temperature gradient. In their work they use the effect of Peltier, consisting of simultaneous heating and cooling of the two opposite sides of TEM [1, 2].

TEM have increasing interest due to simultaneous improving of their economic and technical parameters as well as and widely application which they get.

As a result of this the producer of TEM supplied to the market the wide assortment of modules with different thermoelectrically parameters, shape and sites [3].

Usually in the data sheets for given TEM some converting characteristics and maximum permissible parameters are shown: maximum temperature difference between the sides of the TEM – *ΔTmax,* maximum current – *Imax,* maximum supplying voltage – *Umax,* and maximum absorbed from the cool side of TEM power - Qc_{max} [4,5]

For creating of one high effective thermoelectrically system (TES), except these data it is necessary the optimal parameters of the real module to be known, as well as the base thermoelectrically parameters of the used for modules materials – coefficient of Zeebek *α [V/K]*, specific resistance of the materials *–ρ [Ω.cm]* and the coefficient of thermal conductivity *k [W/cm.K]*.

Unfortunately the producer does not show that information in the data sheets and that is why it is necessary to have a method for calculation of these parameters.

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The goal of this paper is the easy and useful method for calculation of the thermoelectrically parameters of TEM.

These are α_m, ρ_m, k_m, coefficient of conversion η, quality factor Z_0 and the parameters of the materials α, ρ and k, based on the information from the producer for the limited parameters of TEM.

The results observed are presented in table and graphical mode with the help of graph editor MATLAB.

II. MATHEMATICAL ANALYSIS

A. Expression of base dependences for cooling TEM.

Next equation (1÷4) are fundamental and they are described in books and papers [6,7,8,9]:

$$Q_c = 2N\left[\alpha I T_c - \frac{1}{2}I^2\frac{\rho}{G} - kG\Delta T\right], \tag{1}$$

Where:

- N – number of thermocouples in TEM;
- G – factor of geometry expressing the relation between the surface and height of the semiconductor element;
- I – electrical current.

The voltage U is given with:

$$U = 2N\left[I\frac{\rho}{G} + \alpha\Delta T\right] \tag{2}$$

And the consummated power from TEM W is:

$$W = U.I \tag{3}$$

Quality factor Z_0 is the parameter, directly connected with the possibility of TEM to pump thermal power:

$$Z_0 = \frac{\alpha^2}{\rho.k} \tag{4}$$

Definition of the parameters α_m, ρ_m *u* k_m*:*

$$\alpha_m = 2.\alpha.N \tag{5}$$

$$\rho_m = \frac{2.\rho.N}{G} \tag{6}$$

$$k_m = 2.N.k.G \tag{7}$$

Using equation (5-7), the equation (1, 2 and 4) can be presented as:

$$Q_c = \alpha_m I T_c - 0,5 I^2 \rho_m - k_m \Delta T \qquad (8)$$

$$U = \alpha_m \Delta T + I \rho_m \qquad (9)$$

$$Z_0 = \frac{\alpha_m{}^2}{\rho_m . k_m}, \qquad (10)$$

B. Calculating thermal electrical parameters of TEM.

After reading the parameters from the producer data sheet: *ΔTmax, Imax, Umax u* Qc_{max}, the thermoelectrically parameters of TEM – Z_0, α_m, ρ_m and k_m could be calculated.

This method uses three of the limit parameters - *ΔTmax, Imax u Umax.*

$$Z_0 = \frac{2\Delta T_{\max}}{(T_h - \Delta T_{\max})^2} \qquad (11)$$

$$\alpha_m = \frac{U_{\max}}{T_h} \qquad (12)$$

$$k_m = \frac{(T_h - \Delta T_{\max}) U_{\max} I_{\max}}{2 T_h \Delta T_{\max}} \qquad (13)$$

$$\rho_m = \frac{(T_h - \Delta T_{\max}) U_{\max}}{T_h I_{\max}}, \qquad (14)$$

Where:

- T_h is the temperature of the hot side of TEM.

After the calculation of thermoelectrically parameters of the module, this easy the thermos physic parameters of the semiconductors to be calculated, from which the thermocouples are created – coefficient of Zeebec α, specific resistivity of the materials ρ and coefficient the thermal conductivity k.

It is done with the help of equation 5÷7, but only of the number of thermocouples N and geometry actor G which are known.

For calculation of the converting coefficient η and thermal resistivity of the hot radiator R_h next equation are used:

$$\eta = \frac{Q_c}{W} \qquad (15)$$

$$R_h = \frac{T_h - T_a}{Q_c + W} \qquad (16)$$

III. Discussion and Results

The algorithm which is used for calculation of thermoelectrically parameters is shown on Fig. 1.

After the initially definition of the conditions and checking for correct their import the next calculation are done:

- Physical characteristics of the chosen thermoelectric module – quality factor Z, (K^{-1}); coefficient of Zeebec α_m *(V/K)*; the resistance of the module ρ_m *(Ω)* and coefficient of module resistivity k_m *(W/K)*;
- The base physical characteristics of the used for TEM thermoelectrically elements: coefficient of Zeebec α_m (V/K); specific resistance ρ *(Ω.cm)* and coefficient of thermal conductivity k *(W/cm.K)*.

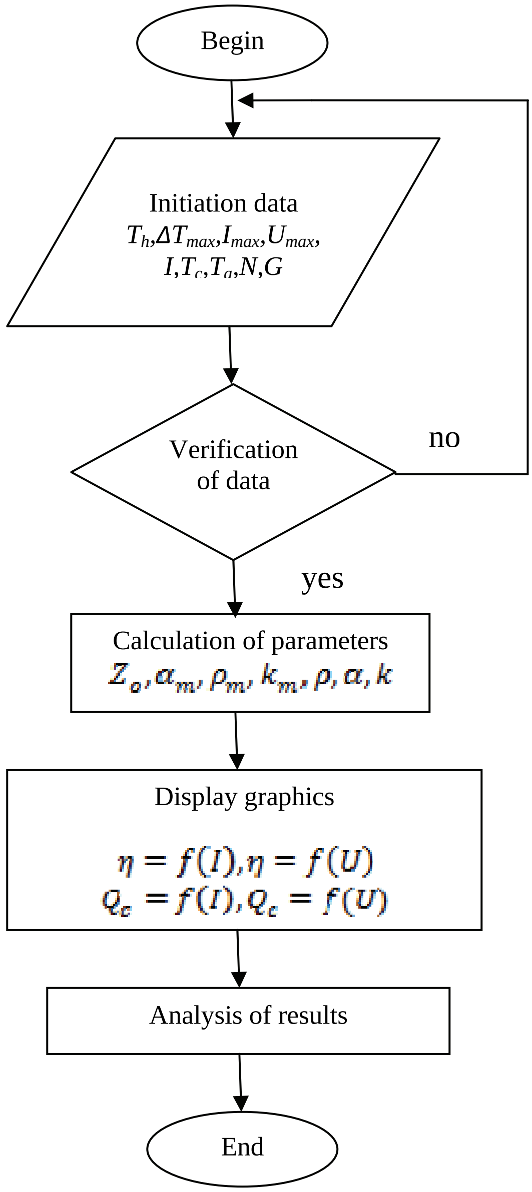

Fig. 1. The algorithm of modeling.

The following is the presentation of the results in the table and graphical mode and their analyses.

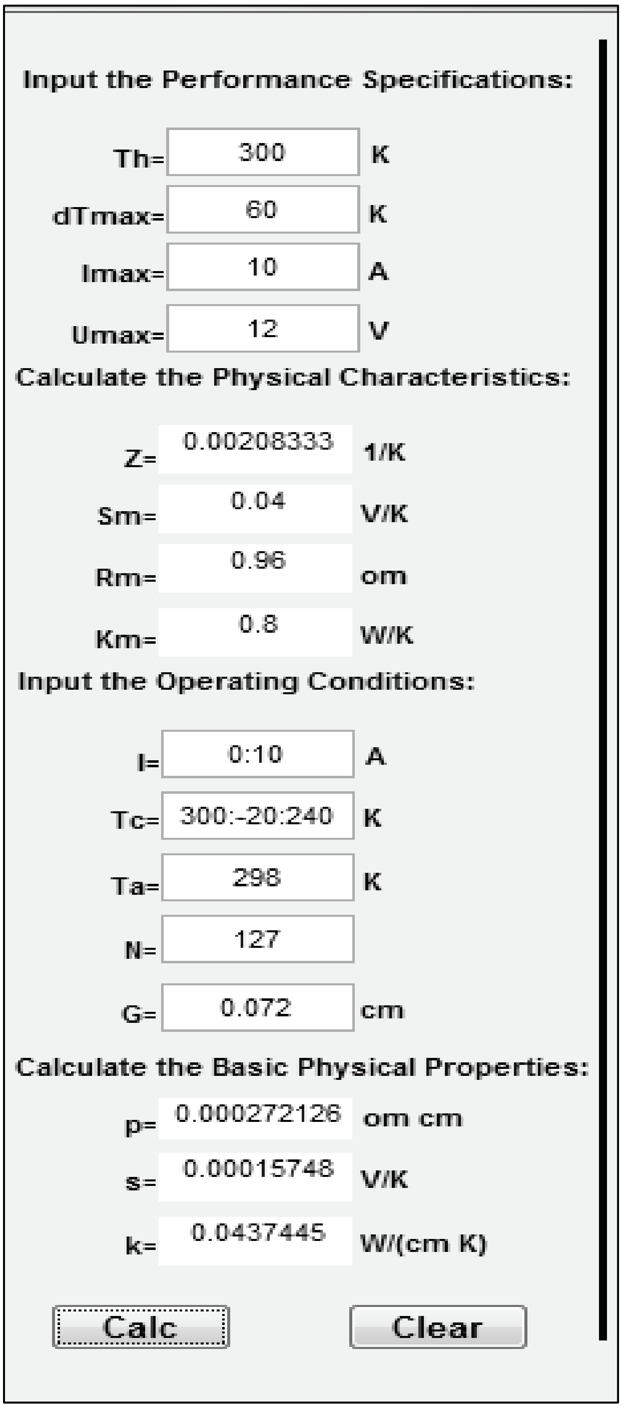

Fig. 2. Input and calculated part of the program interface.

The visualization of the results from the modeling of the thermal electrical module is performed with the help of the program product realized on the base the graphical editor MATLAB.

The program gives wide possibilities for the user, who can import high number parameters: limited parameters of TEM, shown in the producer data catalogue - *ΔTmax*, *Imax*, *Umax*, and well the working condition in which thermoelectrically module will be used.

They are:

- Input current I (in determined limits);
- Temperatures of the hot and cool side of the module – T_h и T_c,. The hot T_h is firmly determined, but T_c is determined limits;
- The ambient temperature T_a;
- The number of the semiconductors thermocouples *N;*
- Geometry factor *G*.

Practically it can be simulated the work of every one arbitrary chosen TEM, if for all input data.

On Figure 2 the table the input/exit part of the working interface is shown.

The results, except in the table, are presented in graphical type.

The program proposes possibility four type of dependence to look at on and analyzed.

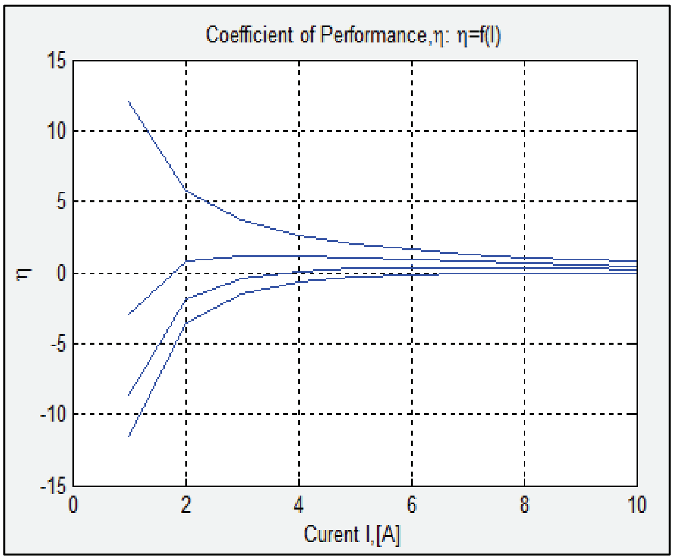

Fig. 3. Dependence of the performance coefficient η on the input current I: η = f (I).

- Coefficient of performance η as a function of the input current *I*: $\eta = f(I)$ – Fig.3;
- Coefficient of the performance η as a function of the voltage supply *U*: $\eta = f(U)$ –Fig. 4;

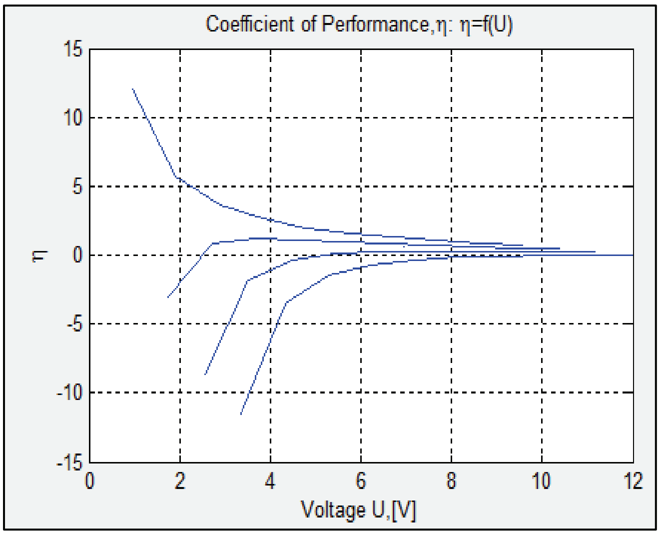

Fig. 4. Dependance of the performance coefficient η on the voltage supply U: η = f (U).

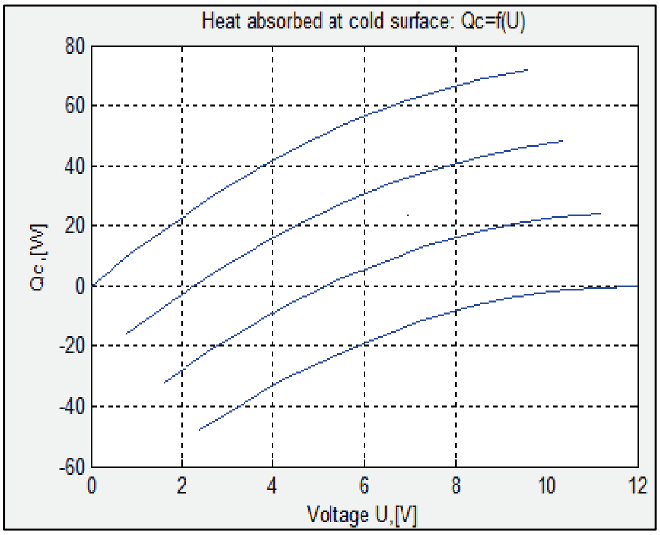

Fig. 6. Dependance of the absorbed thermal power Q_c on the voltage supplye U: $Q_c = f(I)$.

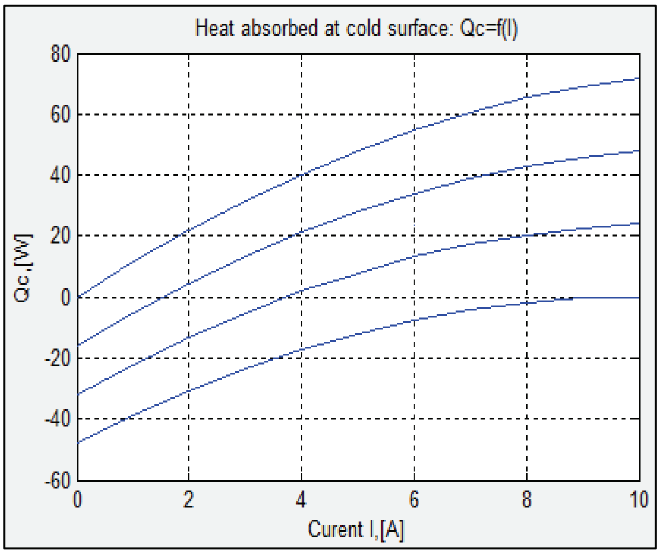

Fig. 5. Dependance of the absorbed thermal power Q_c on the input current.

- The absorbed thermal power Q_c from the cool side of TEM as a function of input current *I:* $Q_c=f(I)$ – Fig. 5;
- The absorbed thermal power Q_c from the cool side of TEM as a function of voltage supply *U:* $Q_c=f(U)$ – Fig. 6;

From Fig. 3 and Fig. 4 the efficiency of the given TEM can be estimated in dependence on DC mode of work.

The graphics from Fig. 5 and Fig. 6 show in which optimal values of input current and voltage the maximum values of the absorbed thermal power from the cool side of TEM could be reached.

IV. Conclusion

The realized mathematical model of thermoelectrically cooling module is a method with the user easily can calculate the base thermal physical parameters of TEM and for the semiconductors thermocouples, and in graphical way to report the absorbed thermal power Q_c in dependence on input current and voltage using a catalog information.

On the base of the received results it can select suitable cooling TEM in the design of thermoelectrically cooling – heating system.

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