Statistical Characteristics of Optimal Divergence of Laser Beam at FSO Systems

Tsvetan Mitsev[1]

Abstract – **In this paper a methodology for calculating the statistical characteristics of the optimal divergence of the laser beam at free space optics systems is developed. It estimates the probability density function of meteorological visibility through the use of statistical material on the likelihood of different weather conditions. The statistical characteristics of the optimal divergence of the laser beam using a deterministic relationship between it and the meteorological visibility are calculated. A simplified model for calculating the volume coefficient of the atmospheric extinction is proposed. Its spectral dependence is taken into account**

***Keywords* – FSO, optical wireless communication systems, probability density function, laser beam optimum divergence.**

I. INTRODUCTION

The main problem with the reliability of the work of free spase optics (FSO) systems occurs in the presence of fog and haze. In a dense fog meteorological distance of visibility *V* [km] may be reduced and reach values of a few tens of meters to several meters. Relative to attenuation *b* (respectively to volume extinction coefficient α [1/km]) in the communication channel, this corresponds to the values of the order of hundreds of dB/km.

In most works related to the model description of the communication channel, it is assumed homogeneous channel, i.e. constant attenuation along the length of the communication channel. This is due mainly to convenient for use in theoretical analyzes semi-empirical relationship between attenuation and meteorological visibility (the meteorological visibility reflects integrally the status of the communication channel, along it's entire length) [1], [2]. In modeling the communication channel attenuation *b* is determined mostly by empirical data on meteorological visibility [3], [4]. Most works devote attention to the wavelength of the optical radiation λ, i.e. to the spectral attenuation depending on the weather conditions [5], [6]. There are also models for prediction and analysis of the effects of fog on the operation of FSO communications system. [7], [8], [9].

In recent years, much attention is paid to the stochastic nature of fading of optical radiation into fog and to the study of statistical characteristics of the atmospheric channel. In contrast to the statistical characteristics of the communication channel of radio frequency wireless communication systems, which long ago are well studied, only recently more attention is paid to the study of statistical characteristics of communication channel of FSO systems [10]. Taking into account the stochastic nature of attenuation of optical radiation in the atmospheric channel of communication is the main factor that contributes to increasing the reliability of operation of these systems. Of particular importance is the modeling of optical communication channel in terms of fog and smoke or haze. This involves a lot of experimental work and theoretical research. An important step towards overcoming the impact of weather conditions and increasing opportunities for data transmission with FSO systems is statistical modeling of atmospheric communication channel.

In our previous work we consider the possibility of increasing the quality indicators of FSO systems through correct choice of divergence of optical radiation θ_t [rad] of the transmitter during process of instalation [11], [12]. Also it is possible automatic adjustment of the divergence of optical radiation of the system during the working process. In [13] formula for calculating the optimum value of divergence angle $\theta_{t,opt}$ depending on various parameters of the system and of the atmospheric communication channel is derived. Studies were made on the assumption of uniform and stationary atmospheric communication channel.

In this work we start building of statistical description of the communication channel of the FSO system. We use statistical data on the histogram of meteorological distance of visibility. The data used are for years. They refer to the northern hemisphere. A model of probability density function (PDF) of meteorological distance of visibility is built. The resulting PDF is used to calculate the statistical characteristics of meteorological visibility. A deterministic relationship between the optimal divergence of optical radiation of the transmitter $\theta_{t,opt}$ of FSO system and meteorological distance of visibility is presented. Through this connection expressions to calculate the statistical characteristics of $\theta_{t,opt}$ are derived and are calculated for a particular case.

II. THEORY

As we noted in the introduction, in recent years more and more attention is paid to the statistical nature of the work of FSO systems. This is in direct relation to the increase of their reliability of work.

In our works [14] and [15] we showed that serious importance to increasing the time during which the system will operate with values of *BER* under specified maximum value is setting the divergence of optical radiation of the transmitter of its optimal value $\theta_{t,opt}$ [rad]. Its value depends on the value of the random variable meteorological visibility distance *V* [km]. Therefore $\theta_{t,opt}$ also is a random variable. To determine its statistical characteristics we need to know those on *V*.

[1]Tsvetan Mitsev is with the Faculty of Telecommunications at Technical University of Sofia, 8 Kl. Ohridski Blvd, Sofia 1000, Bulgaria, E-mail: mitzev@tu-sofia.bg.

In the literature is published for a given area and for a chosen season (or time period, usually annual) average statistical data on the percentage of the total time during which there is a particular weather conditions. Because surroundings are varied, mists, fogs, rain or snow, general characteristics, which can be summarized is meteorologival visibility *V*. In fact, this is the characteristic important for the work of the FSO system. It is associated with volume extinction coefficient α [km^{-1}], and through it to the transparency of the communication channel τ_a. The last characteristic is fundamental to energy balance (the change of power from the transmitter to the receiver) of the FSO systems.

A. Visibility probability density function (PDF)

Usually we have a histogram of the distribution of meteorological distance of visibility *V*. These are presented in tabular form statistical data on average time as a percentage of total in the year *p* [%] in which there is a particular weather conditions. Any weather conditions are characterized by an appropriate range of values of meteorological distance of vision (visibility) ΔV_i [km] = $V_{i+1} - V_i$, $i = 1, 2, 3 \ldots$.

Based on these data can be calculated statistical characteristics of meteorological visibility *V*, viewed as a random variable: probability density function (PDF) $f(V)$ [km^{-1}], mathematical expectation m_V [km], dispersion σ_V^2 [km^2], average square deviation σ_V [km]. To make a step shaped line $p(V)$ at a probability density function $f(V)$ we must comply with the condition [16]

$$\int_{-\infty}^{\infty} f(V).dV = 1 . \qquad (1)$$

In the case of a stepped (step shaped) function with finite range (limited by physical considerations above and below) the condition (1) is converted to

$$\sum_i f_i \Delta V_i = 1, \quad f_i = f\left(V \in [V_i, V_{i+1}]\right) = \text{const} . \qquad (2)$$

The last equality and the condition

$$\sum_i K . p_i \Delta V_i = 1$$

give us the relationship

$$f_i = K . p_i , \quad K = \frac{1}{\sum_i p_i . \Delta V_i} . \qquad (3)$$

By determining the probability density function *fi* we easily determine other statistical characteristics of *V* [15],

$$m_V = \int_{-\infty}^{\infty} V . f(V) dV \qquad (4)$$

and

$$\sigma_V^2 = \int_{-\infty}^{\infty} [V - m_V]^2 . f(V) dV = \int_{-\infty}^{\infty} V^2 . f(V) dV - m_V^2 . \qquad (5)$$

Of course for the practical calculations of m_V and σ_V^2 we also replace integrals in (4) and (5) with sums like in (1).

B. Relationship between volume extinction coefficient and meteorological distance of vision (visibility

Deterministic relationship between $\theta_{t,opt}$ and *V* is implemented through transparency of the communication channel τ_a and volume extinction coefficient α. In the literature, the most commonly used semi-empirical relationship is

$$\alpha\left[km^{-1}\right] = \frac{3{,}92}{V[km]} \left(\frac{\lambda[\mu m]}{0{,}55} \right)^{-0,585 \sqrt[3]{V[km]}} . \qquad (6)$$

This relationship is characterized by: it has relatively inconvenient form for theoretical analysis; it applies to the $V \leq 10$ km. Therefore, the work offer more convenient for mathematical calculations relationship

$$\alpha = A . V^{-Q} , \qquad (7)$$

where the coefficients *A* and *Q* are spectrally dependent. In Tablet I are attached values of *A* and *Q* for three values of the wavelength of the optical radiation λ. The last column is a maximum relative error between the values of α calculated by formulas (7) and (6). Calculations were made for a range of values of meteorological distance of visibility *V* [km] $\in$ [0,02; 100].

TABLE I
VALUES OF SPECTTRAL COEFFICIENTS *A И Q*

λ [μm]	*A*	*Q*	$\Delta\alpha_{max}$ [%]
0,85	2,87	1,08	7,28
1,31	2,11	1,16	15,53
1,55	1,87	1,2	22,97

C. Calculation of the statistical characteristics of $\theta_{t,opt}$ by numerical integration

In our work [11] is taken out the deterministic relationship between random quantity $\theta_{t,opt}$ and random quantity *V*, namely

$$\theta_{t,opt} \equiv \theta = \frac{1}{z} \sqrt{\frac{2.\tau_t . \Phi_L}{\pi . e . I_{min}}} \sqrt{e^{-\alpha(V).z}} , \qquad (8)$$

where *z* [km] is the length of the communication channel; τ_t is the transmission of transmitter optical antenna; Φ_L [W] is the power of the laser pulses (i.e. bit entity in IM / DD and

OOK modulation); e is the base of natural logarithm; I_{min} [W/m^2] is the minimum intensity of the optical radiation into the aperture of the photodetector.

The minimum intensity I_{min} depends on many parameters of the receiver and different physical constants. It is selected so that satisfies the information capacity of the systems C_I [bps] and and the specified maximum value for bit error BER_{max}, which will operate the system.

If it is known a PDF of V and using equation (3) and connection (8), we can determine the statistical characteristics of θ [16]

$$m_\theta = \int_{-\infty}^{\infty} \theta(V).f(V).dV \,, \quad (9)$$

$$\sigma_\theta^2 = \int_{-\infty}^{\infty} [\theta(V) - m_\theta]^2 . f(V).dV = \int_{-\infty}^{\infty} \theta^2(V).f(V).dV - m_\theta^2 \,. \quad (10)$$

Of course, here, similarly to (1), due to the step shaped dependence $f(V)$, replacing the solution of the integrals with the sums. In this case the inner integrals for the different members of the sum, i.e., within the ranges ΔV_i, we solve numerically.

III. Results and Discussions

To test developed methodology for calculating the statistical characteristics of optimum divergence of the laser beam at FSO systems we will use enclosed in [17] tabulated statistical average data on the percentage of time during the year in which there is a different weather conditions. The data refer to the northern hemisphere of the Earth (Tab. II, columns 2 and 4). The values of meteorological distance of visibility V are divided into 11 spaces. The probability of a fall of V into the final two spaces is zero. On the basis of these data has been obtained the probability density function of V in column 6.

TABLE II
PROBABILITY DENSITY FUNCTION OF METEOROLOGICAL DISTANCE OF VISIBILITY V

i	V [km]	ΔV [km]	P [%]	$\Delta V.P$	$f(V)$ [1/km]	$\Delta V. f(V)$
1	2	3	4	5	6	7
1	< 0,05	0,05	0	0	0	0
2	0,05÷0,2	0,15	0,4	0,06	3,0256E-4	4,538E-5
3	0,2÷0,5	0,3	0,6	0,18	4,538E-4	1,362E-4
4	0,5÷1	0,5	1,6	0,8	1,21E-3	6,051E-4
5	1÷2	1	4,2	4,2	3,177E-3	3,177E-3
6	2÷5	3	0,6	1,8	4,538E-4	1,362E-3
7	5÷10	5	43,4	217	3,282E-2	1,641E-1
8	10÷20	10	19,8	198	1,498E-2	1,498E-1
9	20÷50	30	28,5	855	2,156E-2	6,467E-1
10	50÷100	50	0,9	45	6,808E-4	3,404E-2
11	> 100	∞	0	0	0	0
Σ	–	–	100	1322,04	–	1

On the Fig. 1 is shown $f(V)$, as step-like character of the $f(V)$ is replaced with a line. The line connects neighboring values fi. The values fi are built for the midpoint of each interval ΔV_i.

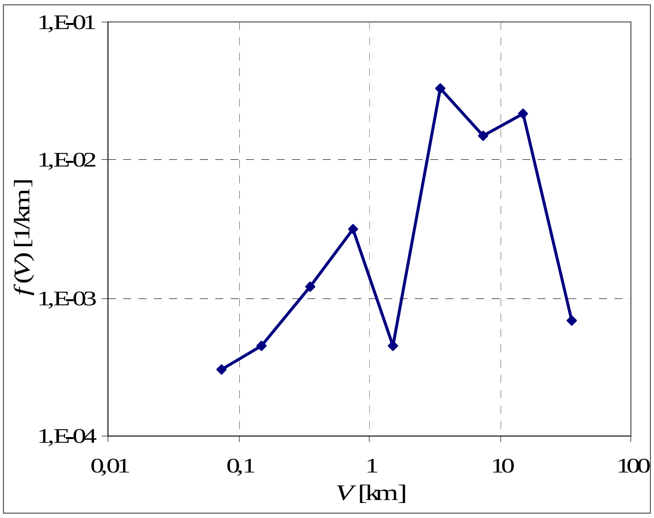

Fig. 1. Probability density function $f(V)$ of meteorological distance of visibility V

Fig. 2 shows used in the work dependencies α(V) calculated for the three lengths of λ. Calculation based on the expression (7) and corresponding coefficients from the Table I.

It is seen a big difference in the values of the volume extinction coefficient α for various wavelengths with increasing the value of meteorological distance of visibility V.

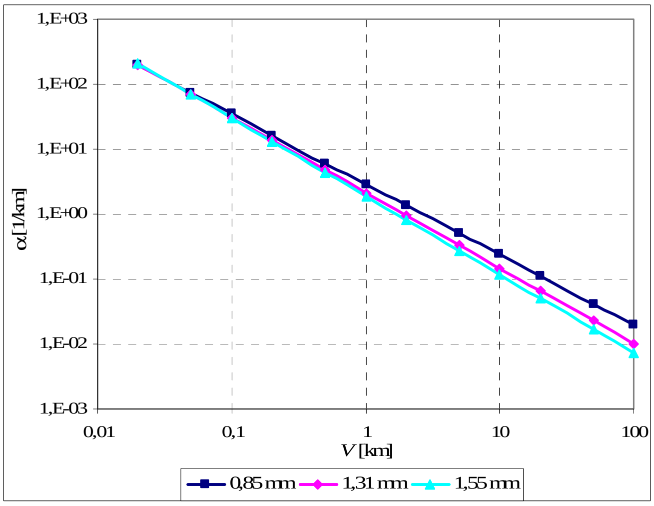

Fig. 2. Dependence on the volume extinction coefficient α of meteorological distance of visibility V

For the calculation of the statistical characteristics through (8) and (9) we choose typical values of variables involved: $z = 1$ km; λ = 1,55 μm; τ_t = 0,85; Φ_L = 20 [mW]; I_{min} = 1,023.10^{-4} W/m^2. The value of I_{min} is calculated for the average statistical parameters of the receiver. In the calculation is selected $C_I = 100$ Mbps and $BER_{max} = 10^{-8}$. In the value of I_{min} the presence of background optical radiation is reported. Calculated by the method above described values of statistical characteristics of V and of $\theta_{t,opt}$ are mapped in Table III.

TABLE III
VALUES OF STATISTICAL CHARACTERISTICS OF VISIBILITY AND OF OPTIMAL DIVERGENCE OF OPTICAL RADIATION OF TRANSMITTER

	m	σ^2	σ
V	28,7 km	261,5 km^2	16,2 km
$\theta_{t,opt}$	6,046 mrad	36,62 $mrad^2$	6,052 mrad

IV. CONCLUSION

Over the past decade has increased the number of scientific publications devoted to the possibilities of predicting and improving the reliability of operation of FSO systems. The reliability of the work is directly dependent on the parameters of the atmospheric channel of communication and the choice of the main parameters of the systems. The random character of the the parameters of the communication channel leads to random character of the basic parameters of the system. One of the main parameters is the optimal divergence of optical radiation of the transmitter.

The first step to a statistical analysis of reliability of FSO systems is to build real statistical model of atmospheric channel of communication, finding its statistical characteristics and related statistical characteristics of optimal divergence of optical radiation of the transmitter.

The work shows that in typical for northern hemisphere weather conditions in a communication channel with a length of 1 km and average statistical parameters of the system, the average optimal divergence is 6,05 mrad with dispersion 36,62 $mrad^2$.

REFERENCES

[1] Oliver Bouchet, Herve Sizum, Christian Boisrobert, Frederique de Foruel, *Free-Space Optics: Propagation and Communications*, John Wiley&Sons, 2010.

[2] Roberto Ramirez-Iniguez, Sevia M. Idrus, Ziran Sun, *Optical Wireless Communications - IR for Wireless Connectivity*, CRC Press, An Auerbach book, 2008.

[3] F. Nadeem, T. Javornik, E. Leitgeb, V. Kvicera, and G. Kandus, "Continental fog attenuation empirical relationship from measured visibility data", Radioengineering, vol. 19, p. 596-600, Dec. 2010.

[4] Heinz A. Willebrand, Baksheesh S. Ghuman, *Free-Space Optics: Enabling Optical Connectivity in Today's Networks*, Indianapolis, Sams Publishing, 2002.

[5] I. Kim, B. McArthur, E. Korevaar, "Comparison of l aser beam propagation at 785 nm and 1550 nm in fog and haze for optical wireless communications," Proc. of SPIE, vol. 4214, pp. 26-37, 2001.

[6] K. Fischer, M. Witiw, and E. Eisenberg, "Optical attenuation in fog at a wavelength of 1.55 micrometers", Atmospheric Research, vol. 87, pp. 252-258, March 2008.

[7] J. Gebhart, M. Leitgeb, E. Bregenzer, "Atmospheric effects on optical wireless links", in 7-th International Conference on Telecommunication, vol.2, pp. 395-401, Zagreb, 2003.

[8] M. Al Naboulsi, H. Sizun, F. de Fornel, "Fog attenuation prediction for optical and infrared waves", Journal of SPIE, Optical Engineering, vol 43, pp. 319-329, 2004.

[9] M. Ijaz, Z. Ghassemlooy, J. Pesek, O. Fiser, H. Le Minh, and E. Bentley, "Modeling of fog and smoke attenuation in free space optical communications link under controlled laboratory conditions", *IEEE Journal of Lightwave Technology*, vol. 31, pp. 1720-1726, April 2013.

[10] M. Khan, M. Awan, E. Leitgeb, F. Nadeem, and I. Hussain, "Selecting a distribution function for optical attenuation in dense continental fog conditions", International Conference on Emerging Technologies, pp. 142-147, Oct. 2009.

[11] Tsvetan Mitsev, Kalin Dimitrov, Hristo Ivanov, Nikolai Kolev, Optimum divergence of laser radiation in FSO systems, 7-th International Conference on Communications, Electromagnetics and medical Applications (CEMA'12), pp. 42-45, Athens, Greece, 2012.

[12] Mitsev Ts., N. Kolev, Hr. Ivanov, K. Dimitrov, Optimum Divergence of the Transmitter Optical Radiation in FSO Systems, XLVII Intern. Scientific Conf. on Inform., Communication and Energy Systems and Technol. (ICEST 2012), pp. 55-58, Veliko Tarnovo, Bulgaria, 2012.

[13] Yordan Kovachev, Tsvetan Mitsev, FSO Availability Depending on the Meteorological Conditions, 9-th International Conference on Communications, Electromagnetics and Medical Applications (CEMA'14), pp. 19-23, Sofia, 2014.

[14] Tsvetan Mitsev, Nikolay Kolev, Optimal Divergence of Laser Beam in Optical Wireless Communication Systems, Elektrotechnica & Elektronica, Vol. 49, No 11-12, pp. 15-20, 2014.

[15] Tsvetan Mitsev, Yordan Kovachev, Availability of MFSO Using Optimal System Parameters, Proceedings of 14th Conference on Microwave Techniques "COMITE 2015", April 22-23, Pardubice, Czech Republic, pp. 131-134, ISBN 978-1-4799-8121-2, 2015.

[16] E. Ferdinandov, B. Pachedjieva, Probabilistic and statistical methods in communications – Part I, Sofia, Siela, 2005. (in bulgarian)

[17] E. Ferdinandov, Laser radiation in Radio-technics, Sofia, Technics, 1981. (in bulgarian)