Comparative analysis of more commonly used recursive methods for parameter estimation in adaptive systems

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Abstract – **The adaptive control covers a set of methods that provide a systematic approach for automatic control in real-time. Recursive methods for parameter estimation have to meet the requirements for identification algorithms in real time. This is determined from the fact that the adjustment of the model after the submission of new data from monitoring, and the development of new control action should be made in a single cycle of discretization. In this article will be made an analysis of some of more commonly used recursive methods for parameter estimation in adaptive systems and their variations.**

Keywords – **adaptive system, instrumental variable method, least squares method, recursive methods for parameter estimation**

Mandatory condition in the control of one system is understanding the dunamic features of the object.[3] Recursive methods for parameter estimation are used in the identification in real time. During the work on system together with the management can continue the specification of the model based on new data received. This is particularly important when the characteristics of the object change over time. These changes are recorded in the elaboration of the control signal from the so-called adaptive systems. Often used in the transmission and processing of signals.

Recursive methods are independent control tool in real time.[1]

1. Recursive versions of least squares method for parameter estimation.

1.1. Recursive weighted least squares

To obtain a recursive estimates on the weighted least squares is used

$$\hat{\theta}_{N+1} - \hat{\theta}_N + \frac{C_N f_{N+1}}{\frac{1}{w_{N+1}} - f^T_{N+1} C_N f_{N+1}} \left(y_{N+1} - f^T_{N+1} \hat{\theta}_N \right)$$

According to (1) the old valie $\hat{\theta}_N$ is adjusted proportionally to the difference $y_{N+1} - \hat{y}_{N+1}$ with a vector coefficient of proportionality

$$\Gamma_{N+1} = \frac{C_N f_{N+1}}{\frac{1}{w_{N+1}} - f^T_{N+1} C_N f_{N+1}} \qquad (2)$$

The value $\hat{y}_{N+1} = f^T_{N+1} \hat{\theta}_N$ is the predicted value for y_{N+1}, calculated on the basis of the coefficients of the previous iteration $\hat{\theta}_N$.

Using remainder instead of prediction errors

In formula (2) is involved by the prediction error

$$e_{N+1} = y_{N+1} - \hat{y}_{N+1}(\hat{\theta}_N) = y_{N+1} - f^T_{N+1} \hat{\theta}_N \, (3)$$

For improving the accuracy of prediction instead of e_{N+1} is used the remainder

$$r_{N+1} = y_{N+1} - f^T_{N+1} \hat{\theta}_{N+1} \qquad (4)$$

From (2) and (3) follows

$$r_{N+1} - e_{N+1} = -f^T_{N+1} (\hat{\theta}_{N+1} - \hat{\theta}_N) \quad (5)$$

If from (2) is determined the difference $(\hat{\theta}_{N+1} - \hat{\theta}_N)$ and substituted in (5), follows

$$r_{N+1} = \frac{\frac{1}{w_{N+1}} e_{N+1}}{\frac{1}{w_{N+1}} - f^T_{N+1} C_N f_{N+1}} \qquad (6)$$

Starting the calculation procedure

To obtain the estimates by the method of weighted least squares are needed $\hat{\theta}_N$ and C_N from previous calculations.

$N \geq k$ observations are collected and with non-recursive method of weighted least-squares and the initial estimates $\hat{\theta}_N$ and C_N are. Then continue the calculations in (1) and (7)

$$C_{N+1} = C_N - \frac{C_N f_{N+1} f^T_{N+1} C_{N+1}}{\frac{1}{w_{N+1}} - f^T_{N+1} C_N f_{N+1}} \qquad (7)$$

The algorithm of the recursive weighted least squares underlies a number of other recursive procedures. It is as follows:

- $N \geq k$ observations are collected and with non-recursive method of weighted least-squares and the initial estimates $\hat{\theta}_N$ and C_N are.
- The new estimates are calculated by the formula

$$\hat{\theta}_{N+1} = \hat{\theta}_N + \frac{C_N f_{N+1}}{\frac{1}{w_{N+1}} - f^T_{N+1} C_N f_{N+1}} \left(y_{N+1} - f^T_{N+1} \hat{\theta}_N \right)$$

- Recalculating matrix C_{N+1} to prepare for the next iteration

$$C_{N+1} = C_N - \frac{C_N f_{N+1} f_{N+1}^T C_{N+1}}{\frac{1}{w_{N+1}} - f_{N+1}^T C_N f_{N+1}} \qquad (7)$$

- With The new estimates $\hat{\theta}_{N+1}$ and C_{N+1} starts the next iteration from point 2 of the algorithm.

1.2. Recursive ordinary least squares.

Recursive least squares can be seen as a special case of RWLS at $W = I$. This means all weights to adopt equal unit, which is possible only if ρ = 1. Therefore, the recursive least squares (RLS) can be obtained, as in the procedure of paragraph 1.1. to lay everywhere ρ = 1. [1,3]

2. Recursive method of instrumental variables.

Finding estimates by the method of instrumental variables much like the least squares

The Instrumental vector v_{N+1} is choosen from (8), or (8 ')

$$v_t^T == [-\hat{y}_{t-1} \quad -\hat{y}_{t-2} \ldots - \hat{y}_{t-n} \quad u_{t-1} \quad u_{t-2} \ldots u_{t-m}], (8)$$

$$v_t^T = [u_{t-1} \quad u_{t-2} \ldots u_{t-m}], \qquad (8')$$

Recursive algorithm method of instrumental variables follows:

- The evaluation starts with recursive weighted LS. This increases the resistance to initial estimates. After accumulating a certain number $N > k$ observations we pass to the recursive method of instrumental variable. N is typically chosen to be equal to 3 to 5 times the number of coefficients.
- The estimated coefficients of $N+1$st iteration are calculated by the formula

$$\hat{\theta}_{N+1} = \hat{\theta}_N + \frac{C_N v_{N+1}}{\rho + f_{N+1}^T C_N v_{N+1}} \left(y_{N+1} - f_{N+1}^T \hat{\theta}_N\right)$$

- The next iteration is prepared by recalculating C_{N+1} by the formula

$$C_{N+1} = \frac{1}{\rho}\left(C_N - \frac{C_N v_{N+1} f_{N+1}^T C_N}{\rho + f_{N+1}^T C_N v_{N+1}}\right) \qquad (10)$$

- Calculations start from item 2 with the new estimates. If it is not necessary to weigh the observations in (9) and (10) is applied $\rho = 1$.[1]

3. Experimental research and results

Studies here relate to the performance, capabilities and reliability of already described recursive methods for parameter estimation in adaptive control systems.

For this purpose, the study used a random input signal dispersion unit and zero mathematical expectation.

Studies are made by using System Identification Toolbox in Matlab\Simulink. For this purpose are made blocks for estimation in real time for the recursive least squares(RLS), recursive least squares with using the remainder instead of the prediction error(RLSr) and recursive instrumental variables (IVu) using S-functions.

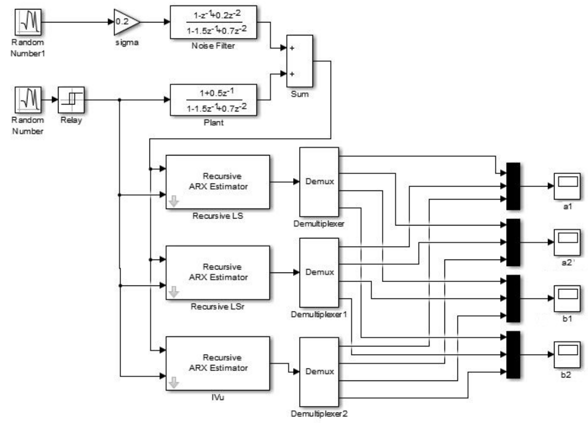

Fig. 1 - Block-diagram for recursive estimation with recursive estimator (RLS,RLSr,IVu)

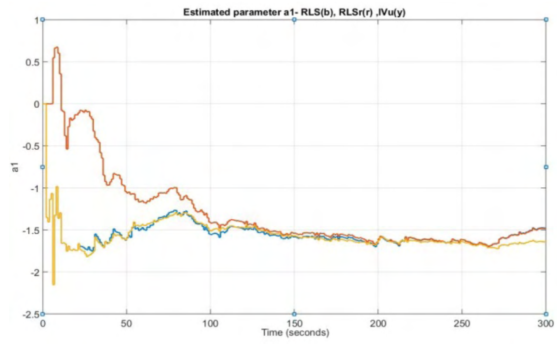

Fig. 2 – Estimated parameter a1- RLS(b),RLSr(r),IVu(y)

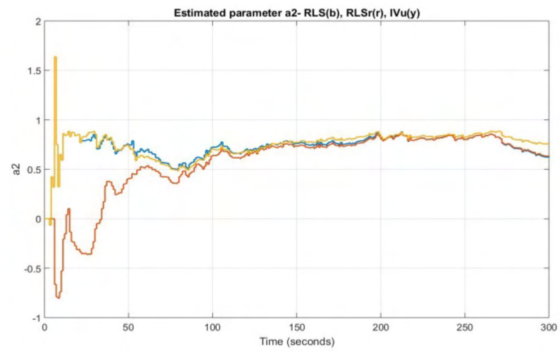

Fig. 3 – Estimated parameter a2- RLS(b),RLSr(r),IVu(y)

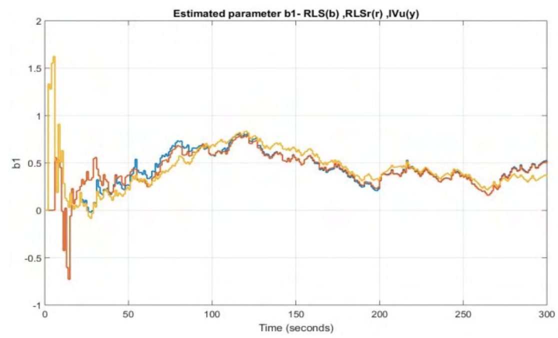

Fig. 4 – Estimated parameter b1- RLS(b),RLSr(r),IVu(y)

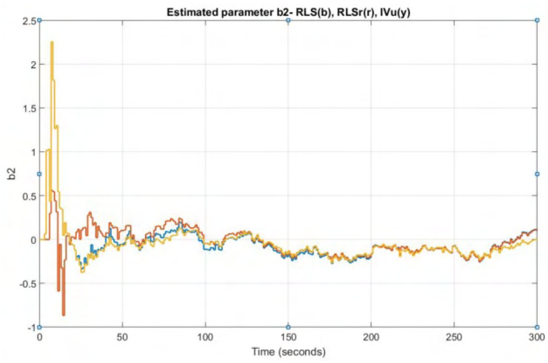

Fig. 5 – Estimated parameter b2- RLS(b),RLSr(r),IVu(y)

4. Analysis and conclusions

- Recursive least squares method (blue line), recursive least squares with using the remainder instead of the prediction error (red line) and recursive instrumental variables (yellow line) are ones of the most reliable methods for parameter estimation and they can be modified for better performance and added to the adaptive control systems in real time
- When selecting a different weighting can be obtained robust variety of assessments that can be noise resistant.
- Recursive methods for parameter estimation in adaptive systems shown in this paper can be used in the field of reliability and diagnostics for fault detection and correction.

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Acknowledgments:

The studies presented in this article were conducted in connection with the project NP6 at the Technical University of Varna, financed from the state budget for 2016