

# A Comparison of Techniques for Characterizing Varied Microstrip Lines

Biljana P. Stošić, Zlata Cvetković

**Abstract** — The varied microstrip taper is fully characterized in frequency-domain. A comparison of different techniques for the line characterization is done. The accuracy of the used methods is studied using analysis of linearly tapered microstrip line. Results of numerical simulations compared with the analytical solutions have shown good agreement.

**Keywords** — Transmission line, Microstrip line, Linearly tapered line, Characterization.

## I. INTRODUCTION

Tapered microstrip lines play an important role in microwave engineering. This nonuniform transmission lines have been extensively used in many applications, e.g., in impedance matching [1], pulse shaping [2, 3], couplers [4], [5], antennas [6], and filters [7]. Namely, impedance matching is an important aspect in the design of microwave and millimeter wave circuitry since impedance mismatches may severely deteriorate the overall performance of electronic systems.

The tapered transmission line performs matching of different impedances, i.e. a physical transition between parts of circuit with different impedances. The taper profile can be chosen in many ways. By changing the type of taper, one can obtain different passband characteristics. Several taper profiles may be considered: linear, exponential, triangular, logarithmic, and so on. In this paper, linearly tapered microstrip line is characterized. Three analysis methods are used and compared here:

- (1) analytical equations based on the transmission line theory,
- (2) numerical simulations in Advanced Design System (ADS),
- (3) numerical simulations obtained by applying the wave digital filter theory.

Analytical method for analyzing tapered transmission lines has been introduced in [8-14]. This method is based on transmission line theory. Here, the method is applied to characterize microstrip taper designed on low-loss substrate.

Wave digital filter theory and its application in modeling and analyzing microstrip circuits of different geometries is explained in details in many papers [15, 16]. The calculating of the wave transfer matrix polynomials for the wave digital networks (WDNs) at hand is discussed in [15]. Here, modification of the existing approach is done in order to find frequency response in WDN which represents model of circuit with different port impedances.

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## II. TAPERED TRANSMISSION LINE

The aim of the analysis given here is matching impedance  $Z_0$  to the impedance  $Z_L$ . A general circuit with included tapered line of length  $L$  is shown in Fig. 1. In the global model, a taper is fed with a voltage source of amplitude  $E$  and internal impedance  $Z_0$ , and is terminated with an impedance  $Z_L$ .

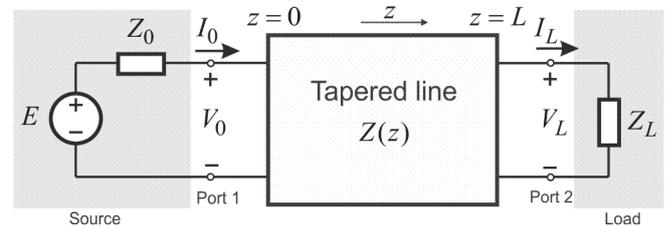


Fig. 1. A circuit with tapered line

In this paper, the terms „linear” refer to the geometrical profile of the tapered line along its length. Line width as a function of the distance  $z$  for linearly tapered line is

$$w(z) = w_{\min} + \frac{w_{\max} - w_{\min}}{L} \cdot z, \quad 0 < z < L, \quad (1)$$

where  $w_{\min} = w(0)$  represents minimum and  $w_{\max} = w(L)$  maximum width of tapered line.

## III. ANALYSIS TECHNIQUES

The methods are applied here to linearly tapered microstrip line. The line is characterized in the frequency domain. Comparison of the analysis results of all three methods explained in the text below is done on case of this line.

(1) Starting up with the transmission line theory, the analytical equations for linear taper are developed. Taper can be treated as lossless transmission line. One can consider an infinitely small portion of uniform line which can be regarded as a two-port circuit with distributed series impedance  $Z(z, \omega)$  and distributed shunt admittance  $Y(z, \omega)$ . Without loss of generality, the series impedance is made of a resistance  $R'dz$  and an inductance  $L'dz$ , where  $R'$  is the resistance per unit length and  $L$  is the inductance per unit length. The shunt admittance is made up of a parallel resistance (of admittance  $G'dz$ ) and a capacitance  $C'dz$ , where  $G'$  is the conductance per unit length and  $C$  is the capacitance per unit length.

Elementary circuit theory shows that the differential equations of transmission line are [5-7]

$$\frac{dV(z, \omega)}{dz} = -Z(z, \omega) \cdot I(z, \omega), \quad (3)$$

$$\frac{dI(z, \omega)}{dz} = -Y(z, \omega) \cdot V(z, \omega), \quad (4)$$

where  $Z(z, \omega) = R(z, \omega) + j\omega L(z, \omega)$  represents the impedance per-unit-length,  $Y(z, \omega) = G(z, \omega) + j\omega C(z, \omega)$  represents the admittance per-unit-length, and  $\omega$  is the angular frequency in radians/second.

Classic transmission line theory states that the characteristic impedance  $Z_0$  of a transmission line can be derived from a knowledge of the resistance  $R$ , inductance  $L$ , conductance  $G$ , and capacitance  $C$  per unit length of the transmission line from the expression

$$Z_0 = \sqrt{\frac{Z(z, \omega)}{Y(z, \omega)}}. \quad (5)$$

The propagation constant is

$$\gamma = \sqrt{Z(z, \omega) \cdot Y(z, \omega)}. \quad (6)$$

Consider a lossless and linearly varied single transmission line with the distributed primary parameters

$$L'(z) = L_0 \cdot (1 + m \cdot z),$$

$$C'(z) = C_0 / (1 + m \cdot z),$$

$$R'(z) = G'(z) = 0,$$

where  $m = (Z_L / Z_0 - 1) / L$ .

The differential equations for voltage and current are

$$\frac{d^2 V(z, \omega)}{dz^2} - \frac{m}{1 + m \cdot z} \cdot \frac{dV(z)}{dz} + \gamma^2 \cdot V(z) = 0,$$

and

$$\frac{d^2 I(z, \omega)}{dz^2} - \frac{m}{1 + m \cdot z} \cdot \frac{dI(z)}{dz} + \gamma^2 \cdot I(z) = 0,$$

where  $\gamma^2 = -Z(z, \omega) \cdot Y(z, \omega) = (\omega / c)^2 \cdot \epsilon_r^{eff}$ .

The solutions of these equations, i.e. the voltage and current distributions across the line, are obtained as

$$V(z) = (1 + m \cdot z) \cdot \{K_1 \cdot J_1(\gamma(1 + mz) / m) + K_2 \cdot Y_1(\gamma(1 + mz) / m)\},$$

and

$$I(z) = \frac{j}{Z_0} \cdot \{K_1 \cdot J_0(\gamma(1 + mz) / m) + K_2 \cdot Y_0(\gamma(1 + mz) / m)\},$$

respectively.  $K_1$  and  $K_2$  are constants found from the boundary conditions

$$V(z=0) = E,$$

and

$$V(z=L) / I(z=L) = Z_L = Z_0 \cdot (1 + mL).$$

Bessel functions of the first kind are denoted by  $J_n(z)$ , and Bessel function of the second kind by  $Y_n(z)$ . Index  $n$  shows their order.

If some expressions are assigned as  $u_1 = u(z=0) = \gamma / m$  and  $u_2 = u(z=L) = \gamma \cdot (1 + mL) / m$ , then constants are

$$K_1 = [Y_1(u_2) - j \cdot Y_0(u_2)] \cdot E \cdot$$

$$\left\{ \begin{array}{l} J_1(u_1) \cdot Y_1(u_2) - J_1(u_2) \cdot Y_1(u_1) + \\ + j \cdot [J_0(u_2) \cdot Y_1(u_1) - J_1(u_1) \cdot Y_0(u_2)] \end{array} \right\}^{-1}$$

and

$$K_2 = -[J_1(u_2) - j \cdot J_0(u_2)] \cdot E \cdot$$

$$\left\{ \begin{array}{l} J_1(u_1) \cdot Y_1(u_2) - J_1(u_2) \cdot Y_1(u_1) + \\ + j \cdot [J_0(u_2) \cdot Y_1(u_1) - J_1(u_1) \cdot Y_0(u_2)] \end{array} \right\}^{-1}$$

Input impedance of the line is

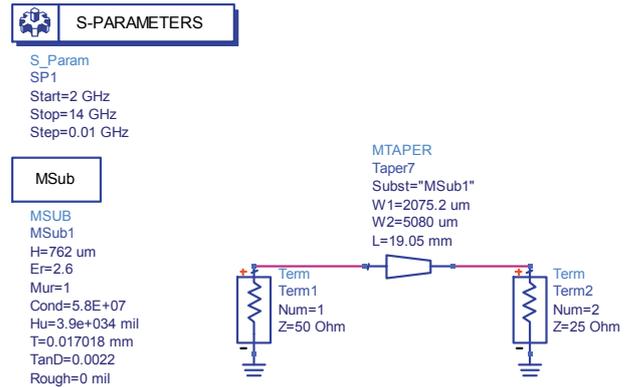
$$Z_{in} = V(z=0) / I(z=0),$$

and the reflection coefficient is

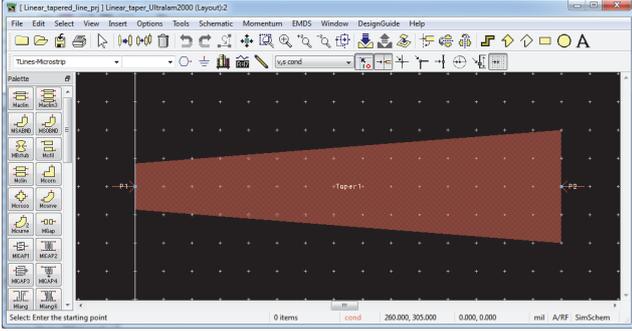
$$\Gamma_{in} = (Z_{in} - Z_0) / (Z_{in} + Z_0).$$

This method can be easily implemented in the MATLAB environment. MATLAB functions *besselj* and *bessely* are employed to find the response. *besselj(nu,Z)* computes the *Bessel function of the first kind*, for each element of the array  $Z$ . *bessely(nu,Z)* computes the *Bessel function of the second kind*. The order  $nu$  need not be an integer, but must be real.

(2) Numerical simulations of the line is done in Advanced Design System (ADS) by schematic shown in Fig. 2a. In ADS, component *Microstrip Width Taper* (MTAPER) is used for representing varied line. The frequency-domain analytical model is a microstrip line macro-model developed by Agilent. The taper is constructed from a series of straight microstrip sections of various widths that are cascaded together. The microstrip line model is the MLIN model. The number of sections is frequency dependent. Dispersion, conductor loss, and dielectric loss effects are included in the microstrip model. Fig. 2b shows layout of tapered line which is designed on low-loss Ultralam substrate.



(a)



(b)

Fig. 2. (a) ADS schematic of varied taper,

(b) Layout of varied taper in ADS

(3) Numerical simulations is done by applying the wave digital filter theory. In Fig. 3, profile of the linear taper as a function of the longitudinal distance is shown. In order to characterized the line that has length  $L$ , the wave digital approach can be applied if the line is approximated with several segments of different widths and equal lengths. Line width  $w(z)$  is a function of distance along line, where  $z$  is distance from the  $Z_0$  side of the line [15, 16]. A good approximation is achieved if the line width is determined at the half of the segment. In other words, for this taper, one can write for the width of the  $i^{th}$  subsection

$$w_i(z) = w_i^{begin} + \frac{w_i^{end} - w_i^{begin}}{2}, \quad i = 1, 2, \dots, N. \quad (2)$$

Here,  $w_i^{end}$  and  $w_i^{begin}$  denote the line width at the beginning and the end, respectively, calculated from Eq. (1).  $N$  is the total number of subsections in which the entire transmission line is broken, and it can be chosen arbitrarily.

A very simple algorithm for direct calculation of polynomial coefficients of rational functions  $S_{21}$  and  $S_{11}$ , developed in  $z$ -domain, is described in [15]. In that paper, one case of circuit consisting of two cascaded linear tapers which forms circuit with equal port impedances is consider. In order to find response of the circuit which port impedances differs one from another, the equations given there are modified. When determine the  $S_{21}$  parameter in wave digital model, one have to take care of ratio  $Z_0 / Z_L$ , and should calculate new value of parameter  $S_{21}$  as  $S_{21} \cdot \sqrt{Z_0 / Z_L}$ .

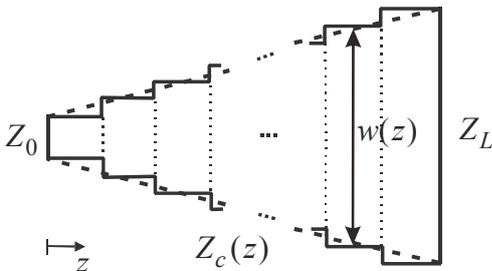


Fig. 3. Linearly tapered line and its approximation for wave digital approach

#### IV. ANALYZED EXAMPLE CIRCUIT

The accuracy of the used methods is studied using analysis of linearly tapered microstrip line. The aim of the analyzed circuit is matching impedance  $Z_0 = 50 \Omega$  to the impedance  $Z_L = 25 \Omega$ . The microstrip line is designed for an Ultralam substrate and considered to be lossless. The parameters are: minimum line width  $w_{min} = 599.44 \mu m$  at  $Z_0$  side of the line, maximum line width  $w_{max} = 5080 \mu m$  at  $Z_L$  side of the line, line length  $L = 19.05 mm$ , relative dielectric constant of substrate  $\epsilon_r = 2.6$  and substrate high  $h = 762 \mu m$ .

Scattering parameters of the linearly varied taper obtained by use of different methods are pictured in Figs. 4-6. They show responses both simulated in MATLAB by proposed approaches and obtained in ADS simulator.

In the case of WDNs, results are obtained in 0.056753 s. If the taper is analyzed directly by using analytical equations, much more simulation time is required, 20.439872 s.

Results of numerical simulations compared with the analytical solutions have shown good agreement.

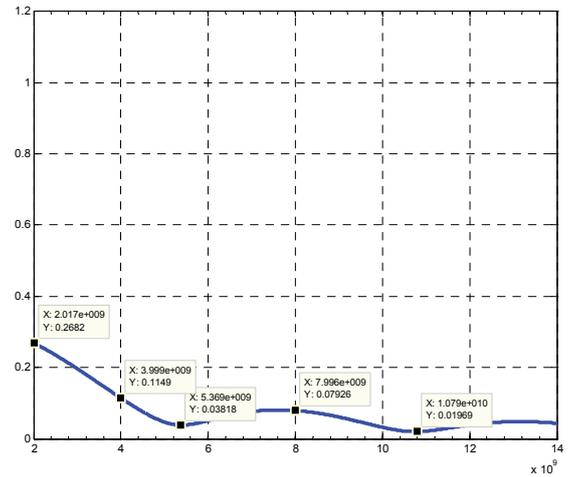


Fig. 4. Reflection coefficient vs. frequency (Method (1))

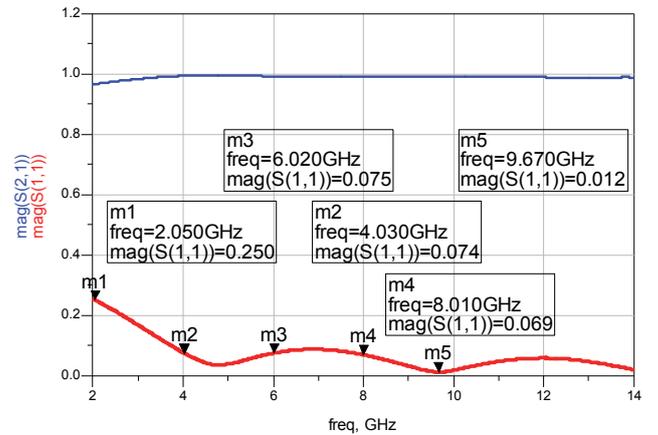


Fig. 5. Reflection and transmission coefficients vs. frequency (Method (2))

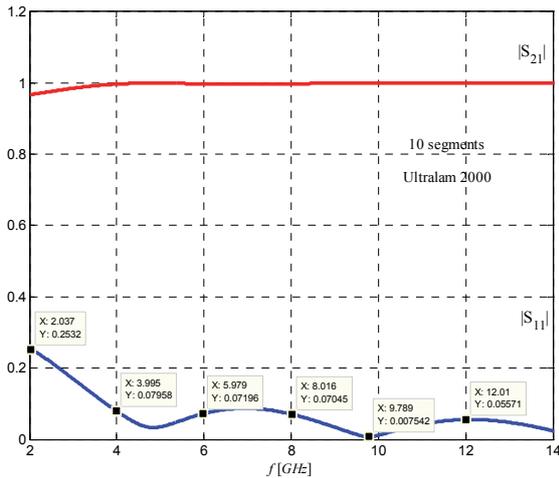


Fig. 6. Reflection and transmission coefficients vs. frequency (Method (3))

## V. CONCLUSION REMARKS

A modification of the wave digital approach is proposed, whose aim is to enlarge types of microstrip structures that can be modeled into WDNs. Types of structures that can be analyzed through the ideas proposed in the paper [15] are those with different port impedances.

The  $S$  parameters are important in microwave design because they are easier to measure and work with at high frequencies. The proposed approach is implemented on a processor Intel(R) Core(TM) i5-3470 CPU @ 3.2 GHz. The analysis of wave digital networks is a very efficient, because there is no high memory request and a very short time is needed for response calculation in the frequency domain directly (a few seconds or even less). This approach provides the faster structure simulation versus complex and time consuming 3D models.

A comparison of different techniques (numerical and analytical solutions) for characterizing the linear taper is done. The advantages and disadvantages of the used techniques are demonstrated by one example and some comparisons (frequency response and estimated time). The computer simulated results obtained by WDNs and analytical equations are compared to those simulations obtained in ADS (simulation based on circuit-level (Circuit-level) and electromagnetic 2.5D MoM simulation (Momentum)). One can observe that results obtained by described approach have good agreement with ADS data in whole frequency band.

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