On Wavelet Based Modeling of EMC Test Chambers – Economic Prediction of the Refined Expansion Coefficients

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Abstract – **Electromagnetic compatibility (EMC) is a key issue in the present smart world, and most of EMC standards require measurement in either fully anechoic chambers or semi anechoic chambers or reverberation chambers. In order to give a sufficiently precise approximation on the measurement uncertainty, it is important to know, i.e., to measure and to model the electromagnetic field distribution inside the chamber around the device under test and the test antenna.**

A one dimensional simplified model is presented for electromagnetic field distribution modelling with a straightforward possibility to extend to higher dimensions. Wavelets are ideal tools for modelling such environments, where the length scale of the obstacles vary, like the test chambers with different sizes of devices under test.

Keywords – **EMC, Fully Anechoic Chamber, Semi Anechoic Chamber, Wavelet, Electromagnetic field modelling.**

I. Introduction

The measurement uncertainty of emission conformity or antenna measurements in fully anechoic chambers (FAC) with 3 m measurement distances is higher, compared to the chambers allowing 10 m measurement distance or to the open area test site (OATS). To understand the phenomenon and the reasons of this effect, simulation methods provide great help. Results computed from simulations can be the key in finding solutions for reducing measurement uncertainty.

The most commonly used tools for modelling anechoic chambers are the finite element method [1], beam-tracing [2], finite differences [3,4], circuital methods [5] or the method of moments [6]. In case of the often-changing devices under test which have various sizes, a model where the adaptivity is a built-in feature can be advantageous. Wavelet based differential equation solver can provide adaptivity [7,8,9]: if the solution is not precise enough, new resolution levels can be added to the already existing solutions thus improving the result.

In the following considerations, after a short introduction to wavelet theory, and wavelet-based differential equation solving methods in Section II, a more detailed summary of a very economic prediction of the finer resolution wavelet coefficients is presented in Section III. The prediction is based on the Ritz variation principle. A one-dimensional model system is built to demonstrate the applicability of the method. As a last step, the conclusions are drawn in Section IV.

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II. Wavelet Based Modelling of Electromagnetic Fields

Wavelet analysis is an effective tool for data analysis, however it can be used for solving differential equations, too [7]. In wavelet analysis and wavelet based function synthesis the space of the functions is divided to subspaces corresponding to different resolution levels, or different detailedness of the function. The key point of wavelet theory that each of these resolution level has the same shaped basis function set, i.e., the wavelets, which arise from one common mother wavelet ψ_0 by simply shifting and shrinking or stretching, thus the mth resolution level basis function at the shift position k can be given as $\psi_{mk}(x)= 2^{m/2}\psi_0(2^{-m}x - k)$.

There exists another type of basis function family; the scaling functions expand sub-spaces of the whole function space that include all the resolution levels up to a given refinement index m. Scaling function arise similarly to wavelets: the mth resolution level basis function at shift index position k can be written as $\phi_{mk}(x)= 2^{m/2}\phi_0(2^{-m}x - k)$, where ϕ_0 is the mother scaling function.

This duality means that any function, like a component of the electromagnetic field, can be expressed both ways, either with only scaling functions – in this case very high resolution level is needed for the whole spatial domain if the function contains fine details at some spots –, with only wavelets – in this case infinitely rough resolution level wavelets might also be necessary –, or in a mixed expansion – starting from a moderately rough resolution level set of scaling functions and adding refinements to only those domains where the function includes finer details.

Both the wavelets and the scaling functions can be generalized to higher dimensions, in those cases the indices are composite resolution level and shift indices, and the variable is also a multi-dimensional variable.

In order to find the possible standing waves and their frequencies, a wave equation

$$\nabla^2 F - \frac{1}{v^2}\cdot\frac{\partial^2}{\partial t^2}F = 0 \qquad (1)$$

has to be solved for the coupled electric and magnetic fields. Here the 3-dimensional field function F can mean either magnetic (H) or electric (E), both having 3 spatial and one

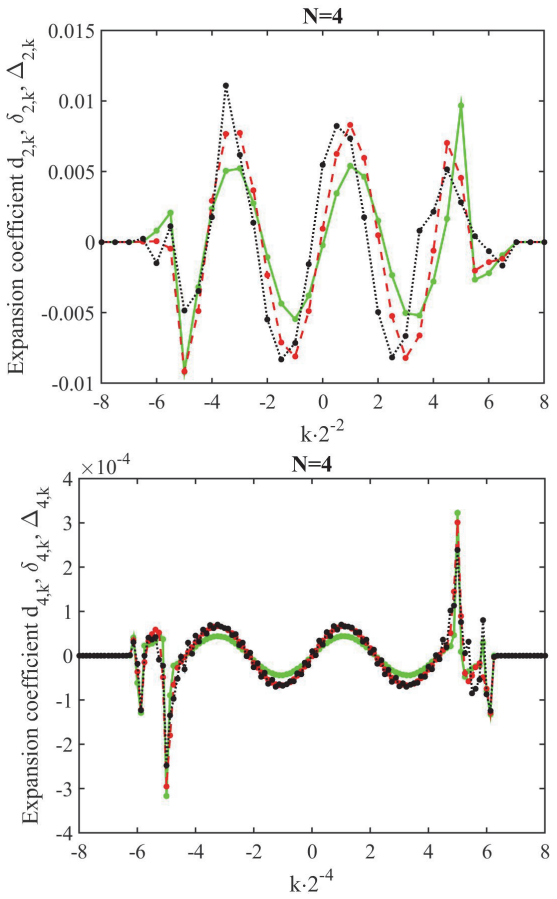

Fig. 1. Coefficients of the eigenfunctions of mode with N=4 nodes (d_{mk} , green line), of the first prediction (δ_{mk} , red dashed line) and the second prediction (Δ_{mk} , black dotted line) for resolution levels m=2 and m=4. The shift index k is scaled so that the position of the coefficients would meet the beginning of the support of wavelet ψ_{mk} in the real space. Arbitrary units. The space of the chamber in the given dimension is 10 units (−5 to 5), the permittivity of the 1 unit thick covering absorbent layer is set to 3, whereas the metallic wall is modelled with permittivity10000 at spatial positions below −6 and above 6 units. Daubechies wavelets and scaling functions with 6 coefficients were used in the calculations

temporal variable. Temporal dimension is usually treated with one of the finite differences methods in the wavelet solver techniques, thus resulting in a simple elliptic equation with a source or an eigenvalue equation

$$\nabla^2 F - S = 0 \tag{2}$$

where the notation S can cover either the source as a function of the space variables at the given time step, or an eigenvalue. An mth resolution level wavelet approximation of the field leads to either a pure scaling function expansion, i.e., to

$$F^{[m]}(r) = \sum_k c_{mk}\phi_{mk}(r), \tag{3}$$

or to a combination of a rough level scaling function expansion refined with as many wavelet resolution levels as necessary

$$F^{[m]}(r) = \sum_k c_{m_0k}\phi_{m_0k}(r) + \sum_{n=m_0}^{m-1}\sum_k d_{nk}\psi_{nk}(r). \tag{4}$$

Using these expansions the differential equations can be straightforwardly transformed into matrix equations: the solution vectors of these matrix equations consist of the wavelet and scaling function expansion coefficients c_{nk} and d_{nk}. Although the matrix equation is simple compared to the differential equation, its solution still has a high computational demand, especially in case of eigenvalue equations. Moreover, in positions, where the solution should be a smooth function, the expansion coefficients d_{nk} are zero, thus calculating them is unnecessary.

III. Predicting Higher Resolution Level Coefficients

If an mth resolution level approximation of the eigenvector solution is already given, naturally arises the question whether it is possible to predict the finer resolution coefficients from the solution in a more economic way than solving the differential equation itself at a higher resolution level. Knowing the approximate next level coefficients can have the following advantages: a) they can be used to determine, whether a wavelet with a given shift index has to be included into the more precise calculation, b) they can be used for calculating the error of the already existing solution, c) they can be used as a last refinement step.

Adding just one wavelet with a variable coefficient δ_{mk} to the already existing solution $F^{[m]}$, as

$$F^{[m+1]pred} = F^{[m]} + \delta_{mk}\psi_{mk} \tag{5}$$

and applying the Ritz variation principle to the thus arising eigenvalue leads to the approximate coefficient

$$\delta_{mk} = -\frac{S^{[m]} - W}{2R} \pm \left(\left(\frac{S^{[m]} - W}{2R}\right)^2 + 1\right)^{\frac{1}{2}}, \tag{6}$$

where the W and R are calculated the same way as the matrix elements in the discretization of the differential equation [8]: W contains only the new wavelet, whereas R both the new wavelet and the already used ones. $S^{[m]}$ is the eigenvalue corresponding to the eigenvector $F^{[m]}$, which is to be refined.

The resulting predicted coefficients δ_{mk} approximate the real coefficients very well. In a 1-dimensional model system consisting of a large air-filled space, a thinner absorber layer and a metallic wall on both sides, we calculated the exact, eigenvector coefficients as well as the predicted ones and plotted the results in Fig. 1. In the plot the expansion coefficients of the eigenfunction with 4 nodes is plotted at two resolution levels. Clearly, the approximation is getting better, as the starting solution is improving with the resolution as it can be seen in Fig. 2.

Seeing the success of the prediction method we have applied it to an already predicted starting coefficient set, i.e., we applied the variation principle to coefficient Δ_{mk} of

$$F^{[m+1]pred2} = F^{[m-1]} + \delta_{m-1k}\psi_{m-1k} + \Delta_{mk}\psi_{mk}\,. \tag{7}$$

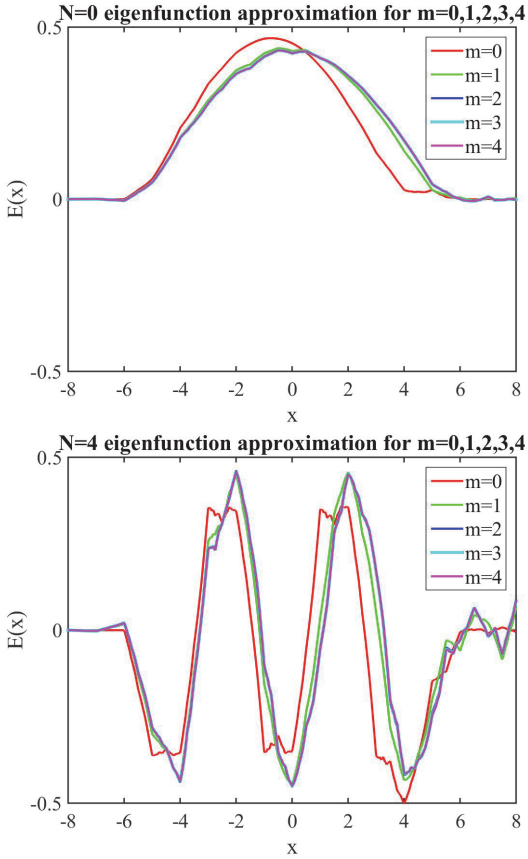

Fig. 2. Wavelet expansion eigenfunctions $E(x)$ for increasing resolution levels in case of the basic mode standing wave (N=0) and the four-node mode (N=4). Arbitrary units. The model is the same as in Fig. 1. Notation m covers the last applied wavelet resolution level, thus the overall resolution is m+1

The resulting formula is very similar to (6)

$$\Delta_{mk} = -\frac{S^{[m]p} - W^{p}}{2R^{p}} \pm \left(\left(\frac{S^{[m]p} - W^{p}}{2R^{p}} \right)^{2} + 1 \right)^{\frac{1}{2}}, \qquad (6)$$

only the matrix elements W^p and R^p, as well as the eigenvalue $S^{[m]p}$ contains not only wavelets from $F^{[m-1]}$, but the ones with the 1st predicted coefficients, too. The second predicted coefficients can be seen in Fig. 1, too, in black dotted lines.

These second predicted coefficients are significantly worse than the first prediction; they tend to oscillate around the real values. This property manifest also in the functions arising from the calculated coefficients. In Fig. 3 these functions are given at two resolution levels; Already at the 4th resolution the functions with the predicted and the eigenvector coefficients are indistinguishable. Their difference is given in Fig 4 for better visibility.

A rather simple averaging method can make the second predicted coefficients better [10].

IV. Conclusion

An early stage one-dimensional forerunner of a wavelet based solver for the standing waves appearing in EMC test chambers like FACs or SACs are presented in the paper. As

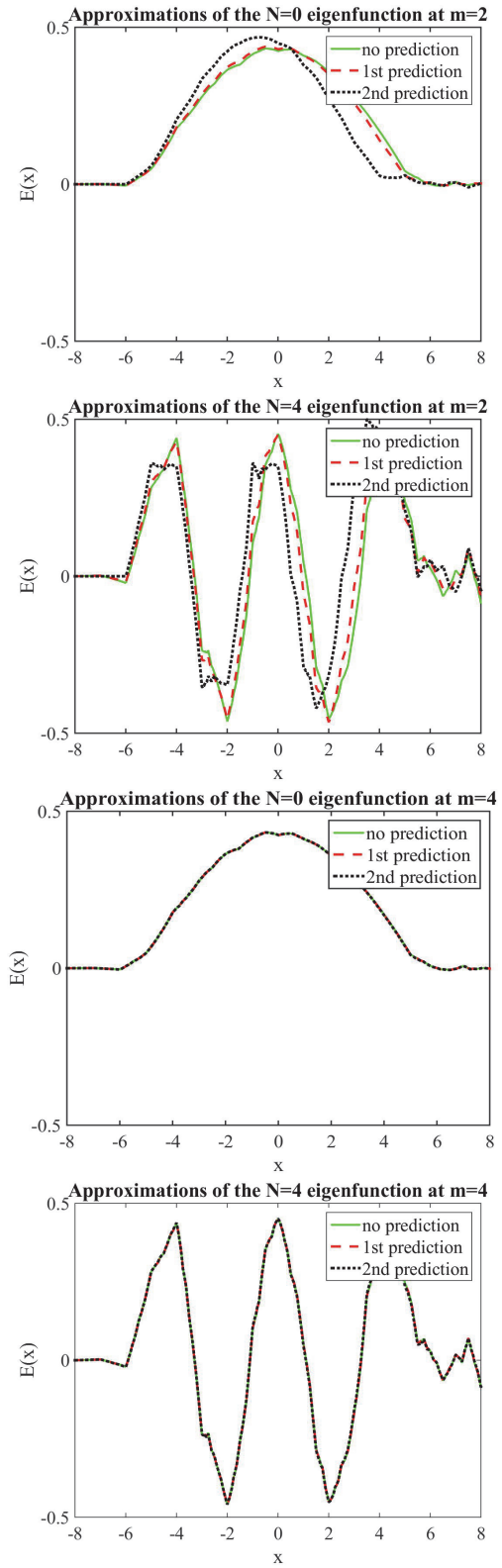

Fig. 3. Approximations of the eigenfunctions with N=0 and N=4 nodes, at scaling function resolution levels m=2 and m=4. Arbitrary units. The model is the same as in Fig. 1. The black lines are the same as the first and third applied resolution level eigenfunctions in Fig. 2 (red line and blue line), the other two lines are predicting the next resolution level results (green and cyan in Fig. 2)

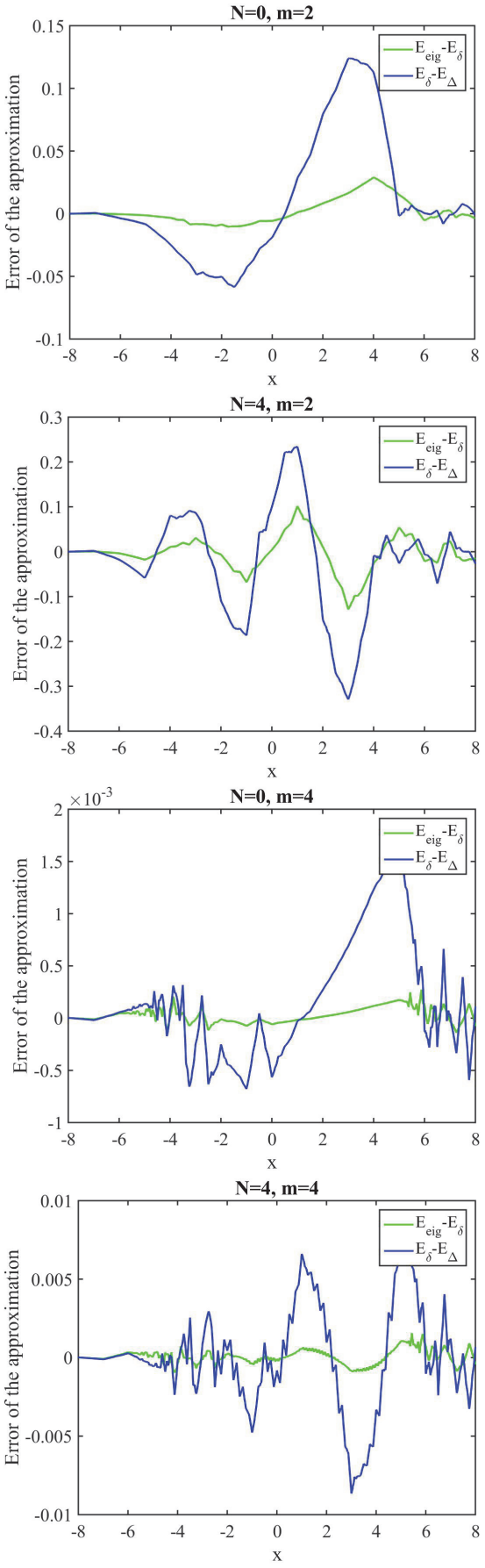

Fig. 4. Difference of the functions from Fig. 3, with predicted wavelet coefficients and the eigenfunctions with N=0 and N=4 nodes, at resolution levels m=2 and m=4. Same model as in Fig. 1. Arbitrary units

the calculation of the wavelet expansion's coefficients is an expensive task, two steps of computationally economic refinement predictions are studied in a simplified model consisting of two various layers around the cavity of the chamber. The result show that the first prediction gives very precise results with less than 0.1% of error eigenvector's coefficients, already at the 4th refinement step (the second refinement level results in approximately 10% of error, and the 3rd refinement level around 1%). All the errors are calculated compared to the given resolution level's wavelet-based eigenfunctions.

At the same time, the second prediction provides much higher errors, usually about 3 to 5 times larger than the first predictions. However, even these second predictions can be used for approximating the magnitude of the error made if the given resolution level is omitted from the calculations completely.

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