Energy and One to Minimum Eigenvalue Spectrum Sensing Algorithm

Amir Eslami[1] and Gunes Karabulut Kurt[2]

Abstract –**Covariance based spectrum sensing methods of cognitive radio technology are known for their non-reliance property to signal and noise power. Using Tracy-Widom distribution of minimum eigenvalue, energy and one to minimum eigenvalue spectrum algorithm is proposed. Unlike energy detection,the performance of the proposed approach does not rely on noise power. Limiting distribution approach to minimum eigenvalue causes the detection algorithm to perform much better in lower SNR ranges. Our proposed method performs better not only in low SNR range but also in low smoothing factors when compared to well known maximum to minimum eigenvaluemethod. The performance improvement is evaluated analytically and compared with other well known algorithms in a wide SNR range and different smoothing factors.**

Keywords –**Spectrum sensing, Cognitive radio technology, Maximum to minimum eigenvalue method, Energy detection, Energy and one to minimum eigenvalue method.**

I. INTRODUCTION

IEEE 802.22 wireless regional area networks are designed to operate over the vacant TV band for broadband access to Wi-Fi devices by using cognitive radio (CR) technology. One the key factors in CR technology is spectrum sensing and reliably detecting the status of the spectrum band being sensed. Researchers have been trying to find new sensing algorithms or to determine ways to increase the performance and reliability of existing methods. There are a wide array of sensing algorithms in the literature including energy detection (ED) [3], [5], matched filter, wavelet based spectrum sensing, cyclostationary based and covariance based detection methods [2]. Each method has its own requirements for the detection process, for instance, the cyclic frequency of primary user is needed for cyclostationary method, waveform information is needed for matched filter method and the noise variance is needed for ED method where the noise uncertainty effects its performance significantly [1]. In this paper by using a limiting approach to the minimum eigenvalue of covariance matrix, we introduce a new method of energy and one to minimum eigenvalue (EOME), inheriting both energy detection and one to minimum eigenvalue effects at once without reliance to any information about the primary user's signal or the noise power. Tracy-Widom distribution of minimum eigenvalue gives EOME a much higher performance in low SNR ranges. There is a method in literature named energy to minimum eigenvalue method (EME) with decision fraction, similar to our proposed method that uses asymptotic approach to the minimum eigenvalue [2], [4], and [9]. EME method has discussed in Section III and it has a much lower performance compared to EOME and maximum to minimum eigenvalue (MME) methods as shown in literature [2], [9] and Section IV of this letter. Rest of the letter is organized as follows. In Section II, system background of cognitive radio systems and in Section III, a brief overview of MME and EME methods are provided. In Section IV, the EOME method is introduced. In Section V, numerical and simulation results are given followed by concluding remarks in Section VI.

Amir Eslamiand Gunes Karabulut Kurt are with the Faculty of Electrical and Electronics Engineering at Istanbul Technical University, Istanbul, Turkey, E-mails: {eslami15, gkurt}@itu.edu.tr

II. SYSTEM BACKGROUND

Assuming that we are interested in sensing a frequency band with central frequency of f_c and W bandwidth, received signals are effected by noise, small and large scale fading effects [2] which can be modeled by different distributions in literature. We assume that additive white Gaussian noise (AWGN) shown as $\eta_c(t)$ in this paper, is stationary process satisfying $E[\eta_c(t)]=0$ and $E[\eta_c(t),\eta_c(t+\tau)]=0$ for any $\tau\neq 0$ with variance σ_η^2. The procedure of detection in cognitive radio systems is a binary hypothesis process. In the case that the channel is being used by primary user, H_1 hypothesis is selected whenever the channel is in idle status, hypothesis H_0 is selected. These two hypotheses for discrete signals which are being sampled with the rate of f_s, compatible with the Nyquist sampling theorem can be shown as follows[2]:

$$\begin{aligned} &H_0 : Y(n) = \eta(n), \\ &H_1 : Y(n) = h(n)X(n) + \eta(n), \end{aligned} \tag{1}$$

where $Y(n)$ is the signal received by detector, $X(n)$ is the signal used by primary user and $h(n)$ is the channel response from source signal to detector. Smoothing factor is used to make time diversity in covariance based spectrum sensing methods by using consecutive signal samples in detection process. Smoothing factor is shown as L in this paper and its effect on received signal samples shown below [2]:

$$\hat{Y}(n) = [Y(n), Y(n-1),..., Y(n-L+1)]^T. \tag{2}$$

In case of hypothesis H_0, where there is no signal sent over the frequency band, the received signal samples with L smoothing factor can be modeled as follows:

$$\hat{\eta}(n) = [\eta(n), \eta(n-1),..., \eta(n-L+1)]^T. \tag{3}$$

Statistical covariance matrices of any matrix like $\hat{Y}(n)$can be defined as:

$$R_{\hat{Y}}(n) = E\left[\hat{Y}(n)\hat{Y}^T(n)\right]. \tag{4}$$

III. Brief Overview of MME and EME Methods

Maximum to minimum eigenvalue is based on the ratio of the maximum eigenvalue to the minimum eigenvalue of the covariance matrix of the received signal. This ratio is then compared to a pre-calculated threshold. If the fraction is larger than the threshold, busy status is concluded and idle is decided otherwise [4]. When the channel is in idle status and no primary user is using the sensed spectrum band, $\boldsymbol{R}_{\hat{Y}}(n)$ is equal to covariance matrix of the noise matrix [10].

$$R_{\hat{\eta}}(N)=\frac{1}{N}\sum_{n=L-1}^{L-2+N}\hat{\eta}(n)\hat{\eta}^{T}(n). \qquad (5)$$

It is shown in the literature that $\frac{e_{max}(Z(N))-\beta}{\Omega}$ converges to Tracy-Widom distribution of order 1 if noise is real and order 2 if noise is complex. e_{max} is the maximum eigenvalue, $Z(N)$, β and Ω are defined as follows:

$$Z(N)=\frac{N}{\sigma_{\eta}^{2}}R_{\hat{\eta}}(N), \qquad (6)$$

$$\beta=\left(\sqrt{N-1}+\sqrt{L}\right)^{2}, \qquad (7)$$

$$\Omega=\left(\sqrt{N-1}+\sqrt{L}\right)\times\left(\frac{1}{\sqrt{N-1}}+\frac{1}{\sqrt{L}}\right)^{\frac{1}{3}}. \qquad (8)$$

Considering N large enough, asymptotic approach in the literature shows that the minimum eigenvalue of $\boldsymbol{R}_{\hat{\eta}}(N)$ is equal to $\frac{\sigma_{\eta}^{2}}{N}(\sqrt{N}-\sqrt{L})^{2}$.

Using the asymptotic value of minimum eigenvalue shown as e_{min} and Tracy-Widom distribution of e_{max}, probability of false alarm in MME detection method can be written as follows [2], [10].

$$P_{fa}=P\left(\frac{e_{\max}}{e_{\min}}>\delta\mid H_{0}\right)=1-F_{1}\left(\frac{\delta\left(\sqrt{N}-\sqrt{L}\right)^{2}-\beta}{\upsilon}\right), \qquad (9)$$

where

$$\delta=\left(\sqrt{\frac{2}{N}}Q^{-1}(P_{fa}+1)\right)\times\left(\frac{N}{(\sqrt{N}-\sqrt{L})^{2}}\right). \qquad (10)$$

Energy to minimum eigenvalue (EME) method is based on the fraction of average energy to asymptotic value of minimum eigenvalue. If the fraction is greater than a pre-defined threshold, method decides busy and idle status is decided otherwise. The probability of false alarm in this case is obtained as:

$$\begin{aligned}P_{fa}&=P\left(\frac{E(t)}{e_{\min}}>\kappa\mid H_{0}\right)\\&=P(E(t)>\kappa\frac{\sigma_{\eta}^{2}}{N}\left(\sqrt{N}-\sqrt{L}\right)^{2}\mid H_{0})\\&\approx Q\left(\frac{\kappa\left(\sqrt{N}-\sqrt{L}\right)^{2}-N}{\sqrt{2N}}\right).\end{aligned} \qquad (11)$$

The expression of κ can be calculated using (11) as:

$$\kappa=\frac{Q^{-1}(P_{fa})\sqrt{2N}+N}{\left(\sqrt{N}-\sqrt{L}\right)^{2}}. \qquad (12)$$

IV. Energy and One to Minimum Eigenvalue Spectrum Sensing Method

It is shown in the literature that minimum eigenvalue can be modeled with a Tracy-Widom distribution. By considering number of samples, N, large enough, $\frac{e_{min}(Z(N))-\Psi}{\upsilon}$converges to Tracy-Widom of order 1 if noise is real and order 2 if noise is complex [6], [8]. e_{min} is minimum eigenvalue, $Z(N)$ is the same as equations 6. Ψ and υ are defined as:

$$\psi=(\sqrt{N}-\sqrt{L})^{2}, \qquad (13)$$

$$\upsilon=(\sqrt{L}-\sqrt{N})\times\left(\frac{1}{\sqrt{L}}-\frac{1}{\sqrt{N}}\right)^{\frac{1}{3}}. \qquad (14)$$

Then, the probability of false alarm of one to minimum eigenvalue (OME) can be defined as:

$$P_{fa}=P\left(\frac{1}{e_{\min}}>\theta\mid H_{0}\right)=P\left(1>\theta e_{\min}\mid H_{0}\right). \qquad (15)$$

By using limiting the distribution of minimum eigenvalue theorem and assuming noise as real, (15) can be written as:

$$\begin{aligned}P_{fa}&=P\left(1>\theta\frac{\sigma_{\eta}^{2}}{N}Z(N)\right)=P\left(\frac{N}{\theta\sigma_{\eta}^{2}}>Z(N)\right)\\&=P\left(\frac{\left(N/\theta\sigma_{\eta}^{2}\right)-\psi}{\upsilon}>\frac{Z(N)-\psi}{\upsilon}\right)\\&=F_{1}\left(\frac{\left(N/\theta\sigma_{\eta}^{2}\right)-\psi}{\upsilon}\right).\end{aligned} \qquad (16)$$

So we obtain:

$$\frac{N}{\theta\sigma_\eta^2} = \upsilon F_1^{-1}(P_{fa}) + \psi \; . \tag{17}$$

Here, θ can be calculated as:

$$\theta = \frac{N}{\sigma_\eta^2(\upsilon F_1^{-1}(P_{fa}) + \psi)} \; . \tag{18}$$

In the case of complex noise, the order of Tracy-Widom distribution in the (16) is 2. From equation (18), it is obvious that threshold of OME relies on noise power. Probability of thenormalized energy of received signals, $E(N)$, be bigger than a threshold named ϕunder hypothesis H_0 can be shown as [1], [3]:

$$P_{fa} = P\big(E(N) > \phi \mid H_0\big). \tag{19}$$

Assuming signal samples large enough and using central limit theorem, probability density function of $E(N)$ under hypothesis H_0, becomes a normal distribution with mean equal to σ_η^2 and variance equal to $\frac{2\sigma_\eta^4}{N}$. So, probability of false alarm can be calculated as follows:

$$P_{fa} = P\left(\frac{E(N) - \sigma_\eta^2}{\sqrt{2\sigma_\eta^4 / N}} > \frac{\phi - \sigma_\eta^2}{\sqrt{2\sigma_\eta^4 / N}} \mid H_0\right)$$

$$= Q\left(\frac{\phi - \sigma_\eta^2}{\sqrt{2\sigma_\eta^4 / N}}\right), \tag{20}$$

where, ϕ can be calculated as [3]:

$$\phi = \sigma_\eta^2\left(1 + \frac{\sqrt{2}Q^{-1}(P_{fa})}{\sqrt{N}}\right). \tag{21}$$

Considering that the values of normalized energy, ϕ and one to minimum eigenvalue are always positive, and combining equations (15) and (19), we can obtain:

$$P_{fa} = P\left(\frac{1}{e_{\min}} \times E(N) > \theta \times \phi \mid H_0\right)$$

$$= P\left(\frac{E(N)}{e_{\min}} > \lambda \mid H_0\right), \tag{22}$$

where λ can be calculated as:

$$\lambda = \theta \times \phi = \frac{N}{\left(\upsilon F_1^{-1}(P_{fa}) + \psi\right)} \times \left(1 + \frac{\sqrt{2}Q^{-1}(P_{fa})}{\sqrt{N}}\right). \tag{23}$$

By simplifying λ, threshold of EOME method can be evaluated as:

$$\lambda = \frac{N + \sqrt{2N}Q^{-1}(P_{fa})}{(\upsilon F_1^{-1}(P_{fa}) + \psi)} \; . \tag{21}$$

EOME sensing method is given in algorithm 1.

The complexity of covariance based spectrum methods stem from the computational of covariance matrix, $R_{\hat{Y}}(N)$ and eigenvalue decomposition of the covariance matrix. For covariance matrix computations, LN multiplications and $L(N-1)$ additions are needed [2]. For eigenvalue decomposition, fast SVD symmetric algorithm [7] can be used with $O(L^2 logL)$ multiplications and additions which makes the total computational complexity equal to $LN + O(L^2 logL)$.

V. Numerical Results and Discussion

The performance of proposed spectrum sensing algorithm is evaluated with the help of numerical results obtained using MATLAB software simulations and compared with very well-known methods. All simulations are using modulated random primary user signals and i.i.d. Gaussian distributed noise samples are used. It is assumed that channel is slow fading and doesn't change during the sampling period. P_{fa} is chosen as 0.1, 10^3 signal samples are used and the stopping criterion is set at 10^4 iterations.

First we have plotted the probability of detection of EOME method vs. SNR passing through Gaussian channel in figure 1 compared with EME and MME methods with smoothing factors of 4 and 16 and ED method without and with noise uncertainty 0.5dB. ED has a good performance but highly reliable to accuracy of noise power measurement in a way that with an only 0.5dB uncertainty in noise power estimation, performance of ED decrease in a high range that makes this method unusable in practice.

Algorithm 1. Energy and one to minimum eigenvalue spectrum sensing method.

Input: λ, P_{fa}, L
Output: D_i
1: X_n = received signal samples, n=1,2,3,…,N
2: Create $R_{\hat{Y}}(N)$ matrix using L consecutive samples
3: Calculate the eigenvalues of covariance matrix $R_{\hat{Y}}(N)$
4: Calculate normalized energy of received signals, $E(N)$
5: Find the minimum eigenvalue, e_{min}
6: $\zeta = \frac{E(N)}{e_{min}}$
7: **if** $\zeta<\lambda$
8: $D_i \leftarrow H_0$
9: **else**
10: $D_i \leftarrow H_1$
11: **return** D_i
12: **end** for

EME has a much lower performance compared to other mentioned blind methods such as MME and EOME because of using asymptotic value of minimum eigenvalue in EME threshold measurement. In smoothing factor of 4, EOME method performs better compared to MME in all SNR ranges. Higher smoothing factors increase the performance of covariance based spectrum sensing methods with the cost of

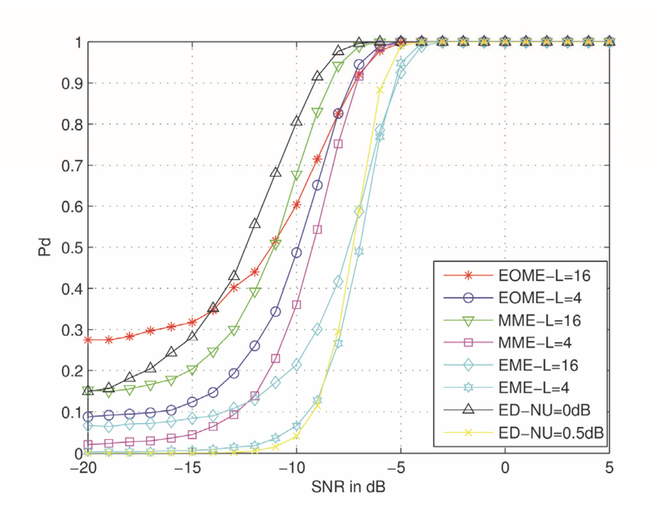

Fig. 1. Simulation results for EOME, EME, MME, all with L=4 and 16 and ED with 0 and 0.5 dB noise uncertainty sensing methods

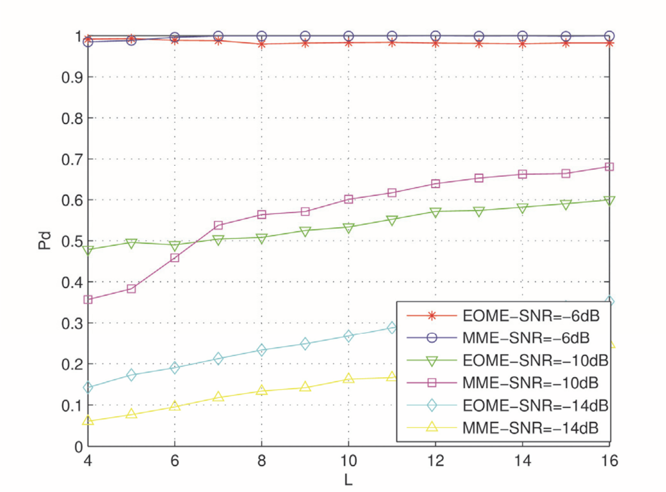

Fig. 2. Simulation results for EOME and MME sensing methods with different smoothing factors in SNRs equal to -6dB, -10dB and -14dB

higher computational complexity as shown in section IV. In high smoothing factors, MME has a better performance in SNRs higher than -11dB but in lower SNRs, EOME performs so much better than MME method passing through Gaussian channel.

Figure 2 shows the effect of smoothing factors between 4 and 16 on MME and EOME in different SNRs equal to -6dB, -10 dB and -14dB passing through Gaussian channel. In high SNR ranges, as mentioned earlier, both MME and EOME methods performs good in all smoothing factors. In -10dB for example, both methods performs almost like each other. In low SNRs such as -14dB, EOME performs much higher than MME method in the specified smoothing factors but in SNRs near to -10dB, performance of EOME is better in smoothing factors lower than 5.

VI. Conclusion

We introduced EOME, a blind sensing algorithmperformance of doesn't rely on any information about primary signal or noise power, by jointly using Tracy-Widom distribution of minimum eigenvalue and energy detection effects at once. Limiting distribution approach to minimum eigenvalue has made EOME perform better when compared to MME and EME methods in low SNR ranges. For devices with processing shortage, EOME performs better with low smoothing factors causing reduced computational complexity which helps making its implementation possible in practice. We also suggested to use fast SVD symmetric algorithm to have a faster process compared to classical methods. This paper provides findings to detail the effects of using limiting distribution of minimum eigenvalue combined with energy detection in covariance based spectrum sensing approaches, addressing a guideline towards optimal solutions to fulfill fundamental sensing requirements of IEEE802.22 WRAN.

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