Symmetry in Quasi-Static Analysis of Transmission Lines by Using Strong FEM Formulation

Žaklina Mančić[1] and Vladimir Petrović[2]

Abstract – **In the quasi-static analysis of transmission lines, symmetry should be taken into account whenever possible, in order to reduce the number of unknowns and thus the calculation time. In this paper, it is described how to include the symmetry in the calculation of quasi-static lines by using the strong FEM formulation.**

Keywords – **Quasi-static analysis, Finite element method, Strong FEM formulation, Symmetry, Capacitance per unit length.**

I. INTRODUCTION

In recent years, the authors are paying more attention to the strong FEM formulation for analysis of electrostatic problems (Galerkin variant, [1]). While weak FEM formulation is widespread in the literature [2,3] strong FEM formulation can rarely be found and to the authors' best knowledge, no one is researching its application to electromagnetic problems in the way that will be here briefly exposed, except the authors.

Unlike weak FEM formulation, whose basis functions have just C^0 continuity, strong formulation exploits basis functions that have continuity of both function and its first derivative on the element boundaries (C^1 continuity). In electrostatics, this fact allows satisfaction of the boundary condition for continuity of the potential V on the boundaries between elements as well as continuity of the normal component of the vector electric displacement field (D_n) on the boundary between elements. By using the properties of the strong basis functions in the analysis of the symmetric structures, symmetry can be simply taken into account.

In this paper, it will be shown how to solve problems which include symmetry using strong FEM formulation considering examples of square coaxial line, shielded symmetric stripline and shielded microstrip with conducting strip of the zero thickness on the isotropic substrate. The observed examples can be solved with high accuracy. It will be shown that by taking symmetry into account, the required number of unknowns is significantly reduced while maintaining accuracy, as it could be expected. Also, we will show that in some cases it is necessary to discard some basis functions on certain finite elements in order to satisfy both boundary conditions.

FEM prodecure applied here is described in details in [4-8]. According to the FEM procedure, closed 2D domain is divided into subdomains, finite elements of rectangular shape. Unknown potential functions of finite elements are approximated as the sum of basis functions with unknown coefficients. These basis functions are polynomials which satisfy both boundary conditions for potential continuity V (C^0 continuity) and vector D_n (C^1 continuity). Potential V is approximated as:

$$V \approx \sum_{j=1}^{N} a_j f_j \, . \tag{1}$$

In this expression a_j are unknown coefficients and f_j are basis functions. 2D closed domain is considered, bounded by contours C_1 (where is $V = V_0$) and C_2 (where is $D_n = D_{n_0}$, $D_{n0} = D_{1n0} - D_{2n0} = \rho_s$ and $\rho_s = 0$). The final system of equations for analysis of 2D problems is [7]:

$$\sum_{j=1}^{N} a_j \left(-\int_{C_1} w_i \varepsilon \frac{\partial f_j}{\partial n} dl + \int_{S_{reg}} \varepsilon \mathrm{grad}_S f_j \mathrm{grad}_S w_i dS + C \int_{C_1} f_j w_i dl \right) = C \int_{C_1} V_0 w_i dl + \int_{C_2} w_i D_{2n0} dl, \; i = 1, 2, \ldots N \tag{2}$$

In Galerkin variant is $w_i = f_i$. This system of equations is valid for both strong and weak formulation and ε is relative permittivity. C is the constant.

In the case when applied basis functions automatically (and exactly) satisfy Dirichlet's condition (for known potential), in the previous equation $C = 0$ should be set for finite elements which have boundary on the conductor. In these elements, the set of basis functions are limited to those which are equal to zero on the conductor surface. Next, additional basis functions which give a constant potential on the conductor surface are added to this set. In the case of node-based functions, these additional basis functions are node-based functions for nodes which are on the conductor. In the case of non node-based basis functions (as in this work), these additional functions are doublets and quadruplets [4] (Figs. 1 and 2). Then, the constant non-zero potential on conductor boundaries is provided by parts of quadruplets and doublets. The corresponding coefficients a_j of these functions are known in advance and in the system of equations all terms of that type are moved to the right-hand side. Now, when the Dirichlet's boundary condition are automatically satisfied, the system of equations is finally:

$$\sum_{j=1}^{N} a_j \left(-\int_{C_1} w_i \varepsilon \frac{\partial f_j}{\partial n} dl + \int_{S_{reg}} \varepsilon \mathrm{grad}_S f_j \mathrm{grad}_S w_i dS \right) = \int_{C_2} w_i D_{2n0} dl, \; i = 1, 2, \ldots N \tag{3}$$

[1]Žaklina Mančić is with the Faculty of Electrical Engineering University of Niš, Aleksandra Medvedeva Niš 18000, Serbia, E-mail: zaklina.mancic@elfak.ni.ac.rs

[2]Vladimir Petrović is with the Robert Bosch, GmbH, Reutlingen, Germany, E-mail: vladimir.petrovic3@de.bosch.com

The Neumann boundary condition (known D_{n}) is applied in symmetrical cases, when the field is tangential to the symmetry line. In this case $D_{\mathrm{n}} = 0$.

II. STRONG BASIS FUNCTIONS AND SYMMETRY

In the case of analyzed 2D problems, strong basis functions are formed as a product of one-dimensional strong basis functions which are proposed in [9].

Two-dimensional strong basis functions consist of quadruplets (Fig. 1), doublets (Fig. 2) and singlets (Fig. 3). There are four types of quadruplets; quadruplets shown in Fig. 1*a* provide continuity of the potential V on the boundaries between finite elements, whereas quadruplets shown in Fig. 1*b* and Fig. 1*c* provide continuity of D_{nx} and D_{ny}, respectively, while quadruplet shown in Fig. 1*d* provides continuity of both D_{nx} and D_{ny}. Quadruplets have four parts.

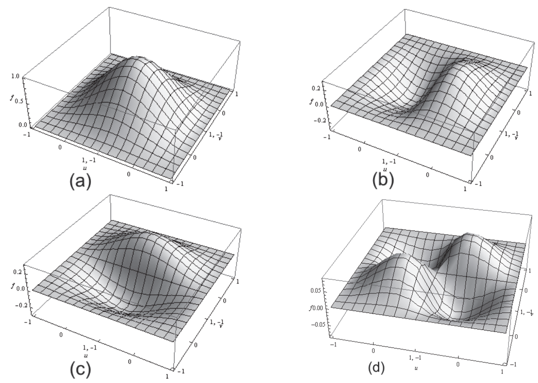

Fig. 1. Quadruplets for the strong FEM formulation in homogeneous media. They provide continuity (a) of V, (b) of D_{nx}, (c) D_{ny}, and (d) both D_{nx} and D_{ny}

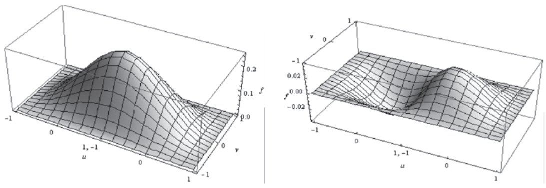

Fig. 2. Doublets for the strong FEM formulation

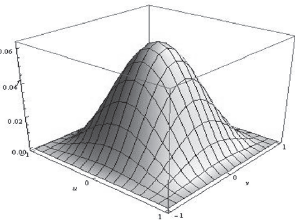

Fig. 3. Typical singlet for the strong FEM formulation

The first quadruplet in Fig. 1 can be considered as node-based basis function. This quadruplet and its parts are used for the implementation (satisfaction) of Dirihlet's boundary condition. The remaining quadruplets in Fig. 2 and their parts are used for satisfying Neumann boundary condition. They give rise to D_n component between the four quadruplet elements. Where, due to symmetry, is $D_n = 0$ at boundaries between the four elements, 1. quadruplet in Fig. 1*d* and 2. either quadruplet in Fig. 1*b* or the one in Fig. 1*c*, depending on the orientation of the symmetry line, are excluded from the set of basis function, as they give rise to D_n. In this way, the symmetry is involved in the system of equations. When the symmetry is taken into account, doublets are excluded from Fig. 2*b* for the same reason as for quadruplets. Singlets are not excluded from the set because $D_n = 0$ is always valid on the boundaries for them.

A. Examples

We analyze examples of square coaxial line with air dielectric, $\varepsilon_r = 1$ (Fig. 4), shielded stripline with air dielectric and zero strip thickness (Fig. 5), and shielded microstrip line with zero metal thickness (Fig. 6).

In the first example (Fig. 4) symmetry lines are denoted as contour 2 in Fig. 4*b*. On this contour, the field is tangential and the Neumann boundary condition $D_n = 0$ is satisfied. On the contour 1 (on the conductor) the Dirichlet's boundary condition is satisfied.

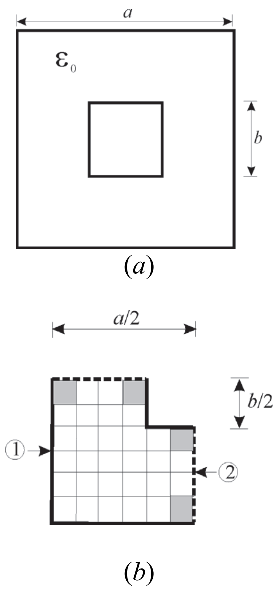

Fig. 4. (a) Square coaxial cable with air ($\varepsilon_r = 1$) and (b) quarter structure

For square coaxial line considered without symmetry, $a/b = 3$, for the number of finite elements $n = 288$, required number of unknowns in the matrice of the system is $N = 1152$ (Fig. 4*a*). When the symmetry is taken into account, Fig. 4*b*, it is enough to consider just a quarter of the structure, thus the number of elements is $n = 72$ (=288/4) whereas the number of unknowns $N = 288$. In cases without symetry it is obtained characteristic impedance $Z_c = 60.375\,\Omega$, for $, n_x = n_y = 3$ for the aforementioned number of finite elements and $Z_c = 60.505\Omega$, for

$n_x = n_y = 4$. In the case when symetry is taken into account, it is obtained $Z_c = 60.344\Omega$, for $n_x = n_y = 3$ and $Z_c = 60.468\Omega$ for $n_x = n_y = 4$, for the aforementioned number of finite elements (agreement of these results with benchmark solutions is already shown in [4], for $b/a = 3$, benchmark solution is $Z_c = 60.6109\Omega$. In this case some basis functions on particular finite elements have to be discarded, in order to satisfy adequately boundary conditions (parts of quadruplets or doublets over finite elements denoted using gray color, Fig. 4*b*). This example has already been discussed in [4], but it is not explained how symmetry should be introduced. The emphasis of the paper [4] is placed on discussion about the convergence of the results for the characteristic impedance depending on the both number of finite elements and the order of the basis function. In previous work [5-8], the comparison of the results obtained for this as well as other similar geometries with the results obtained using different numerical methods is performed, while advantages and disadvantages of the proposed method are highlighted, too. However, the purpose of this paper is to discuss how the symmetry should be implemented to satisfy both boundary conditions. The number of finite elements (mesh density) and the order of basis function are the result of previous research of the authors, obtained by comparing the obtained results with the other methods. The same is valid for the following examples, as the structures we analyze have well-known solutions of the high accuracy.

In the second example (Fig. 5, $a/w = 5$, $b/w = 4$) it is also enough to analyze just the quarter of the structure too. Inside the structure is air, $\varepsilon_r = 1$.

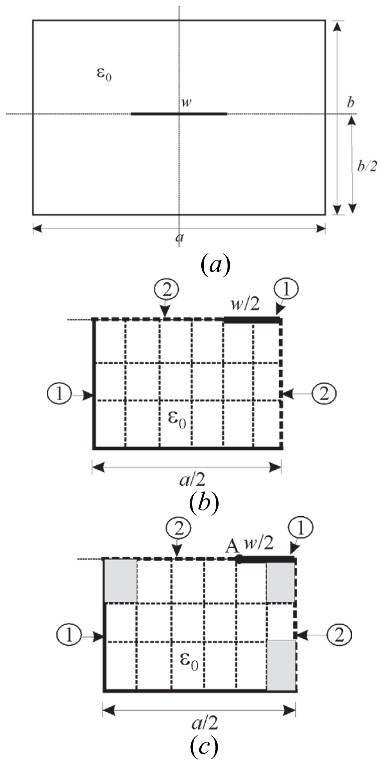

Fig. 5. (*a*) Shielded stripline with zero-thickness of the conducting strip, and (*b*) and (*c*) quarter structure

For observed geometry, software package FEMM [3] gives $Z_c = 94.2789\Omega$ (for 20 nodes and 26 triangular finite elements). Some basis functions on particular finite elements have to be discarded, in order to satisfy adequately boundary conditions, too. Such are, for example, parts of quadruplets or doublets over finite elements denoted using gray color in Fig. 5*c*, which have parts on two contours, so they should satisfy at the same time Dirichlet's condition on the one edge and Neumann's boundary condition on the other one. If it is not possible at the same time, they should be discarded from the set. For such configuration without symmetry for $N = 1536$ unknowns and $n = 384$ mesh elements, $Z_c = 95.91\,\Omega$ is obtained for $n_x = n_y = 3$ order of basis functions. When symmetry is considered for $N = 596$ of unknowns and $n = 96$ mesh elements, it is obtained $Z_c = 94.92\Omega$. For the order of basis function $n_x = n_y = 4$, when the symmetry is taken into consideration (the number of finite elements remains the same and it is $n = 96$) the number of unknowns is raising to $N = 1621$ while the characteristic impedance is $Z_c = 95.808\,\Omega$. This small difference can be explained by the aforementioned exclusion of some basis functions in the solution with symmetry.

As for the third example, microstrip structures with single-layered and multi-layered isotropic, biisotropic and anisotropic substrate and strip of the finite as well as zero thickness are analyzed in [5-8]. Thus, for example, for microstrip of zero thickness of the conducting strip on istotropic substrate, ($a/b = 5$, $b/w = 4$), Fig. 6*a*, it is enough to analyze one half of the structure (Fig. 6*b*).

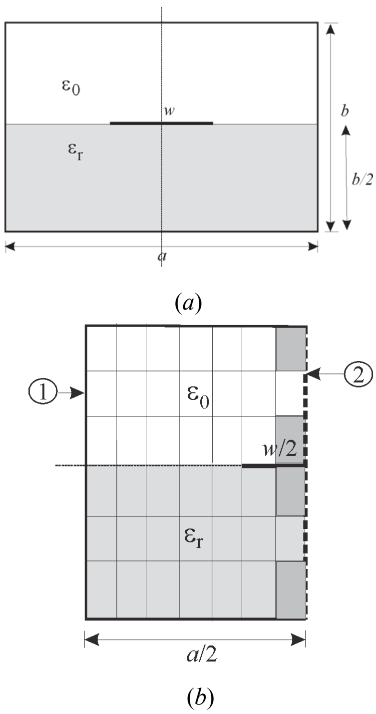

Fig. 6. (*a*) Shielded microstrip, $a/w = 5, b/w = 4$, and (*b*) half structure

Software package FEMM [3] gives $Z_c = 41.5442\Omega$ for 20 nodes and 26 triangular finite elements. For $n = 216$ mesh elements and $N = 864$ unknowns, without symmetry, for the substrate of $\varepsilon_r = 9.3$ it is obtained $Z_c = 41.7298\,\Omega$. When the symmetry is taken into consideration and just the half of the configuration is observed, i.e. $n = 108$ finite elements, it is obtained $Z_c = 41.7615\,\Omega$. If we consider this configuration with the air medium, $\varepsilon_r = 1$, for $n = 216$ mesh elements it is obtained $Z_{c_0} = 95.386\,\Omega$ while if the symmetry is taken into consideration as in Fig. 7, i.e. if it is considered the half of geometry, $n = 108$ mesh elements and $N = 429$ unknowns, it is obtained $Z_{c_0} = 95.9188\,\Omega$. Those small differences between results obtained with and without symmetry can also be explained by the exclusion of some of basis functions in the symmetry case. In the observed case in Fig. 5*a*, Fig. 5*b* and Fig. 6 the order of basis function is $n_x = n_y = 3$ and there are $n = 216$ mesh elements. Besides the case of Fig. 5*a* where it was demonstrated that quarter of the structure is enough to take symmetry into consideration in the case of homogeneous dielectric (air in the observed case), the structure in Fig. 7 demonstrates how to take symmetry into consideration by observing the half of the structure for the same example as in the Fig. 5*a*.

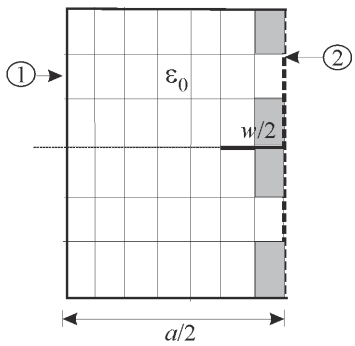

Fig. 7. Half of structure which represents shielded stripline with air

III. Conclusion

As FEM has sparse matrix, it is convenient to take symmetry into account whenever it is possible as the size of the system matrix can be reduced and thus the computing time.

In this paper, it is explained how the symmetry is applied for the strong form of FEM. Because of the special (strong) basis functions, that automatically satisfy C^1 continuity, the use of symmetry leads to exclusion of some basis function on the edges of the reduced calculation domain. Application of this method is shown in three characteristic examples. Because of the exclusion of some basis functions, small differences in results with and without symmetry was observed. It is shown on the few examples that the influence of symmetry does not significantly affect the accuracy of the results but it increases the computation speed and it significantly decreases the number of unknowns.

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