Outage Probability Performance of Hybrid RF/FSO System with SSC Receiver

Milica Petkovic[1] and Goran T. Djordjevic[1]

Abstract – **In this paper, we present the analysis of hybrid radio frequency (RF) / free-space optical (FSO) system. The switch-and-stay (SSC) diversity technique is employed at the receiver. The intensity fluctuations of the optical signal are modelled by Gamma-Gamma distribution, while the RF fink is affected by Rayleigh fading. Novel outage probability expression for hybrid RF/FSO system is derived. The effect of atmospheric turbulence strength and SSC implementation on the hybrid RF/FSO system performance is observed.**

Keywords –**Atmospheric turbulence, free-space optics (FSO), Gamma-Gamma distribution, outage probability, radio frequency (RF) system, Rayleigh fading, switch-and-stay (SSC) diversity technique.**

I. Introduction

As a license-free and cost-effective modern technique, free-space optical (FSO) system has become a good substitution to traditional wireless radio frequency (RF) systems. The FSO provides high-data rates and wide bandwidth, and represents a good solution for the "last mile" problem [1-3]. Beside an idea to be an alternative, the FSO is also suitable to be a complement technique to the RF systems. The FSO signal transmission represents optical wireless communication between transmitter laser and receiver photodetector via atmospheric channel. Hence, the atmospheric conditions have strong influence on the quality of system performance. Due to variations in atmospheric temperature, pressure and altitude, the random changes in the refractive index occurs, which results in the existence of the atmospheric turbulence [1]. Although many statistical models have been proposed to define the effect of this phenomenon, the Gamma-Gamma distribution is accepted as an optimal model, since it gives a good agreement in experimental and theoretical data in wide range of atmospheric conditions [1-4].

In order to alleviate the optical signal degradation due to atmospheric turbulence, many techniques have been borrowed from RF systems. Firstly, spatial diversity techniques have been considered within FSO systems at the transmitting and/or receiving part [5-8]. The extension of the cover area has been accomplished by applying relying technology within FSO systems [9-11]. Furthermore, the idea of mixed RF/FSO systems, where the first hop is RF link and the second hop represents the FSO signal transmission to overcome a connectivity gap between the backbone and last mile access networks, has been proposed in [12]. In addition, the hybrid system, which consists from FSO and RF link, was observed in [13-16]. In the hybrid RF/FSO systems, the same data are transmitted over both links simultaneously. The received signals are combined by some of diversity techniques.

In this paper, we analyze the hybrid RF/FSO system, when the switch-and-stay (SSC) diversity technique is applied at the receiver. The well-known Gamma-Gamma distribution is used to model the effect of atmospheric turbulence. The RF channel experiences Rayleigh fading. Novel expression for the outage probability of the system under investigation is derived. Numerical results are presented, which are used to illustrate and observe the effect of atmospheric turbulence strength and SSC implementation on the overall system performance.

The rest of the paper is organized as follows. The system and channel models are presented in Section II. The outage probability analysis is described in Section III. Numerical results and discussions are given in Section IV, while concluding remarks are presented in Section V.

II. System and Channel Models

In the hybrid RF/FSO system presented in Fig. 1, the information data are transmitted at the same time via both RF and FSO links, i.e., both FSO and RF links will be simultaneously active. At the receiver, dual branch SSC spatial diversity technique is employed, which is the simplest form of switched diversity. In dual-branch SSC based system, when the instantaneous signal-to-noise ratio (SNR) of the active branch falls below a predetermined switching threshold, γ_T, the receiver switches and stays to the other alternative branch, regardless of whether or not the SNR of the other branch is above or below the threshold [17].

A. FSO Channel Model

In the FSO subsystem, we adopt intensity modulation and

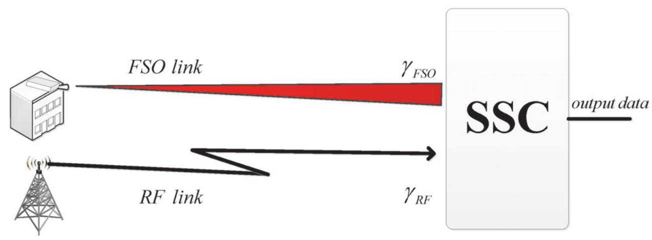

Fig. 1. System model of hybrid RF/FSO system

[1]Milica I. Petkovic and Goran T. Djordjevic are with University of Nis, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mails: milica.petkovic@elfak.ni.ac.rs; goran@elfak.ni.ac.rs

direct detection (IM/DD) with on/off keying (OOK). It is assumed that the intensity fluctuations of the received optical signal are modeled by Gamma-Gamma distribution, thus the probability density function (PDF) optical signal irradiance, I, is given as [1]

$$f_I(I)=\frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)}I^{\frac{\alpha+\beta}{2}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right), \tag{1}$$

where $\Gamma(.)$ is the gamma function defined by [18, Eq. (8.310.1)], and $K_\nu(.)$ is the ν-th order modified Bessel function of the second kind defined by [18, Eq. (8.432.2)]. The atmospheric turbulence parameters α and β are related to the atmospheric conditions. When the plane wave propagation is assumed, the parameters α and β are defined as [1]

$$\begin{aligned}\alpha&=\left(\exp\left(\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{7/6}}\right)-1\right)^{-1},\\ \beta&=\left(\exp\left(\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{12/5}\right)^{5/6}}\right)-1\right)^{-1},\end{aligned} \tag{2}$$

where the Rytov variance σ_R^2 is defined as

$$\sigma_R^2=1.23C_n^2k^{7/6}L^{11/6}. \tag{3}$$

The wave-number is defined as $k=2\pi/\lambda$, where λ is the optical wavelength, L is the FSO link length, and C_n^2 is the index of refraction structure parameter, which is used as a metric for the atmospheric turbulence strength.

The instantaneous SNR, γ_{FSO}, is defined as

$$\gamma_{FSO}=\frac{R^2P_t^2I^2}{\sigma_{n_1}^2}=\frac{R^2P_t^2I^2}{\sigma_{n_1}^2}, \tag{4}$$

where R denotes the detector responsivity, P_t is the average transmitted optical power, and the $\sigma_{n_1}^2$ is the additive white Gaussian noise (AWGN) variance.

The electrical SNR, μ_{FSO}, is determined as

$$\mu_{FSO}=\frac{R^2P_t^2}{\sigma_{n_1}^2}\mathrm{E}^2[I]=\frac{R^2P_t^2}{\sigma_{n_1}^2}, \tag{5}$$

since I is normalized for the Gamma-Gamma distributed atmospheric turbulence $\left(\mathrm{E}[I]=1\right)$.

Based on Eqs. (1), (4) and (5), the PDF of the instantaneous SNR is expressed as

$$f_{\gamma_{FSO}}(\gamma)=\frac{(\alpha\beta)^{\frac{\alpha+\beta}{2}}\gamma^{\frac{\alpha+\beta}{4}-1}}{\Gamma(\alpha)\Gamma(\beta)\mu_{FSO}^{\frac{\alpha+\beta}{4}}}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{\gamma}{\mu_{FSO}}}}\right). \tag{6}$$

The cumulative distribution function (CDF) of γ_{FSO} is determined as

$$F_{\gamma_{FSO}}(\gamma)=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}G_{1,3}^{2,1}\left(\alpha\beta\sqrt{\frac{\gamma}{\mu_{FSO}}}\,\middle|\,\begin{matrix}1\\ \alpha,\ \beta,\ 0\end{matrix}\right), \tag{7}$$

where $G_{p,q}^{m,n}(\cdot)$ is the Meijer's G-function [18, Eq. (9.301)].

B. RF channel model

The signal transmission via RF link is affected by Rayleigh fading, and the instantaneous SNR, γ_{RF}, over RF link is defined as

$$\gamma_{RF}=\frac{P_rh^2}{\sigma_{n_2}^2}, \tag{8}$$

where h is the signal fading amplitude with $\mathrm{E}\left[h^2\right]=1$, where $\mathrm{E}[\cdot]$ is mathematical expectation, P_r represents the RF transmit power, and $\sigma_{n_2}^2$ is the AWGN variance.

The average SNR, μ_{RF}, over RF link is determined as

$$\mu_{RF}=\mathrm{E}[\gamma_{RF}]=\frac{P_r}{\sigma_{n_2}^2}\mathrm{E}[h^2]=\frac{P_r}{\sigma_{n_2}^2}. \tag{9}$$

The PDF of the instantaneous SNR over RF link is expressed as [17]

$$f_{\gamma_{RF}}(\gamma)=\frac{1}{\mu_{RF}}\exp\left(-\frac{\gamma}{\mu_{RF}}\right), \tag{10}$$

while the CDF is

$$F_{\gamma_{RF}}(\gamma)=1-\exp\left(-\frac{\gamma}{\mu_{RF}}\right). \tag{11}$$

$$F_{ssc}(\gamma)=\begin{cases}\dfrac{F_{\gamma_{FSO}}(\gamma_T)F_{\gamma_{RF}}(\gamma_T)}{F_{\gamma_{FSO}}(\gamma_T)+F_{\gamma_{RF}}(\gamma_T)}\left(F_{\gamma_{FSO}}(\gamma)+F_{\gamma_{RF}}(\gamma)\right), & 0<\gamma\le\gamma_T,\\[2ex] \dfrac{F_{\gamma_{FSO}}(\gamma_T)F_{\gamma_{RF}}(\gamma_T)}{F_{\gamma_{FSO}}(\gamma_T)+F_{\gamma_{RF}}(\gamma_T)}\left(F_{\gamma_{FSO}}(\gamma)+\dfrac{F_{\gamma_{FSO}}(\gamma)}{F_{\gamma_{FSO}}(\gamma_T)}+F_{\gamma_{RF}}(\gamma)+\dfrac{F_{\gamma_{RF}}(\gamma)}{F_{\gamma_{RF}}(\gamma_T)}-2\right), & \gamma>\gamma_T,\end{cases} \tag{13}$$

III. Outage Probability Analysis

The outage probability of the SSC based system is defined as the probability that the instantaneous SNR at SSC output, denoted by γ_{SSC}, drops below a predetermined outage threshold, denoted by q. For the considered scenario of the hybrid RF/FSO system, the outage probability can be obtained as [17]

$$P_{out} = F_{ssc}\left(q\right), \tag{12}$$

where $F_{ssc}\left(q\right)$ is the CDF of the instantaneous SNR at SSC output, γ_{SSC}, defined in Eq. (13) at the bottom of the previous page, and $F_{\gamma_{FSO}}\left(\cdot\right)$ and $F_{\gamma_{RF}}\left(\cdot\right)$ are the CDFs defined by Eqs. (7) and (11), respectively.

IV. Numerical Results

Numerical results obtained based on the outage probability expression in Eqs. (12) and (13) are presented. The atmospheric turbulence parameters α and β are determined by Eq. (2). The atmospheric turbulence strength is determined by the index of refraction structure parameter with values $C_n^2 = 6\times10^{-15}$ m$^{-2/3}$, $C_n^2 = 2\times10^{-14}$ m$^{-2/3}$ and $C_n^2 = 5\times10^{-14}$ m$^{-2/3}$ in weak, moderate and strong turbulence conditions, respectively.

Fig. 2 presents the hybrid RF/FSO system outage probability dependence on the average and electrical SNRs in different atmospheric turbulence conditions. System performs better when the FSO link is affected by weak atmospheric turbulence, i.e., when the index of refraction structure parameter is lower. Beside the hybrid RF/FSO system, the outage probability of the dual branch RF system with SSC receiver is also observed, which is obtained based on Eqs. (12) and (13) considering that $F_{\gamma_{FSO}}\left(\cdot\right) = F_{\gamma_{RF}}\left(\cdot\right)$. It can be noticed that the hybrid RF/FSO system has better outage probability performance compared to RF system, only when the FSO link is affected by favorable conditions. On the other hand, when the FSO signal transmission suffers from moderate and strong turbulence, dual branch RF system has better performance.

The outage probability dependence on the average and electrical SNRs in weak and strong atmospheric turbulence conditions is presented in Fig. 3, together with the outage probability results of the dual branch SSC based FSO system, which is obtained based on Eqs. (12) and (13) considering that $F_{\gamma_{RF}}\left(\cdot\right) = F_{\gamma_{FSO}}\left(\cdot\right)$. When the FSO links are affected by weak atmospheric turbulence, which correspond to clear air terms (sunny days), system with only FSO links has better performance than the hybrid RF/FSO system. When signal transmission environment is influenced by foggy weather conditions, atmospheric turbulence is strong, and the hybrid RF/FSO system has better performance, since the RF link is less dependent on the fog than the FSO link. Since FSO

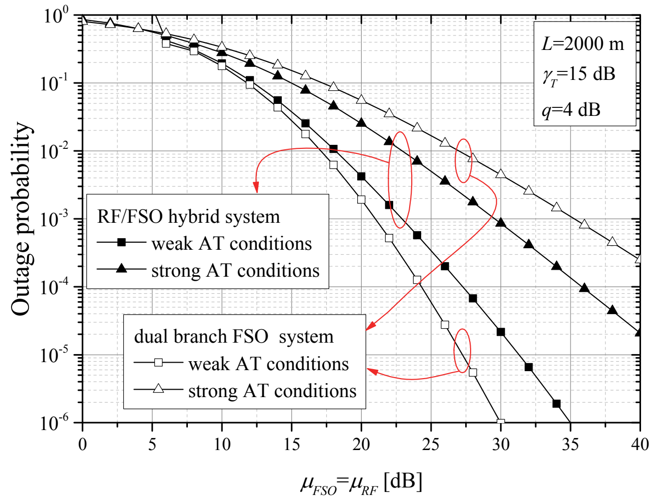

Fig. 3. Outage probability versus μ_{FSO}=μ_{RF} for hybrid RF/FSO and dual branch FSO system

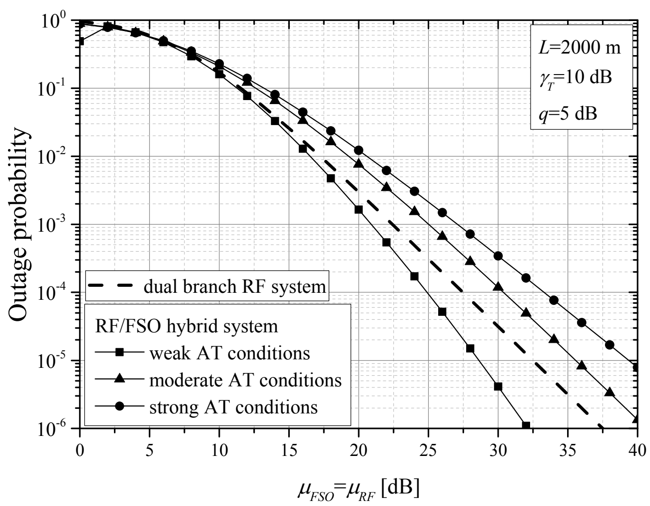

Fig. 2. Outage probability versus μ_{FSO}=μ_{RF} for hybrid RF/FSO and dual branch RF system

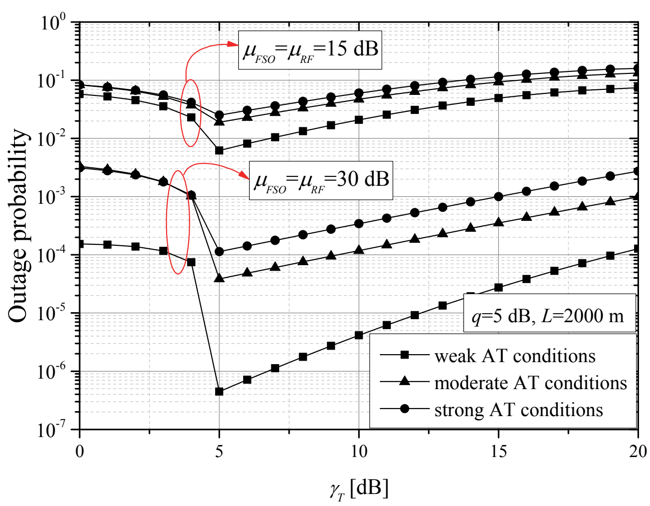

Fig. 4. Outage probability versus the switching threshold in various atmospheric turbulence conditions

system implementation is easier and less expensive than the RF system implementation, the balance between cost and quality of the transmission can be made to provide optimal system performance.

In Fig. 4, the outage probability in function of the switching threshold is observed, considering different atmospheric turbulence conditions. Both average SNR over RF link and electrical SNR over FSO link take values 15 dB or 30 dB. As it is expected, greater values of SNR lead to the better system performance. The optimal value of the switching threshold can be noticed, when the system has the best performance. This optimal value of the switching threshold is equal to the value of the predetermined outage threshold, i.e., $\gamma_T=q$. In this case, the SSC receiver behaves as selection combiner receiver, which selects the channel with greater SNR, leading to the best outage probability performance.

V. Conclusion

In this paper, we have presented the outage probability analysis of the hybrid RF/FSO system. The SSC diversity receiver has been employed at the receiver. The FSO link is affected by the Gamma-Gamma distributed atmospheric turbulence, while the RF link is subject to Rayleigh fading. The outage probability expression has been derived. The effect of atmospheric turbulence strength on the system performance has been observed and compared with the performance of dual branch SSC RF and dual branch SSC FSO systems. It has been concluded that hybrid RF/FSO system performs better than the dual branch SSC based RF system only when the FSO link is affected by weak atmospheric conditions. Furthermore, when the weather conditions are good (sunny days), it is more profitable to implement two FSO links than the hybrid RF/FSO system, since the implementation of the FSO link is easy and low-cost. On the other hand, when the optical signal transmission is under the influence of foggy weather conditions, atmospheric turbulence and visibility are strong and harmful, and the hybrid RF/FSO system performs better than the dual branch FSO system.

Acknowledgement

This work was supported by the Ministry of Education, Science and Technology Development of the Republic of Serbia under Grant TR-32028.

References

[1] L.C. Andrews, and R.N. Philips, *Laser Beam Propagation through Random Media*, 2nd ed., Bellingham, Washington, USA, Spie Press, 2005.

[2] Z. Ghassemlooy, W. Popoola, and S. Rajbhandari, *Optical Wireless Communications: System and Channel Modelling with MATLAB,* Boca Raton, USA, CRC Press, 2013.

[3] M. A. Khalighi, and M. Uysal, "Survey on Free Space Optical Communication: A Communication Theory Perspective", *IEEE Commun. Surveys Tuts.*, vol. 16, no. 4, pp. 2231-2258, 2014.

[4] M.A. Al-Habash, L.C. Andrews, and R.L. Phillips, "Mathematical Model for the Irradiance Probability Density Function of a Laser Beam Propagating through Turbulent Media", *Opt. Eng.*, vol. 40, no. 8, pp. 1554-1562, 2001.

[5] T. A. Tsiftsis, H. G. Sandalidis, G. K. Karagiannidis, and M. Uysal, "Optical Wireless Links with Spatial Diversity over Strong Atmospheric Turbulence Channels", *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 951-957, 2009.

[6] E. Bayaki, R. Schober, and R. K. Mallik, "Performance Analysis of MIMO Free-Space Optical Systems in Gamma-Gamma Fading", *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3415-3424, 2009.

[7] W. Zixiong, W. Zhong, S. Fu, and C. Lin, "Performance Comparison of Different Modulation Formats over Free-space Optical (FSO) Turbulence Links with Space Diversity Reception Technique", *IEEE Photon. J.*, vol. 1, no. 6, pp. 277-285, 2009.

[8] H. Moradi, H.H. Refai, and P.G. LoPresti, "Switch-and-stay and Switch-and-examine Dual Diversity for High-speed Free-space Optics Links", *IET Optoelectron.*, vol. 6, no. 1, pp. 34-42, 2012.

[9] M. Safari, and M. Uysal "Relay-assisted Free-space Optical Communication", *IEEE Trans. Wireless Commun.*, vol. 7, pp. 5441-5449, 2008.

[10] J.Y. Wang, J.B. Wang, M. Chen, Y. Tang, and Y. Zhang, "Outage Analysis for Relay-aided Free-space Optical Communications over Turbulence Channels with Nonzero Boresight Pointing Errors", *IEEE Photon. J.*, vol. 6, 7901815, 2014.

[11] G.K. Karagiannidis, T.A. Tsiftsis, and H.G. Sandalidis, "Outage Probability of Relayed Free Space Optical Communication Systems", *Electron. Lett.*, vol. 42, pp. 994-995, 2006.

[12] E. Lee, J. Park, D. Han, and G. Yoon, "Performance Analysis of the Asymmetric Dual-hop Relay Transmission with Mixed RF/FSO Links", *IEEE Photonic Tech. L.*, vol. 23, no. 21, pp. 1642-1644, 2011.

[13] N.D. Chatzidiamantis, G.K. Karagiannidis, E.E. Kriezis, and M. Matthaiou, "Diversity Combining in Hybrid RF/FSO Systems with PSK Modulation", *ICC 2011 Conference*, pp. 1-6, Kyoto, 2011.

[14] B. He, and R. Schober, "Bit-interleaved Coded Modulation for Hybrid RF/FSO Systems", *IEEE Trans. Commun.*, vol. 57, no. 12, pp. 3753-3763, 2009.

[15] T. Rakia, H.C. Yang, M.-S. Alouini, and F. Gebali, "Outage Analysis of Practical FSO/RF Hybrid System with Adaptive Combining", *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1366-1369, 2015.

[16] M. Usman, H.C. Yang, and M.-S. Alouini, "Practical Switching-based Hybrid FSO/RF Transmission and its Performance Analysis", *IEEE Photon. J.*, vol. 6, no. 5, pp. 1-13, 2014.

[17] M. K. Simon, and M.-S. Alouni, *Digital Communication over Fading Channels*, 2nd ed., New York, John Wiley & Sons Inc., 2004.

[18] I.S. Gradshteyn, and I.M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed., New York, Academic, 2000.