Outage Probability of Cooperative Multi-hop Multiuser Relaying Networks

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***Abstract* – The multi-hop amplify-and-forward (AF) relay systems with larger number of mobile users is considered. In order to improve a signal transmission quality in environment with many obstacles, the multi-hop cooperative relays with maximum-ratio-combining (MRC) scheme are implemented. Opportunistic scheme for signal transmission to end users is applied. Outage probability expression for considered system is derived and numerical results are presented and analyzed.**

***Keywords* – Amplify-and-forward relay, Multi-hop, Multiuser, Rayleigh fading.**

I. Introduction

The cooperative relaying technology provides increased coverage area and balanced quality of service for all users. Moreover, applying relays in wireless infrastructure the benefit from the spatial diversity and signal combining techniques can be utilized, leading to a reduction of the fading and shadowing effects. This kind of systems bring greater diversity gain and decreased pathloss. Increasing the number of relays in series results in the pathloss decreasing, while greater number of parallel relays causes increased diversity gain [1].

Many papers consider the multi-hop system with amplify-and-forward (AF) or decode-and-forward (DF) relay protocols without using cooperative diversity [2-4]. In cooperative relaying system, the spatial diversity is based on multiple information signal copies arrived to the destination. The exact symbol error probability (SER) of network with parallel AF nodes which transmit signal only to the destination is derived in [5]. Multi-hop diversity system in which each nod receives signals form all the previous nodes is considered in [6]. The performance analysis of AF and DF relaying diversity is presented for the instantaneous end-to-end signal-to-noise ratio (SNR) using approximation approach. In [7], the relays process and forward the signals to other relays, as well as to the destination, over Rayleigh fading channels. The maximum-ratio-combining (MRC) scheme is applied to the receivers. However, the derived SER is mathematically difficult to track and performance analysis is complicated. Therefore, the authors in [8] develop an analytical approach for the end-to-end SNR determination of the multi-branch multi-hop system. The analysis in [8] is reduced to the well-known dual-hop system. The MRC technique is applied at both the AF relays and destination in order to combine the signals from the source and previous relays. The average SER expression is presented as a good approximation to the exact simulation results.

In this paper, the multi-hop system discussed in [7-8], is extended to the case when last relay in this network transfers the information signal to a large number of mobile users. In this way, the system exploits the advantages of multi-hop system in the poor non-line-of-sight environment to achieve good signal quality which is then distributed to mobile users. We assume that opportunistic multiuser scheduling scheme is applied in which the mobile user with the strongest channel is selected [9]. The outage probability expression for considered system over Rayleigh fading channels is derived and the impact of number of relays and mobile users are analyzed.

II. System Model

A multi-hop multiuser wireless relay communication network where a source S (R_0) communicates with a destinations D_k ($k=\overline{1,K}$) via AF relays R_i ($i=\overline{1,L}$) is depicted in Fig. 1 (L = 4). Non-line-of-sight environment where direct link between source and relays on one side and destinations on the other side does not exist is explored. Channel gain from the i-th transmission node to the j-th receiving node is denoted as $h_{i,j}$. Index i = 0 is reserved for the source. We assume that amplitude and phase of channel gains are Rayleigh distributed and uniformly distributed over $[0, 2\pi)$, correspondingly. For the Rayleigh fading channel exposed to the influence of a zero mean complex additive white Gaussian noise (AWGN), the instantaneous SNR of link between nodes i and j, denoted as $\gamma_{i,j}$, is modelled as

$$f_{\gamma_{i,j}}(\gamma)=\frac{1}{\overline{\gamma}_{i,j}}\exp\left(-\frac{\gamma}{\overline{\gamma}_{i,j}}\right), \tag{1}$$

with $\overline{\gamma}_{i,j}=E[\gamma_{i,j}]=E\left[\left|h_{i,j}\right|^2\right]E_{si}/\sigma^2$, where E_{si} is the average symbol energy of the i-th transmission node and σ^2 is the variance of the noise.

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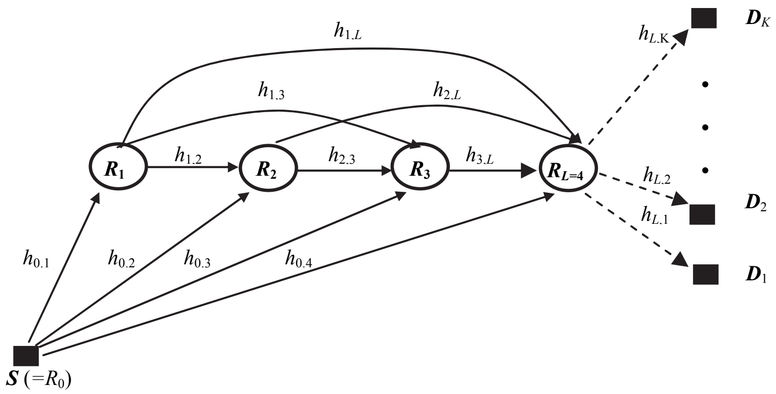

Fig. 1. Multi-hop multiuser relay network

Time resource T is divided into time slots $\Delta t = T / (L+1)$ assigned to the source and each relay. During the time slots, one node acts transmitter and the others act receiver. Namely, data transmission is performed in L+1 steps [8]. The source broadcasts signal to all other relays in the initial (0-th) step.

During the next L steps, let us relate the i-th relay and i-th transmission step ($i = \overline{1, L}$). The i-th relay receives signals from the source and previous i-1 relays, combines them using MRC algorithm, amplifies resulting signal and transmits it to the next nodes. The proposed strategy should ensure interference free transmission. Following the described steps, SNR at the first relay and j-th relay can be expressed as [7]

$$A_1 = \gamma_{0,1} \tag{2}$$

and

$$A_j = \gamma_{0,j} + \sum_{m=1}^{j-1} \frac{A_m \gamma_{m,j}}{A_m + \gamma_{m,j} + 1}, \quad j = \overline{2, L}, \tag{3}$$

respectively. Multi-hop relay system can be approximated with dual-hop relay systems in the way described in details in [8]. The resulting SNR at the last relay, γ_L, can be obtained using expression equivalent to the result given by [8, Eq. (11)].

The L-th relay serves destination with the best channel condition, i.e. the destination that ensure the most advantageous system performance. As we mentioned, K users are linked to the last relay. We assume that all average SNRs of link between the last relay and destinations are equal $\overline{\gamma}_{L,1} = \overline{\gamma}_{L,2} = ... = \overline{\gamma}_{L,K} = \overline{\gamma}_m$. Opportunistic multiuser scheduling which assumes that the relay selects the user which channel has the highest instantaneous SNR out of K destinations, $\gamma_m = \max_{1 \le k \le K} \gamma_{L,k}$, is adopted. Then, the CDF of γ_m is

$$F_{\gamma_m}(\gamma) = \prod_{k=1}^{K} F_{\gamma_{L,k}}(\gamma), \tag{4}$$

where

$$F_{\gamma_{L,k}}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\overline{\gamma}_m}\right) \tag{5}$$

Applying the opportunistic protocol in relay-multiuser communication, the equivalent instantaneous end-to-end SNR is defined as [7]

$$\gamma_{eq} = \frac{\gamma_L \gamma_m}{\gamma_L + \gamma_m}. \tag{6}$$

III. Outage Probability Analysis

In this section, described system is studied in terms of the outage probability which is important and widely accepted system performance indicator. The outage probability is defined as probability that system's end-to-end SNR falls below a protection value γ_{th}, also known as outage threshold, which is determined to satisfy quality of service constraint. The outage probability for considered system can be evaluated as

$$\begin{aligned} F_{eq}(\gamma_{th}) &= \int_0^{\infty} P_r\left(\gamma_{eq} < \gamma_{th} \middle| \gamma_L\right) f_{\gamma_L}(\gamma_L)\, d\gamma_L \\ &= \int_0^{\gamma_{th}} P_r\left(\gamma_{L,k} > \frac{\gamma_L \gamma_{th}}{\gamma_L - \gamma_{th}} \middle| \gamma_L\right)^K f_{\gamma_L}(\gamma_L)\, d\gamma_L \\ &+ \int_{\gamma_{th}}^{\infty} P_r\left(\gamma_{L,k} < \frac{\gamma_L \gamma_{th}}{\gamma_L - \gamma_{th}} \middle| \gamma_L\right)^K f_{\gamma_L}(\gamma_L)\, d\gamma_L = I_1 + I_2, \end{aligned} \tag{7}$$

where I_1 and I_2 are defined in the following way

$$I_1 = \int_0^{\gamma_{th}} f_{\gamma_L}(\gamma_L) d\gamma_L, \tag{8}$$

$$I_2 = \int_{\gamma_{th}}^{\infty} P_r\left(\gamma_{Lk} < \frac{\gamma_L \gamma_{th}}{\gamma_L - \gamma_{th}} \middle| \gamma_L\right)^K f_{\gamma_L}(\gamma_L)\, d\gamma_L, \tag{9}$$

with $f_{\gamma_L}(\gamma_L)$ defined in [8]. Substituting that equation into (8), we get

$$I_1 = \sum_{r=1}^{L} \sum_{p=1}^{N_r} \pi'_{r,p} \left(1 - e^{-\frac{\gamma_{th}}{\overline{\Gamma}_{r,p}}}\right). \tag{10}$$

where parameters $\pi'_{r,p}$ and $\overline{\Gamma}_{r,p}$ can be found in [8]. Introducing a change of variables, $x = \gamma_L - \gamma_{th}$ in (9) and applying binomial expansion, the integral I_2 becomes

$$I_2 = \sum_{k=0}^{K} \binom{K}{k} (-1)^k \exp\left(-k\frac{\gamma_{th}}{\overline{\gamma}_m}\right) \times \int_0^{\infty} \exp\left(-k\frac{\gamma_{th}^2}{\overline{\gamma}_m x}\right) f_{\gamma_L}(x+\gamma_{th})\, dx. \quad (11)$$

Using PDF of γ_L and [10, Eq. (3.324.1)], the closed-form expression of integral is obtained

$$I_2 = \sum_{r=1}^{L}\sum_{p=1}^{N_r}\sum_{k=0}^{K} \binom{K}{k} \sqrt{\frac{k\gamma_{th}^2}{\overline{\Gamma}_{r,p}\overline{\gamma}_m}} 2\pi'_{r,p} (-1)^k \times \exp\left(-\left(\frac{k}{\overline{\gamma}_m} + \frac{1}{\overline{\Gamma}_{r,p}}\right)\gamma_{th}\right) K_1\left(2\sqrt{\frac{k\gamma_{th}^2}{\overline{\gamma}_m \overline{\Gamma}_{r,p}}}\right), \quad (12)$$

where $K_n(\cdot)$ denotes the modified Bessel function of the second kind and n-th order [10, Eq. (8.407.1)]. Finally, the outage probability expression for considered AF relay system can be evaluated as the sum of derived expressions (10) and (12).

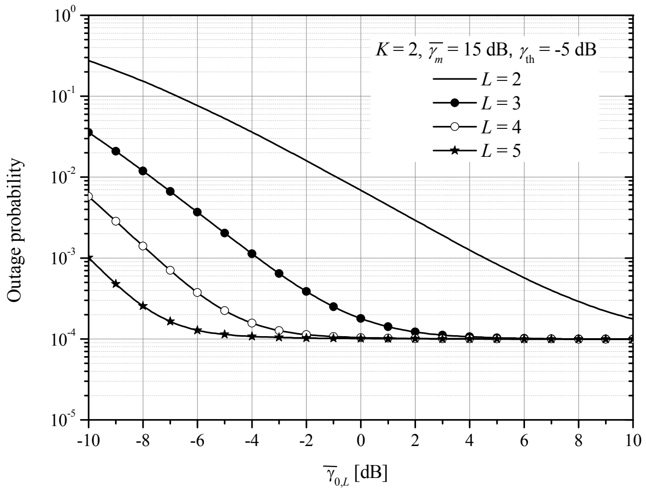

Fig. 2. Outage probability versus average SNR of S-R_L link for different number of relays

We assume that $E_{si}\big|_{i=1}^{L-1} = E_{s0}/(L-1)$ and that relays are uniformly distributed between the source and the last relay, so the channel gain between i-th and r-th relay is

$$E\left[\left|h_{i,r}\right|^2\right] = E\left[\left|h_{0,L}\right|^2\right] \Big/ \left(\frac{r-i}{L}\right)^{\mu} \quad (13)$$

with the path-loss factor μ=3.76 corresponding to the outdoor hotzone model [11]. Numerical results for the outage probability of the system with different number of relays versus average SNR defined for link between the source and the last relay, i.e. $\overline{\gamma}_{0,L} = E\left[\left|h_{0,L}\right|^2\right] E_0 \big/ \sigma^2$, are presented in Fig. 2. The figure depicts that increasing of relays number leads to the outage probability decreasing. Therefore, deployment of relays in multi-hop multiuser wireless networks is a promising solution for good link quality and high data rate in large areas. But, the gap among the curves reduces with increasing number of relays implying that there is no need to increase number of relays uncontrolled. Also, the influence of L is more evident for lower values of average SNR between the source and the last relay. For higher average SNR values, increasing of number of relays which assist in communication between the source and multiple destinations does not lead to the system performance improvement. Furthermore, it is worth observing that the outage probability curves saturate faster for higher L.

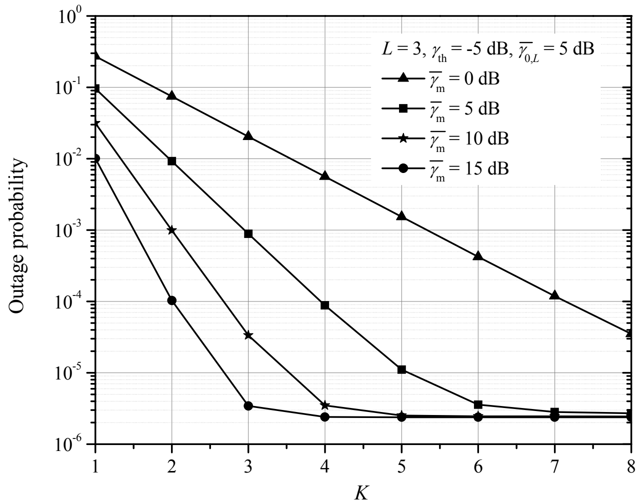

Fig. 3. Outage probability versus number of users

Examination of the impact of destinations number on the outage probability is carried out through Fig. 3. It can be noticed that the outage probability decreases as K increases. By increasing of K, curves for the outage probability enter saturation. This process becomes faster for higher average SNR between the last relay and destinations. Finally, similar to the conclusion regarding the number of relays made from Fig. 2, the relative gain achieved introducing one more destination decreases with increase of K.

IV. CONCLUSION

In this paper, outage performance for multi-hop multiuser network with the AF relays is analyzed. Analytical expression for outage probability is derived for such system in the Rayleigh fading environment. The effect of users and relays number and average SNR values on the overall system performance is examined. Increasing number of mobile users improves the link quality of the second hop, leading to a better outage probability performance.

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