Closed-form Design of New Class of Selective CIC FIR Filter Functions

Biljana P. Stošić, Vlastimir D. Pavlović

Abstract — **This paper deals with design of new CIC (Cascaded-Integrator-Comb) FIR (finite impulse response) filter functions. Closed-form expressions of new class of selective CIC filter functions by introducing spreading of the delays in the CIC filter comb stages are firstly presented here. Then, the authors are focused on the analysis of the proposed CIC filters in comparison with existing classical CIC structures and some recent results reported in the literature. The novel cascaded-filter architecture has valuable benefits: higher insertion loss in stopband, smaller impulse response coefficient values, and better characteristic in passband region compared with classical CIC filters.**

Keywords — **Digital filters, CIC FIR filters, Multiplierless design, Linear phase, Selective filters, Compensator filter.**

I. Introduction

CIC (Cascaded-Integrator-Comb) FIR (finite impulse response) filters introduced by Eugene B. Hogenauer [1], more than three decades ago, are becoming increasingly popular due to their compact integration with modern communication systems, their multiplier free design, etc. Various improvements of classical CIC filters have been reported in the last decade [2]-[10]. However, further improvement is still an issue, e.g. design of new CIC FIR filter functions, improvement of large passband droop which is undesirable in many applications because the original signal can be destroyed, increasing of attenuation in the stopband region, etc.

A CIC filter is cascade connection of simple integrator and comb filter stages. Design of a novel class of selective CIC filter functions based on the classical CIC filters, by spreading the delays in the CIC filter comb stages, is recently shown in the literature [6]-[10]. In [6], and [8]-[10], novel CIC filter functions in the explicit compact form, as well as their frequency responses and performance improvements over the classical CIC filters, are presented. The novel designed class gives higher insertion losses in the stopband region, and higher selectivity. The paper [8] provides graphs which can be used to design a novel class of selective CIC filters given specification which is suggested in [6]. They are very useful for the designers who will be able to do selection of the design parameters of the novel filter functions that they need for the particular design task.

In this paper, the new modified CIC FIR filter functions which preserve the CIC filter simplicity avoiding the multipliers are designed. The novel filter functions are given in recursive and non-recursive forms. The performance analysis in more detail through a few examples is done. It starts by a detailed analysis of frequency response characteristics. A comparative study of the performances is made with that of well-known classical CIC filters in graphical and tabular forms, as well as with some recent improvements of classical CIC filters given recently in the litearture [6]-[10]. Comparisons are for the same level of constant group delay. Also, some parameters of the novel class CIC filter functions (i.e. passband cut-off frequency and minimum attenuation in stopband) and their dependence on free parameters are given. The results illustrate the superiority of the suggested novel CIC filter functions and show that they can be a good alternative instead of classical CIC filters.

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II. Classical CIC Filter Functions

The z-domain expression for the classical CIC FIR filter function with normalized amplitude response characteristic is

$$H(N,K,z) = \left(H(N,z)\right)^K , \tag{1}$$

$$H(N,z) = \frac{1-z^{-N}}{N\cdot(1-z^{-1})} = \frac{1}{N}\sum_{r=0}^{N-1} z^{-r} , \tag{2}$$

where N is the decimation factor, and K is the number of CIC sections [1].

The frequency response of CIC FIR filter function presented in exponential form is

$$H(N,K,z=e^{j\omega}) = e^{-jK(N-1)\omega/2}\cdot\left(\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\right)^K . \tag{3}$$

The magnitude response characteristic is defined as the magnitude of the complex filter frequency response given in (3). The phase response characteristic of the proposed new modified CIC FIR filter functions is defined as the phase angle of the complex filter frequency response given in (3), and has the form

$$\varphi(N,K,\omega) = -(N-1)\cdot K\cdot\omega/2 + 2\cdot\nu\cdot\pi , \ \nu = 0,1,2,... \tag{4}$$

A filter has a linear phase and therefore a constant group delay defined as

$$\tau(N,K,\omega) = -d\varphi(N,K,\omega)/d\omega = (N-1)\cdot K/2 . \tag{5}$$

III. Design Forms of Novel Class of CIC FIR Filter Functions

A. Non-Recursive and Recursive Forms of Novel Filter Class

The novel filter class is designed as cascade of four non-identical CIC FIR filter sections $H(N-2,z)$, $H(N-1,z)$, $H(N+1,z)$ and $H(N+2,z)$ which are repeated L times.

A recursive form of a novel class of CIC FIR filter functions is given as

$$H(N,K,L,z)=\left(\frac{1-z^{-(N-2)}}{(N-2)\cdot(1-z^{-1})}\cdot\frac{1-z^{-(N-1)}}{(N-1)\cdot(1-z^{-1})}\right)^{L}\cdot$$
$$\cdot\left(\frac{1-z^{-(N+1)}}{(N+1)\cdot(1-z^{-1})}\cdot\frac{1-z^{-(N+2)}}{(N+2)\cdot(1-z^{-1})}\right)^{L}$$
and $K=4L$. (6)

The proposed filter function has normalized amplitude response characteristics.

The filter function of a designed novel class of CIC FIR filter functions can be written in non-recursive form as

$$H(N,K,L,z)=\left[\left(\frac{1}{N-2}\sum_{r=0}^{N-3}z^{-r}\right)\cdot\left(\frac{1}{N-1}\sum_{r=0}^{N-2}z^{-r}\right)\cdot\left(\frac{1}{N+1}\sum_{r=0}^{N}z^{-r}\right)\right]^{L}\cdot$$
$$\cdot\left[\left(\frac{1}{N+2}\sum_{r=0}^{N+1}z^{-r}\right)\right]^{L}=\frac{1}{H_0(N,K,L)}\cdot\sum_{r=0}^{(N-1)\cdot K}h(N,r)\cdot z^{-r} \quad (7)$$

where $H_0(N,K,L)$ is the normalized constant for the unit magnitude response at $f=0$, where parameter $K=4L$, and $h(N,r)$ represent the coefficients of the impulse response.

B. Impulse Response Coefficients

Vector **h** is the vector of impulse response coefficients, defined as

$$\boldsymbol{h}(N,K,L)=\{h(N,0),h(N,1),...,h(N,M-1),h(N,M)\} \quad (8)$$

where total number of elements is $M=(N-1)\cdot K+1$. The coefficients satisfy the following symmetry condition, $h(N,r)=h(N,M-r)$. The impulse response coefficients of the classical CIC filters can be also found in [5].

The non-recursive implementation of classical CIC filter functions, $H(N,K,z)$, obtained for odd value of free integer parameter $N=7$ and $K=4$, is observed here in the form of Eq. (7). The normalized constant is 2401. The vector of the impulse response coefficient is

$$\boldsymbol{h}_{CIC}(7,4)=\left\{\begin{matrix}1,4,10,20,35,56,84,116,149,180,206,\\224,231,224,206,180,149,116,84,56,\\35,20,10,4,1\end{matrix}\right\} \quad (9)$$

The non-recursive forms of novel class of CIC filter functions, $H(N,K,L,z)$, obtained for even value of integer parameter $N=7$ and $K=4$ (obtained for $L=1$), satisfied Eq. (7). The constant is $H_0(7,4,1)=2160$. The vector **h** of impulse response coefficients is

$$\mathbf{h}(7,4,1)=\left\{\begin{matrix}1,4,10,20,35,55,79,106,134,160,181,195,200,\\195,181,160,134,106,79,55,35,20,10,4,1\end{matrix}\right\} \quad (10)$$

Notice that the proposed linear phase FIR filter functions in non-recursive form have shown obvious symmetry of coefficients. The normalized constants for proposed filter class are smaller than those of classical CIC FIR filters. Also, impulse response coefficients of proposed filter class have smaller values in comparison to those of classical CIC filters.

C. Frequency Responses

Frequency response of designed FIR filter functions is easily obtained by evaluating the filter function in the z -plane at the sample points defined by setting $z=e^{j\omega}$, where $\omega=2\pi\cdot f$ is angular frequency in radians per second. Using Euler's identity, frequency response characteristic can be written in exponential form as

$$H(N,K,L,z=e^{j\omega})=e^{-jK(N-1)\omega/2}\cdot\left(\frac{\sin((N-2)\omega/2)}{(N-2)\cdot\sin(\omega/2)}\right)^{L}\cdot$$
$$\cdot\left(\frac{\sin((N-1)\omega/2)}{(N-1)\cdot\sin(\omega/2)}\cdot\frac{\sin((N+1)\omega/2)}{(N+1)\cdot\sin(\omega/2)}\cdot\frac{\sin((N+2)\omega/2)}{(N+2)\cdot\sin(\omega/2)}\right)^{L}$$
and $K=4L$. (11)

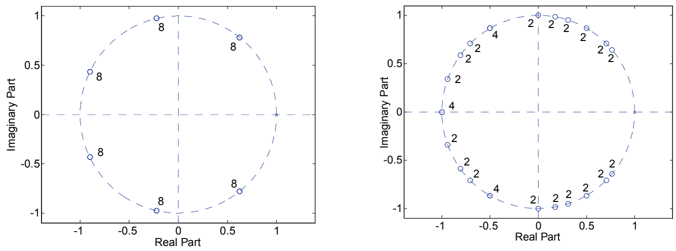
The magnitude response characteristic of the proposed filter functions, $\left|H(N,K,L,e^{j\omega})\right|$, is defined as the magnitude of the complex filter frequency response $H(N,K,L,z=e^{j\omega})$.

The linear phase response characteristic of the proposed novel class of the modified CIC FIR filter functions is
$\varphi(N,K,L,\omega)=-(N-1)\cdot K\cdot\omega/2+2\cdot\nu\cdot\pi$, $\nu=0,1,2,...$, and $K=4L$. (12)

A novel class of the modified CIC FIR filter functions has a the constant group delay response characteristic expressed as
$\tau(N,K,L,\omega)=(N-1)\cdot K/2$ and $K=4L$. (13)

D. Zero-Plots for Filter Functions

The locations of zeros in z -plane along with their multiplicities for the classical CIC and the proposed class of CIC filter functions are shown in Fig. 1. They are shown for case of $N=8$, and $L=2$.



(*a*) Classical CIC filter (*b*) Novel filter class, $L=2$

Fig. 1. Locations and multiplicities of filter function zeros in z -plane for $N=8$, and $K=8$ cascades

Note that the comb stages of the classical CIC filters produce N zeros, equally spaced around the unit-circle, and the integrator stages produce single poles canceling the zeros at $z=1$. Finally, the classical CIC filter function has $N-1$ different zeros being a N^{th} root of unity and located at $z_r=e^{j\cdot 2\pi\cdot r/N}$, $r=1,2,...,N-1$. The total number of zeros is $(N-1)\cdot K$. Note that the classical CIC filters have all multiple zeros with maximum multiplicity equal to the number of cascades K , which is not the case in the proposed solutions. The zeros of the proposed filter class are more evenly distributed with their multiplicities therefore reduced as can be seen in Fig. 1b.

E. Selection of the Design Parameters

The choice of free integer parameters N and L is done in the same way as for CIC filters, there are the same restrictions on the group delay response. The parameter K can take different integer values, $K = 4L$.

The attenuation in the stopband region is closely related to the parameter L. By increasing L for the constant value of N, the higher stopband attenuation is achieved.

The constant group delay τ is equal for the classical CIC filters (Eq. (1)) and the novel modified CIC filter functions (Eq. (6)). The constant group delays for different values of parameters N, L and $K = 4L$ are given in Table I.

TABLE I
GROUP DELAY τ FOR $N \in \{5,6,\ldots,11\}$, $L \in \{1,2,3\}$ AND $K = 4L$

N		5	6	7	8	9	10	11	12
$\tau[s]$	$L = 1$	8	10	12	14	16	18	20	22
	$L = 2$	16	20	24	28	32	36	40	44
	$L = 3$	24	30	36	42	48	54	60	66

IV. DESIGN EXAMPLE AND COMPARISONS OF NOVEL CIC FIR FILTER FUNCTIONS

In order to validate theoretical design, a test example is designed for different filter parameters.

A. Frequency Responses of new CIC Filter Functions Compared with Classical CIC Filters

Like classical CIC filters, new designed CIC FIR filter functions have significant passband droop, which is usually intolerable in many cases, what can be seen in Fig. 2. Hence, it is of great interest to get a flat passband in some way, i.e. by connecting an additional filter (so-called CIC compensator) in cascade with the CIC decimator.

Passband droop compensation is done by use of a multiplerless FIR filter with one free parameter b presented in [3, 4]

$$G(z^N) = B \cdot \left[1 + A \cdot z^{-N} + z^{-2N}\right], \tag{14}$$

where $B = -2^{-(b+2)}$ is a scaling factor ensuring unitary gain at the digital frequency zero, and $A = -\left[2^{b+2} + 2\right]$. The compensator is connected in the cascade with the suggested filter functions.

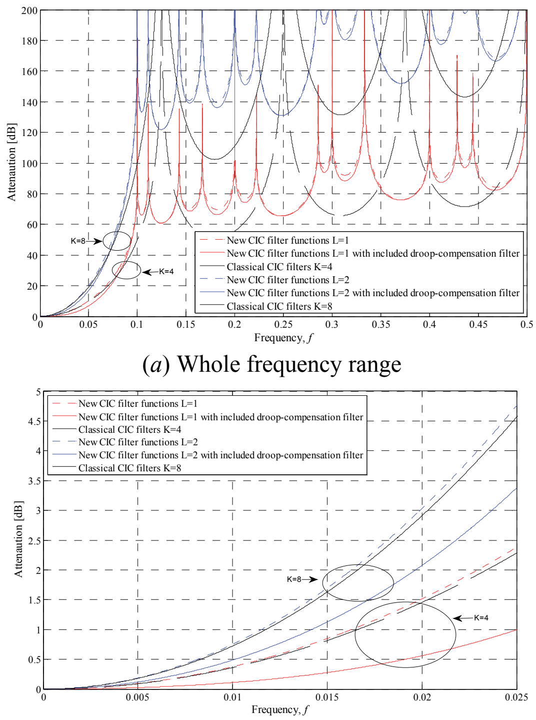
TABLE II
CUT-OFF FREQUENCIES IN PASSBAND AND STOPBAND, CONSTANT GROUP DELAY AND STOPBAND ATTENUATION OF CLASSICAL CIC FILTER FOR $K \in \{4,8\}$ AND $N = 8$

K	f_{cp}	α_{max} [dB]	f_{cs}	α_{min} [dB]
4	0.00881	0.28	0.1008	51.1894
8	0.00623	0.28	0.1008	102.3788

In order to illustrate clearly the achieved improvements of the new class, the normalized magnitude response characteristics in dB of the new class and classical CIC filters are summarized in Fig. 2. It can be concluded that achieved passband droop compensation is similar for both values of parameter $L \in \{1, 2\}$.

TABLE III
CUT-OFF FREQUENCIES IN PASSBAND AND STOPBAND, CONSTANT GROUP DELAY AND STOPBAND ATTENUATION OF PROPOSED CLASS OF CIC FILTER FOR $K \in \{4,8\}$, $L \in \{1, 2\}$, $N = 8$ AND $\alpha_{max}(f_{cp}) = 0.28\,\text{dB}$

Filter type	L	K	f_{cp}	f_{cs}	α_{min} [dB]
no added droop-compensation filter	1	4	0.00864	0.09716	60.8814
	2	8	0.00611	0.09717	121.7628
added droop-compensation filter with $b = 1$	1	4	0.01505	0.09753	60.8812
	2	8	0.00763	0.09717	121.7626



(*a*) Whole frequency range

(*b*) Passband detail

Fig. 2. Magnitude response characteristics in dB of proposed class of CIC FIR filter functions and those functions with included droop-compensation filter for $N = 8$, different values of parameter $L = 1$ and $L = 2$, and $b = 1$

In Tables II and III, parameter values of both the classical CIC filter function $H(N, K, z)$, given in Eq. (1), and the novel CIC filter functions $H(N, K, L, z)$, given in Eq. (6), are presented respectively. They are obtained for chosen parameter values $N = 8$ and $K \in \{4, 8\}$ obtained for $L \in \{1, 2\}$. The given parameters are: passband and stopband cut-off frequencies, f_{cp} and f_{cs}, maximum attenuation in the passband, α_{max} [dB] and miminum attenuation in the stopband region, α_{min} [dB]. The filter functions are designed for the same number of cascaded sections with the difference that the CIC filters have an identical section in all cascades, and the designed novel class has a cascade-connected different CIC filter sections. Also, they have the same level of constant group delay, as well as number of delay elements, but the novel designed filter functions give higher insertion losses in stopband, as well as it has higher selectivity.

Achieved improvement of the stopband attenuation is about 18.93%. Note that the normalized stopband cut-off frequencies for novel filter functions, given in Table III, are practically identical for different values of integer parameter L, but minimum attenuation in the stopband region increase rapidly by increasing its value.

In order to demonstrate the performances of the proposed filter class in the stopband, achieved improvement of the minimum attenuation over classical CIC filters is summarized in Fig. 3.

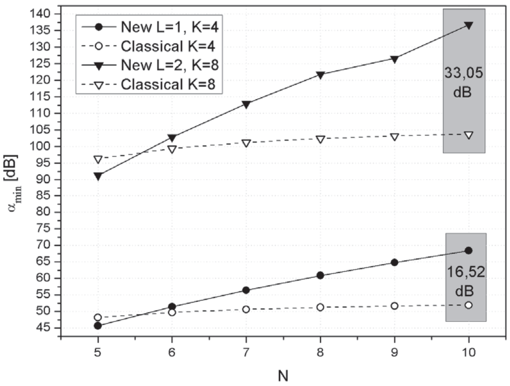


Fig. 3. Attenuation in the stopband of the proposed novel class over classical CIC filters versus parameter N

B. *Frequency Responses of new CIC Filters Compared with Improvements Given in the Recent Literature*

The idea of modified CIC filters is not new, it is proposed by the authors in [6], [8]-[10]. Because of that, these four papers will be considered for comparison. The CIC FIR filter functions which closed-form design equations are presented in these papers are designed to compare feature of each of those classes with the new class of CIC FIR filter functions presented in this paper. Each class is designed for specified group delay $\tau = 36s$. The normalized magnitude response characteristics of these filters are compared in Fig. 4. The new filter classes have droop-compensation filter given in Eq. (14) included during design.

Comparison of new filter functions with functions given in [6], [9], [10], [8] and clasical CIC filters is summarized in Figs. 5-7, respectively. The new class shows higher selectivity. From the passband detail given in Fig. 4, one can observe that passband characteristic of the new filter class is better than those ones of classical CIC filters and filters given in [8]-[10] over a wider frequency range. The passband characteristics given here and in [6] are very similar in general and better than that one of classical CIC filters over a wider frequency range.

Comparison of new filter functions with functions given in [3] and classical CIC filters is pictured in Fig. 8. Technique proposed in [3] includes passband droop compensator and stopband improvement filters. From the passband detail given in Fig. 4, one can observe that passband characteristic of the new filter class is better than those ones of classical CIC filters and filter given in [3] over a wider frequency range. The new class shows higher selectivity in the transition area. The proposed filter class has bigger attenuation in the stopband area without added additional filter for improvements versus solution given in [3] where stopband improvement filter is included.

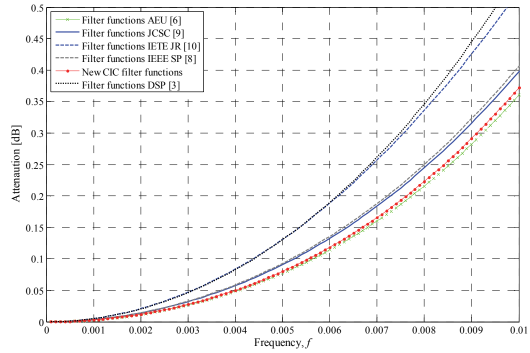


Fig. 4. Comparison of normalized magnitude response characteristics in dB - passband detail (red lines - novel class of CIC filters with droop-compensation filter for $N=10$, $K=8$, $L=2$, $M=10$, $b=0$; dotted black lines - method proposed in [3] for $M=9$, $K=9$, $b=0$, $N_1=4$, $N_2=5$; solid green lines - filters from [6] with passband droop compensator for $N=10$, $L=1$, $K=8$; solid blue lines - filters from [9] with compensator for $N=9$, $L=1$, $K=9$; dashed blue lines - filters from [10] with compensator for $N=4$, $L=3$, $K=24$; dashed gray lines - filters from [8] with compensator for $N=9$, $L=1$, $K=9$)

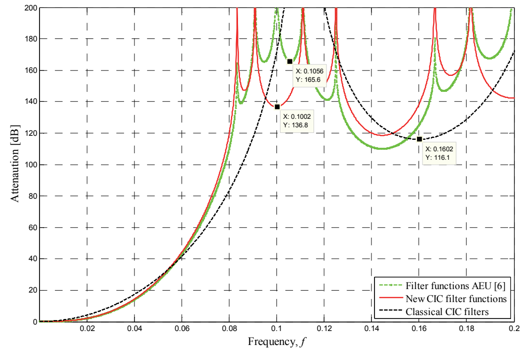


Fig. 5. Comparison of normalized magnitude response characteristics in dB (dashed black lines - classical CIC filter for $K=9$, $N=9$; solid red lines - novel class of CIC FIR filter functions with droop-compensation filter for $N=10$, $K=8$, $L=2$, $M=10$, $b=0$; solid green line - filter functions given in [6] with passband droop compensator for $N=10$, $L=1$, $K=8$

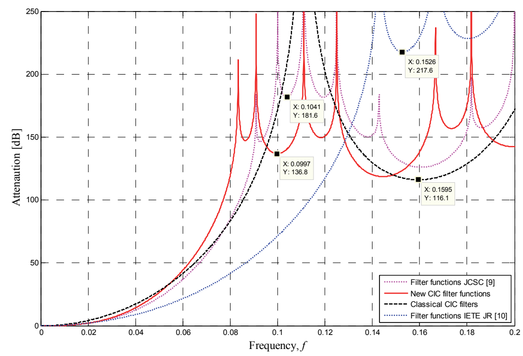


Fig. 6. Comparison of normalized magnitude response characteristics in dB (dashed black lines - classical CIC filter for $K=9$, $N=9$; solid red lines - novel class of CIC filters with droop-compensation filter for $N=10$, $K=8$, $L=2$, $M=10$, $b=0$; dotted magenta line - filter functions given in [9] with passband droop compensator for $N=9$, $L=1$, $K=9$; dotted blue line - filter functions given in [10] with included passband droop compensator for $N=4$, $L=3$, $K=24$

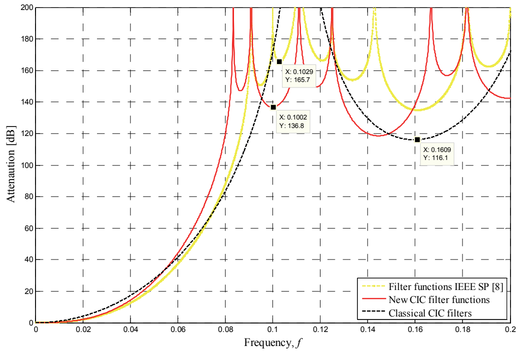


Fig. 7. Comparison of normalized magnitude response characteristics in dB (dashed black lines - classical CIC filter for $K = 9$, $N = 9$; solid red lines - novel class of CIC FIR filter functions with droop-compensation filter for $N = 10$, $K = 8$, $L = 2$, $M = 10$, $b = 0$; solid yellow line - filter functions from [8] with passband droop compensator for $N = 9$, $L = 1$, $K = 9$)

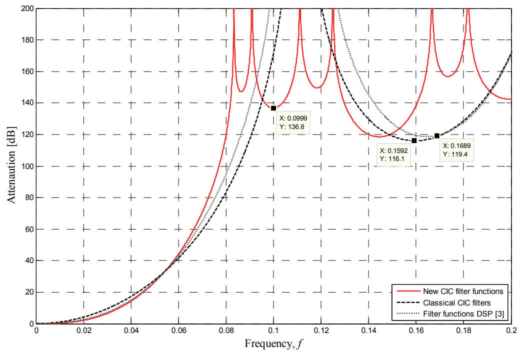


Fig. 8. Comparison of normalized magnitude response characteristics in dB (dashed black lines - classical CIC filter for $K = 9$, $N = 9$; solid red lines - novel class of CIC FIR filter functions with droop-compensation filter for $N = 10$, $K = 8$, $L = 2$, $M = 10$, $b = 0$; dotted black lines - method proposed in [3] for $M = 9$, $K = 9$, $b = 0$, $N_1 = 4$, $N_2 = 5$)

V. Conclusion Remarks

This paper deals with the design of a novel class of linear phase multiplierless finite duration impulse response (FIR) filter functions using several cascaded non-identical CIC FIR sections. The properties of this designed filter class are demonstrated by including several examples and some comparisons.

The filter functions are compared here for the equal group delays. The simultaneous improvements in the passband and stopband of filter functions are achieved. The novel filter class has greatly reduced passband droop by the compensator from [3] that is connected in the cascade with the suggested filter functions. Also, it gives higher insertion losses in stopband, as well as it has higher selectivity. From the simulation results, it was observed that the suggested novel modified CIC filter functions seem as an alternative functions for communication system applications.

In this paper, the implementation aspects have not been considered as the paper is devoted to new classes of modified CIC FIR filter functions and a study of their properties. But, in the literature, the CIC filters are used in a wide array of applications: in modern communication systems, such as software defined radio [11], in sigma-delta analog-to-digital converters [12]-[15], etc.

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