

# Self and Mutual Impedances of Power Cables

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**Abstract** –The review of relations for determining self and mutual impedances of power cables is given in the paper. Afterwards, the accuracy of presented relations is tested. The problems of single-core cables with metal sheets earthed at both or one end, with or without cross-bonding are discussed in more details.

**Keywords** – Power cables, Circulation current, cross-bonding

## I. INTRODUCTION

During the realization and exploitation of power transmission lines, it is of great importance to know values of serial self and mutual impedances of cables. It is well known that serial self impedances of the conductors and metal sheets can be determined as a sum of internal and external impedance values. The relations for determining internal impedance of the conductor with circular cross-section are proposed long time ago. During the time, increasing value of transmitted energy resulted in enlarging conductor cross-sections (3000 mm<sup>2</sup>), as well as in improving constructing solutions for cable realization. In these cases, it is not possible to apply above mentioned, previously proposed relations. On the other hand, main problem in determining external self and mutual impedance is taking into account earth as the return current path. The expressions for determining ground return path impedance for known current frequency are known for almost one century. Those relations are not easy to apply for engineering purposes, since they include calculation of modified Bessel function of complex variable and numerical solving of definite integrals. In order to avoid such difficulties, a large number of simplified engineering relations has been developed. They are often applied without knowledge about their accuracy or about assumptions and neglectings under which these relations are evaluated. A review of relations for calculating self and mutual cable impedances is given in the paper. On one example, the accuracy of presented relations has been tested. The problems of single-core cables with metal sheets placed in the ground, earthed at both or one end, with or without cross-bonding are especially discussed.

## II. THEORETICAL BACKGROUND

### A. Expressions for Three-Phase Transmission Line

The three single-core cables which form three-phase transmission line are observed, Fig. 1. The cables are laid in flat formation, which is usual for high voltage cables. Phase conductors are marked with 1, 2, and 3, while their metal

sheets are labeled with 4, 5, and 6. Having in mind the mentioned marking of phase conductors and metal sheets, the following matrix equation can be formed:

$$-\begin{bmatrix} \frac{\partial \underline{U}_1}{\partial x} \\ \frac{\partial \underline{U}_2}{\partial x} \\ \frac{\partial \underline{U}_3}{\partial x} \\ \frac{\partial \underline{U}_4}{\partial x} \\ \frac{\partial \underline{U}_5}{\partial x} \\ \frac{\partial \underline{U}_6}{\partial x} \end{bmatrix} = \begin{bmatrix} \underline{z}_{11} & \underline{z}_{12} & \underline{z}_{13} & \underline{z}_{14} & \underline{z}_{15} & \underline{z}_{16} \\ \underline{z}_{21} & \underline{z}_{22} & \underline{z}_{23} & \underline{z}_{24} & \underline{z}_{25} & \underline{z}_{26} \\ \underline{z}_{31} & \underline{z}_{32} & \underline{z}_{33} & \underline{z}_{34} & \underline{z}_{35} & \underline{z}_{36} \\ \underline{z}_{41} & \underline{z}_{42} & \underline{z}_{43} & \underline{z}_{44} & \underline{z}_{45} & \underline{z}_{46} \\ \underline{z}_{51} & \underline{z}_{52} & \underline{z}_{53} & \underline{z}_{54} & \underline{z}_{55} & \underline{z}_{56} \\ \underline{z}_{61} & \underline{z}_{62} & \underline{z}_{63} & \underline{z}_{64} & \underline{z}_{65} & \underline{z}_{66} \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \\ \underline{I}_4 \\ \underline{I}_5 \\ \underline{I}_6 \end{bmatrix} \quad (1)$$

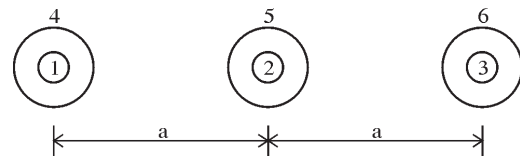


Fig. 1. Three single-core cables laid in a flat formation

Relation (1) has general form and can be applied for arbitrary position of the cables. With  $\underline{z}_{ii}$ ,  $i=1,2,3$ , self impedances per unit length of the loops conductor-return ground path are labeled, while  $\underline{z}_{ii}$ ,  $i=4,5,6$  denote self impedances per unit length of the loops metal sheet-return ground path. Also,  $\underline{z}_{ij} = \underline{z}_{ji}$ ,  $i \neq j$ , denote corresponding mutual impedances per unit length, while  $x$  is coordinate along the cable route. Obviously, for cables from Fig. 1 it is  $\underline{z}_{11} = \underline{z}_{22} = \underline{z}_{33}$  and  $\underline{z}_{44} = \underline{z}_{55} = \underline{z}_{66}$ . The radial density distribution of the current flowing through metal sheets depends on the currents flowing through phase cable and currents having closing path outside metal sheets. For taking into account this influence, instead of (1), expression (2), formed for loops cable-metal sheet and metal sheet-return ground [1], is used:

$$-\begin{bmatrix} \frac{\partial(\underline{U}_1 - \underline{U}_4)}{\partial x} \\ \frac{\partial(\underline{U}_2 - \underline{U}_5)}{\partial x} \\ \frac{\partial(\underline{U}_3 - \underline{U}_6)}{\partial x} \\ \frac{\partial \underline{U}_4}{\partial x} \\ \frac{\partial \underline{U}_5}{\partial x} \\ \frac{\partial \underline{U}_6}{\partial x} \end{bmatrix} = \begin{bmatrix} \underline{z}_{111} & 0 & 0 & \underline{z}_{114} & 0 & 0 \\ 0 & \underline{z}_{122} & 0 & 0 & \underline{z}_{125} & 0 \\ 0 & 0 & \underline{z}_{133} & 0 & 0 & \underline{z}_{136} \\ \underline{z}_{141} & 0 & 0 & \underline{z}_{144} & \underline{z}_{145} & \underline{z}_{146} \\ 0 & \underline{z}_{152} & 0 & \underline{z}_{154} & \underline{z}_{155} & \underline{z}_{156} \\ 0 & 0 & \underline{z}_{163} & \underline{z}_{164} & \underline{z}_{165} & \underline{z}_{166} \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \\ \underline{I}_1 + \underline{I}_4 \\ \underline{I}_2 + \underline{I}_5 \\ \underline{I}_3 + \underline{I}_6 \end{bmatrix} \quad (2)$$

where it is

$$\underline{z}_{111} = \underline{z}_{122} = \underline{z}_{133} = \underline{z}_{ps} + \underline{z}_{iz} + \underline{z}_{eu},$$

$$\underline{z}_{144} = \underline{z}_{155} = \underline{z}_{166} = \underline{z}_{es} + \underline{z}_{eiz} + \underline{z}_{zs},$$

$$\underline{z}_{114} = \underline{z}_{141} = \underline{z}_{125} = \underline{z}_{152} = \underline{z}_{136} = \underline{z}_{163} = -\underline{z}_{lm}, \quad \underline{z}_{145} = \underline{z}_{z12},$$

$$\text{and } \underline{z}_{146} = \underline{z}_{z13}, \quad \underline{z}_{156} = \underline{z}_{z23}.$$

Previous relation is formed for cables with metal sheets, but without armature. The same procedure can be applied for the cables with armature and results in more complex relation for

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determining matrix elements of dimension 9x9. In previous expressions,  $\underline{z}_{ps}$  is internal impedance per unit length of the cable,  $\underline{z}_{iz}$  is insulation impedance per unit length,  $\underline{z}_{eu}$  is internal impedance per unit length of the metal sheet with internal return path,  $\underline{z}_{es}$  is internal impedance per unit length of internal impedance of metal sheet with external return path,  $\underline{z}_{lm}$  is mutual impedance per unit length of the loops cable-metal sheet and metal sheet-return ground path,  $\underline{z}_{zs}$  is self impedance of the ground, while  $\underline{z}_{145} = \underline{z}_{154} = \underline{z}_{212}$ ,  $\underline{z}_{146} = \underline{z}_{164} = \underline{z}_{213}$  and  $\underline{z}_{156} = \underline{z}_{165} = \underline{z}_{223}$  are mutual impedances per unit length of the ground. For system from Fig. 1 it is  $\underline{z}_{145} = \underline{z}_{156}$ . After simple transformations, expression (2) can be reduced to the form (1) where it is

$$\begin{aligned} \underline{z}_{11} &= \underline{z}_{22} = \underline{z}_{33} = \underline{z}_{ps} + \underline{z}_{iz} + \underline{z}_{eu} - 2\underline{z}_{lm} + \underline{z}_{es} + \underline{z}_{eiz} + \underline{z}_{zs}; \\ \underline{z}_{44} &= \underline{z}_{55} = \underline{z}_{66} = \underline{z}_{es} + \underline{z}_{eiz} + \underline{z}_{zs}; \\ \underline{z}_{14} &= \underline{z}_{41} = \underline{z}_{25} = \underline{z}_{52} = \underline{z}_{36} = \underline{z}_{63} = \underline{z}_{es} + \underline{z}_{eiz} + \underline{z}_{zs} - \underline{z}_{lm}; \\ \underline{z}_{12} &= \underline{z}_{21} = \underline{z}_{45} = \underline{z}_{54} = \underline{z}_{15} = \underline{z}_{51} = \underline{z}_{24} = \underline{z}_{42} = \underline{z}_{212}; \\ \underline{z}_{13} &= \underline{z}_{31} = \underline{z}_{46} = \underline{z}_{64} = \underline{z}_{16} = \underline{z}_{61} = \underline{z}_{34} = \underline{z}_{43} = \underline{z}_{213}; \\ \underline{z}_{23} &= \underline{z}_{32} = \underline{z}_{56} = \underline{z}_{65} = \underline{z}_{26} = \underline{z}_{62} = \underline{z}_{35} = \underline{z}_{53} = \underline{z}_{223}. \end{aligned} \quad (3)$$

Since expression (2) can be transformed into form (1), expression (1) can be assumed as general.

### B. Internal Impedance of Conductor and Metal Sheet

The resistance per unit length of the cable conductor for DC current versus temperature can be determined as

$$R' = R'_0(1 + \alpha(\theta - 20)), \quad (4)$$

where  $R'_0$  labels the resistance of the cable conductors for DC current at 20°C [2],  $\alpha$  is temperature coefficient of the resistance, while  $\theta$  is conductors' temperature. For AC current, there are skin and proximity effects, causing the resistance increasing. For determining AC resistance, in [2] the following relation is used,

$$R = R'(1 + y_s + y_p). \quad (5)$$

In (5)  $y_s$  is coefficient which models skin effect influence, while  $y_p$  value models influence of the proximity effect. From complex form of Maxwell's equations system, the following expression for internal impedance per unit length of the conductor cable with circular cross-section of radius  $r_c$  can be derived:

$$\underline{z}_{in} = \frac{\sqrt{2} j^{3/2} \delta_c^{-1} \rho J_0(\sqrt{2} j^{3/2} \delta_c^{-1} r_c)}{2\pi r_c J_1(\sqrt{2} j^{3/2} \delta_c^{-1} r_c)}. \quad (6)$$

In (6),  $J_0$  labels Bessel function of the first kind of zero order, while  $J_1$  is Bessel function of the first kind of order one. With  $\delta_c$  the penetration depth is labeled:

$$\delta_c = \sqrt{\frac{2\rho}{\omega\mu}}, \quad (7)$$

where  $\omega$  is angular frequency,  $\mu$  is magnetic permeability and  $\rho$  is specific resistivity of the conductor. Since

penetration depth at frequency 50 Hz is about 9,35 mm, for cross-sections between 25 mm<sup>2</sup> and 2500 mm<sup>2</sup> it is  $0,3 < r_c/\delta_c < 3$ . Using the first three terms of series expansion of Bessel function, the simplified relation is obtained:

$$\underline{z}_{in} = \frac{\rho}{\pi r_c^2} \left( 1 + \frac{1}{3} \left( \frac{r_c}{2\delta_c} \right)^4 \right) + j \frac{\omega\mu}{8\pi}. \quad (8)$$

For interval  $0,3 < \delta_c < 3$  the following relations provide error less than 3%:

$$y_s = \frac{x_s^4}{192 + 0,8x_s^4}, \quad x_s = 15,9 \cdot 10^{-4} \sqrt{\frac{f k_s}{R'}}. \quad (9)$$

Previous expressions were used until 2014. godine in IEC 60287-1-1, for  $x_s \leq 2,8$ . The increase of power results in enlarging the cable cross-section with strongly present skin effect. In order to neutralize skin effect, various solutions (conductors made of lacquered wires, or isolating conductors' segments) are applied. For calculating these types of cables, in [3] new values of the coefficients for existing expressions are given. For conductors with circular cross-section, for  $x_s \leq 2,8$ , relation (9) is recommended. For other interval it is

$$\begin{aligned} y_s &= -0,136 - 0,0177x_s + 0,0563x_s^2, \quad 2,8 \leq x_s \leq 3,8 \\ y_s &= 0,354x_s - 0,733, \quad x_s > 3,8 \end{aligned} \quad (10)$$

Using previous expressions, the error is less than 0,6% in complete interval. For cables with segment conductors, in IEC60287-1-1, published in 2014, relations (10) are accepted and coefficients values  $c_p$  and  $c_b$  are given.

Proximity effects are modeled in standard IEC60287-1-1 with expression

$$y_p = \frac{x_p^4}{192 + 0,8x_p^4} \left( \frac{d_c}{a} \right)^2 \left[ 0,312 \left( \frac{d_c}{a} \right)^2 + \frac{1,18}{\frac{x_p^4}{192 + 0,8x_p^4} + 0,27} \right], \quad (11)$$

$$x_p = 15,9 \cdot 10^{-4} \sqrt{\frac{f k_p}{R'}}.$$

Internal reactance of the conductor, i.e. internal inductance, can be determined from imaginary part of the expression (6).

Relation (8) provides calculation of internal inductivity with accuracy better than 3% for  $r_p/\delta_c \leq 1,3$  (cross section about 460 mm<sup>2</sup> at 50 Hz). In [4], for  $r_p/\delta_c \leq 2$  the following expression is proposed:

$$l_{in} = \frac{\mu}{8\pi} \left( 1 - \frac{1}{6} \left( \frac{r_c}{2\delta_c} \right)^4 \right), \quad (12)$$

and for  $r_c/\delta_c > 2$  (cross-section larger than 1000 mm<sup>2</sup> at 50 Hz):

$$l_{in} = \frac{\mu}{8\pi} \left( \frac{2\delta_c}{r_c} - \frac{3}{64} \left( \frac{2\delta_c}{r_c} \right)^3 \right). \quad (13)$$

Internal impedance of cylindrical conductor of circular cross-section can be determined as

$$\underline{z}_{in} = \frac{\rho_c u}{2\pi r_o} \frac{N_1(r_i u) J_0(r_o u) - N_0(r_o u) J_1(r_i u)}{N_1(r_i u) J_1(r_o u) - N_1(r_o u) J_1(r_i u)}. \quad (14)$$

It can also be applied for calculate the metal sheet impedance when the return path is trough the armature (labelled as  $\underline{z}_{es}$  in (3)). In [5] the following approximation is proposed:

$$\underline{z}_{in} \approx \frac{\rho m_c}{2\pi r_o} \coth(m_c(r_o - r_i)) - \frac{\rho}{2\pi r_i(r_i + r_o)}. \quad (15)$$

Internal impedance of cylindrical conductor with internal return path can be determined from expression:

$$\underline{z}_{in} = \frac{\rho u}{2\pi r_i} \frac{N_1(r_o u) J_0(r_i u) - J_1(r_o u) N_0(r_i u)}{N_1(r_i u) J_1(r_o u) - N_1(r_o u) J_1(r_i u)}. \quad (16)$$

Applying (24), metal sheet impedance labeled with  $\underline{z}_{es}$  in (3) can be determined. Since the thickness of metal sheet is very small, the non-uniform current distribution in metal sheet can be neglected. Consequently, the resistance of metal sheet can be determined as DC resistance. Insulation impedance in (3) can be determined from expression:

$$\underline{z}_{iz} = j \frac{\omega \mu_0}{2\pi} \ln \frac{r_{ou}}{r_{in}}, \quad (17)$$

where  $r_{ou}$  and  $r_{in}$  are outer and insulation radius, respectively Expression (25) can be used to determine  $\underline{z}_{iz}$  in (3), adopting that  $r_{ou}$  is inner radius of metal sheet,  $r_{ei}$ , while  $r_{in}$  is radius of the conductor  $r_c$ . Also, equation (3) can be applied to determine  $\underline{z}_{eiz}$ , where  $r_{ou}$  is cable radius, while  $r_{in}$  is external radius of metal sheet  $r_{eo}$ . Mutual impedance of the loops conductor-metal sheet and metal sheet-return ground path  $\underline{z}_{im}$  is determined as:

$$\underline{z}_{im} = \frac{\rho_c}{\pi^2 r_{eo} r_{ei}} \frac{1}{N_1(r_{eo} u) J_1(r_{ei} u) - N_1(r_{ei} u) J_1(r_{eo} u)}. \quad (18)$$

### C. External mutual and self impedances

Ther procedure for calculation of external impedances needs to include soil characteristic, as a part of return current path. Basics for determining values of impedance per unigt length of the conductors placed above the ground or in the ground are given in [6] and [7]. Carson's relations from [7] are little bit simpler, but they are formed for a conductor above ground, since the approach from [6] provides possibility for analysis of the conductors placed in the ground. The expressions are formed for lineical infinitely long conductor and infinitely deep ground. The expressions for calculating external self and mutual impedances are also given in [5]. They have different form from the ones in [6], but essentially the expressions are identical. Those expressions are:

$$\underline{z}_{mm} = \frac{j\omega\mu_0}{2\pi} \left[ K_0(mR) - K_0(m2h_m) + 2 \int_0^{+\infty} \frac{1}{\alpha + \sqrt{\alpha^2 + m^2}} e^{-2h_m \sqrt{\alpha^2 + m^2}} \cos(\alpha R) d\alpha \right], \quad (19)$$

and

$$\underline{z}_{mn} = \frac{j\omega\mu_0}{2\pi} \left[ K_0(md) - K_0(mD) + 2 \int_0^{+\infty} \frac{1}{\alpha + \sqrt{\alpha^2 + m^2}} e^{-(h_m+h_n)\sqrt{\alpha^2 + m^2}} \cos(\alpha x) d\alpha \right], \quad (20)$$

In previous expressions  $K_0$  labels the modified Bessel function of the second kind of zero order,  $R$  radius of the conductor or cable, while  $m = \sqrt{j\omega\mu_0\sigma_z}$  is complex propagation constant of EM waves through the ground ( $\sigma_z = \sigma_z + j\omega\epsilon_z$ , where  $\epsilon_z$  is dielectric permittivity of the ground).

If non-uniform current distribution in metal sheets is considered, expressions (1) -(2) in combination with (3) are applied, where  $R$  is cable radius in expression for  $\underline{z}_{zs}$ , while  $\underline{z}_{z12}$ ,  $\underline{z}_{z13}$  and  $\underline{z}_{z23}$  in (3) are determined from expression (20). Dimension  $d$ ,  $D$ ,  $h_m$  and  $h_n$  can be noticed from Fig. 2. (1 denotes ground and 2 denotes air).

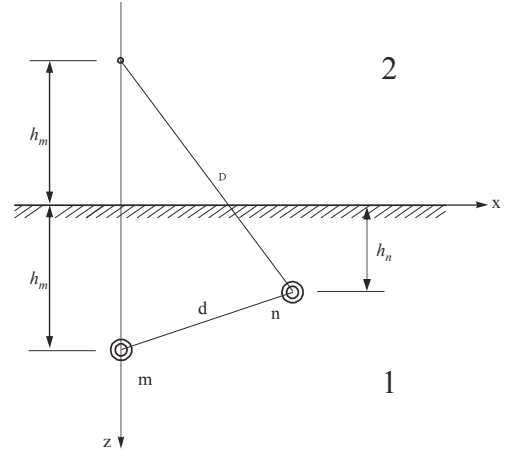


Fig. 2. Two cables in the ground

Integral in (19) and (20) can not be solved in closed form and it is necessary to apply numerical integration. For this reason, there are many published papers with proposed procedures for approximate solution of (19) and (20). Very often, Carson's relation [7] has been applied with integral that is a little bit simpler for approximate numerical solving than the one from [6] (expressions (19) and (20)). Namely Ammetani proved that Pollaczek expression [7] can be reduced to Carson's expression [6] by substituting  $e^{-(h_m+h_n)\sqrt{\alpha^2 + m^2}}$  with  $e^{-(h_m+h_n)|\alpha|}$  in (8), which provided possibility for using Carson's expression for determining impedance of the conductor placed in the ground. Using Carson's' expression also results in simpler approximation which includes complex depth of return path. Expressions for external impedance proposed by Carson include impedance for ideal conducting ground and corrective factors  $\Delta R$  and  $\Delta X$ :

$$\underline{z}_{mm}^s = \Delta R_{mm} + j(\omega) \frac{\mu_0}{2\pi} \ln \frac{2h_m}{r_m} + \Delta X_{mm}), \quad \text{and} \quad (21)$$

$$\underline{z}_{mn}^s = \Delta R_{mn} + j(\omega \frac{\mu_0}{2\pi} \ln \frac{D_{mn}}{d_{mn}} + \Delta X_{mn}). \quad (22)$$

After some numerical procedures for determining approximate value of external self and mutual impedance (in details presented in [7]), it is obtained:

$$\underline{z}_{mm}^s = \frac{\omega\mu_0}{8} + j\omega \frac{\mu_0}{2\pi} \ln \frac{D_e}{r_m} \quad \text{and} \quad \underline{z}_{mn}^s = \frac{\omega\mu_0}{8} + j\omega \frac{\mu_0}{2\pi} \ln \frac{D_e}{d_{mn}}. \quad (23)$$

In (23) it is

$$D_e = \frac{e^{0,6159315}}{4\pi\sqrt{5}10^{-4}} \sqrt{\frac{\rho}{f}} \approx 658 \sqrt{\frac{\rho}{f}}. \quad (24)$$

The relation for determining self impedance, that includes complex return path (i.e. complex depth  $\underline{p}=1/\underline{m}$ ) is given in [9] as:

$$\underline{z}_{mm} = j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h + \underline{p})}{R_m}. \quad (25)$$

Neglecting conductor's height above the ground  $h$  and imaginary part of  $\underline{\sigma}_z$ , expression (25) can be reduced to [10],

$$Z_{mm} = \frac{\omega\mu_0}{8} + j\omega \frac{\mu_0}{2\pi} \ln \frac{D_e}{R_m}, \quad D_e = 711,762 \sqrt{\frac{\rho}{f}} \quad (26)$$

Very accurate approximation for external self and mutual impedance is given in [5]

$$\underline{z}_{mm} = \frac{j\omega\mu_0}{2\pi} \left[ -\ln\left(\frac{\gamma m R}{2}\right) + 0,5 - \frac{4}{3} m h_m \right], \quad (27)$$

$$\underline{z}_{mn} = \frac{j\omega\mu_0}{2\pi} \left[ -\ln\left(\frac{\gamma m d_{mn}}{2}\right) + 0,5 - \frac{2}{3} m (h_m + h_n) \right]. \quad (28)$$

For  $\underline{\sigma}_z = \sigma_z + j\omega\varepsilon_z \approx \sigma_z$  from (27)-(28), it is

$$\underline{z}_{mm} = \frac{\omega\mu_0}{8} + j\omega \frac{\mu_0}{2\pi} \ln \frac{D_e}{R}, \quad \underline{z}_{mn} = \frac{\omega\mu_0}{8} + j\omega \frac{\mu_0}{2\pi} \ln \frac{D_e}{d_{mn}}, \quad (29)$$

where it is

$$D_e = \frac{\sqrt{2}\sqrt{e}}{\gamma\sqrt{\pi\mu_0}} \sqrt{\frac{\rho}{f}} = 658,87160 \sqrt{\frac{\rho}{f}}. \quad (30)$$

### III. NUMERICAL RESULTS

The 400 kV power cable of Milliken type with cross-section 2500 mm<sup>2</sup> with isolated copper conductors is observed. Metal sheets are made of aluminum of cross-section 500 mm<sup>2</sup>. Three single-core cables are placed in a flat formation at the depth 1m, with mutual distance between axis of 0.3 m. The conductor temperature is 90°C, while metal sheet temperature value is assumed as 70°C.

In Table 3, results for internal impedances of metal sheets, obtained in two different ways, are presented. The first group of results is obtained using Bessel functions in expression (3) (complete model). The second group is calculated using simplified relations (29)-(30). Impedances in (3) are calculated by numerical integration of Pollaczek relations

(19)-(20). In both cases, the same value of conductor internal impedance is used.

TABLE I  
SELF AND MUTUAL IMPEDANCE OF THE 400 KV CABLE (2500 mm<sup>2</sup>)  
IN A FLAT FORMATION

Impedance	Complete model	Simplified model
$\underline{z}_{11}$ [Ω/km]	0,060066+j0,661113	0,0599476+j0,661171
$\underline{z}_{44}$ [Ω/km]	0,1177101+j0,600938	0,117266+j0,601210
$\underline{z}_{41}$ [Ω/km]	0,0494655+j0,601135	0,0493480+j0,601210
$\underline{z}_{12}$ [Ω/km]	0,0494646+j0,505118	0,0493480+j0,505152
$\underline{z}_{13}$ [Ω/km]	0,0494646+j0,461566	0,0493480+j0,461601

### IV. CONCLUSION

The review of the relations for determining self and mutual impedances of the cable is given in the paper. Afterwards, the accuracy of presented relations is tested. The problems of single-core cables with metal sheets earthed at both or one end, with or without cross-bonding are especially discussed. Considering the presented results, it is obvious that simplified relation can be applied, since the maximal error, which exists in results for mutual resistance, is less than 0,3%.

### REFERENCES

- [1] H. W. Dommel, *Electromagnetic Transients Program Theory Book* (EMTP Theory Book), Portland: Bonneville Power Administration, 1986.
- [2] IEC Std. 60228, "Conductors of Insulated Cables", 2004.
- [3] CIGRE Working Group B1.03, "Large Cross-Sections and Composite Screens Design", June 2005.
- [4] H. Schunk, *Stromverdrängung*, UTB, Dr. Alfred Hüthig Verlag, 1975.
- [5] L.M. Wedepohl, and D.J. Wilcox, "Transit Analysis of Underground Power-Transmission Systems", *Proc. IEE*, 120, pp. 253-260, 1973.
- [6] J.R. Carlson, "Wave Propagation in Overhead Wires with Ground Return", *Bell System Journal*, vol. 5, pp. 539-554, 1926.
- [7] F. Pollaczek, "Über das Feld einer unendlichen langen wechselstromdurchflossenen Einfachleitung", *E.N.T.*, vol. 3, pp. 339-360, 1926.
- [8] A. Ametani, Y. Miyamoto, T. Asada, Y. Baba, N. Nagaoka, I. Lafaia, J. Mahseredjian, and K. Tanabe, "A Study on High-Frequency Wave Propagation along Overhead Conductors by Earth-Return Admittance/Impedance and Numerical Electromagnetic Analysis", *International Conference on Power Systems Transients (IPST2015)*, Cavtat, Croatia, June 15-18, 2015, <https://goo.gl/HVohBt>.
- [9] C. Gary, "Approche Complete de la Propagation Multifilaire en Haute Fréquence par Utilisation des Matrices Complexes" (Complete Approach to Multiconductor Propagation at High Frequency with Complex Matrices), *EdF Bulletin de la Direction des Etudes et Recherches*, Série B, no. 3/4, pp. 5-20, 1976.
- [10] A. Deri, G. Tevan, A. Semlyen, and A. Castanheira, "The Complex Ground Return Plane, a Simplified Model for Homogeneous and Multy-Layer Earth Return", *IEEE Trans. on Power Apparatus and Systems*, vol. PAS 100, no.8, pp. 3686-3693, August 1981.