

Spectrum Analysis of Analog Signals in MATLAB Environment

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Abstract – This paper presents an approach for studying the spectrum analysis of analog signals with various shape forms, by using trigonometric and complex forms of Fourier transformation. An algorithm and its implementation in MATLAB environment have been developed. The implemented software allows studying the spectrum of different analog signals and the influence of signals parameters on its spectrum. The presented approach can be used in engineering education for studying this process.

Keywords – Communication, signal and systems, signal processing, spectrum analyses, detection and identifying of signals, computer simulation, MATLAB.

I. INTRODUCTION

The spectrum analysis of the signals is a basic part in the theory of the telecommunications. It allows each periodic signal to be presented as sum of an infinite number of sinusoidal signals with different amplitudes and initial phase offsets. The spectrum analysis allows to make a deeper study of the signal (distribution of the amplitudes, power, phase shifts, etc.). It also simplifies the understanding and interpretation of the effects of various time-domain operations. The spectrum analysis is widely used in filtration, recognition of signals and noise, restrictions of bandwidth, etc.

The spectrum analysis is studied in the course "Signals and Systems" at the undergraduate course in specialties "Telecommunications", "Computer and Software Engineering" and "Electronics" at Technical University of Sofia. During the seminar exercises the students will have the opportunity to study the spectrum of different periodic and aperiodic analog signals and analyze the amplitude spectrum and phase spectrum of the signals. They will be able to change the signal parameters and analyze their influence on the signal in time and frequency domain.

In order to be able to generate signals with maximum number of different shape forms and to change as much as possible signal parameters, it is proposed to use a program code in the MATLAB environment [1]. The choice of MATLAB over other software products and programming languages is due to the wide range of built-in functions that

will facilitate students, as there is no need to create all these functions from scratch [2]. Writing source code in MATLAB is preferable to the MATLAB Simulink because of the additional features that provide students with ability to change signals in a way that MATLAB Simulink does not provide [3]-[5]. Writing the source code also leads to the requirement that the students will understand the functions describing the calculation of the signal spectrum.

II. PROBLEM FORMULATION

The main tasks to be solved in the spectrum analysis of signals are associated with the variety of signals as types (periodic and aperiodic) and shapes (sinusoidal, rectangular, triangular, sawtooth, Gaussian, etc.), as well as with the usage of the different forms of the Fourier transformation.

Let $S(t)$ is an aperiodic analogue signal, it has the following CFT (Complex Fourier Transformation), given in Eq.1 [6]:

$$S(j\omega) = \int_{-\infty}^{\infty} S(t)e^{-j\omega t} dt \quad (1)$$

where $S(j\omega)$ is the Fourier Spectral Density. In general case, it's a complex function, described with Eq.2 [6].

$$S(j\omega) = \text{Re}[S(j\omega)] + j \text{Im}[S(j\omega)] = |S(j\omega)|e^{j\phi(\omega)} \quad (2)$$

where $|S(j\omega)|$ is the Amplitude Fourier Spectrum, $\phi(\omega)$ is Phase Fourier Spectrum

Let $S(t)$ is a periodical analogue signal, the complex form of the Fourier series is described as following [7]:

$$S(t) = \sum_{-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad (3)$$

where

$$C_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} S(t)e^{-jn\omega_0 t} dt \quad (4)$$

The short trigonometric form of the Fourier series is given with [8]:

$$S(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n), \quad (5)$$

where

$$A_n = \sqrt{a_n^2 + b_n^2}, \quad (6)$$

$$\phi_n = -\arctg \frac{b_n}{a_n}. \quad (7)$$

where A_n is the Amplitude Spectrum, ϕ_n is Phase Spectrum. In the Eq. (5) a_0 is the constant component of the signal. In the Eq. (6) and (7) with a_n and b_n are labeled the amplitudes of the cosinusoidal and sinusoidal components of the full trigonometric form of the Fourier transformation respectively.

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One of the most commonly used signals is the rectangular pulse train. Its mathematical description is following:

$$s(t) = \begin{cases} A & 0 < t < \tau \\ 0 & \tau < t < T \end{cases} \quad (8)$$

where A , T and τ are the Amplitude, Period and Pulse width of the rectangular pulse train respectively.

The main tasks which can be solved by computer simulations are following:

1. To create an appropriate model for a presentation of the signals.
2. To create algorithms for calculation of the Fourier transformations.
3. To investigate the influence of the waveforms, signal parameters and Fourier series parameters of the signal spectrum.
4. To represent graphically the results of the investigations.

III. BASIC ALGORITHM FOR SPECTRUM ANALYSIS

The block diagram of the proposed algorithm for spectrum analysis of the analog signals is shown in Fig. 1.

Initially the global parameters and variables (number of harmonics that will be calculated, etc.) are set. Then it is necessary to select the form of the signal and to set its parameters (basic parameters such as period, amplitude, etc. as well as specific parameters for the particular signal). Considering all the parameters of the signal, students must create a mathematical description of the researched signal (the studies of the following periodic and aperiodic signals are envisaged: sinusoidal, rectangular, triangular, sawtooth, and Gaussian).

By working with periodic signals, a full trigonometric, a short trigonometric and a complex forms of Fourier transformation will be used. Then determining the spectrum of aperiodic signals a Fourier transformation for aperiodic signals is used. Furthermore, the algorithm provides the solution of the inverse task, associated with reconstruction of the signal. In case of aperiodic signal, the students will not create an algorithm for inverse Fourier transformation due to the inability of MATLAB to give the alphabetic decision of the integral in the inverse Fourier transformation. Created software modules for determining the spectrum of periodic and aperiodic signals allows creating a graphical representation of the signal, its amplitude and phase spectrum. This gives possibilities for investigation and visualization of the impact of certain signal parameters of the signal in time and frequency domain.

In order to see how the reconstructed signal becomes closer to the original with the increase of the number of harmonics in the Fourier transformation, it is proposed to plot in one axis the original signal and the obtained reconstructed signal. The graphic is updating every 0.1 seconds, when an additional harmonic is added to the reconstructed signal. The obtained graphic presentation gives a real idea about the nature of the Fourier transformation, as well as show the Gibbs phenomenon.

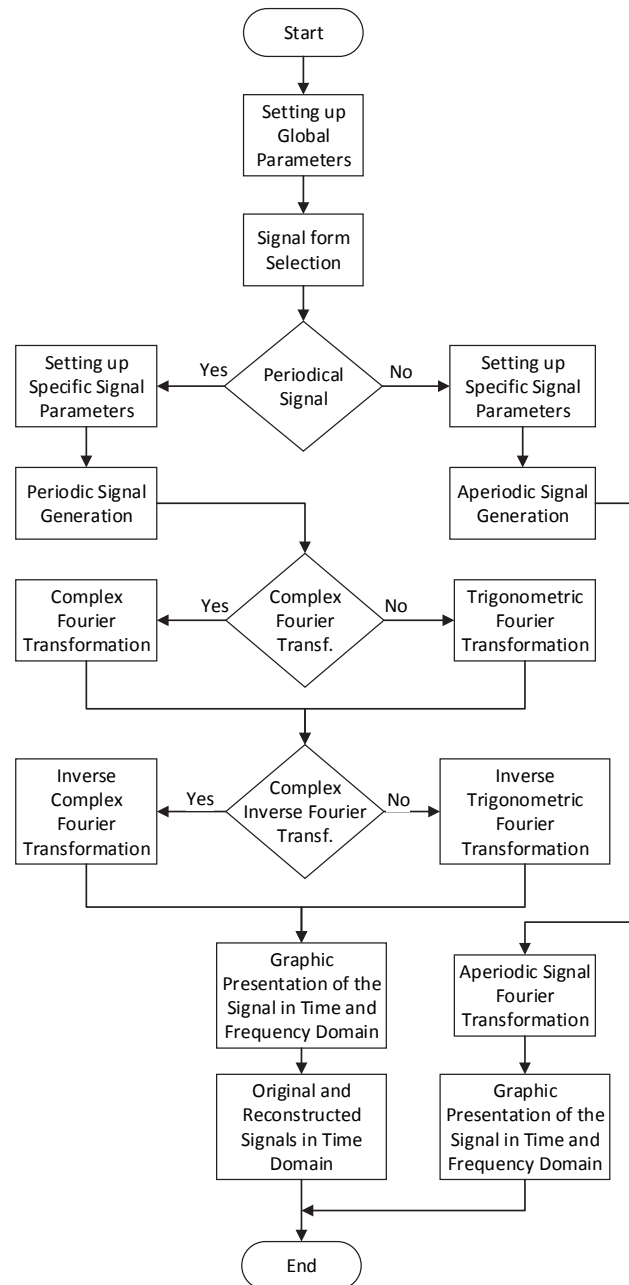


Fig. 1. Block diagram of the basic algorithm

IV. EXPERIMENTAL PART

The formulated problems are solved by computer simulation in MATLAB environment.

Some results from simulations of rectangular pulse train for one period and aperiodical unipolar rectangular impulse are shown in Fig. 2 and Fig. 3, respectively. The results from simulations of periodical and aperiodical Gaussian signal are given in Fig. 4 and Fig. 5, respectively. Each figure shows a graphic presentation of the signal in the time domain (original and the reconstructed after inverse Fourier transformation), magnitude and phase spectrum.

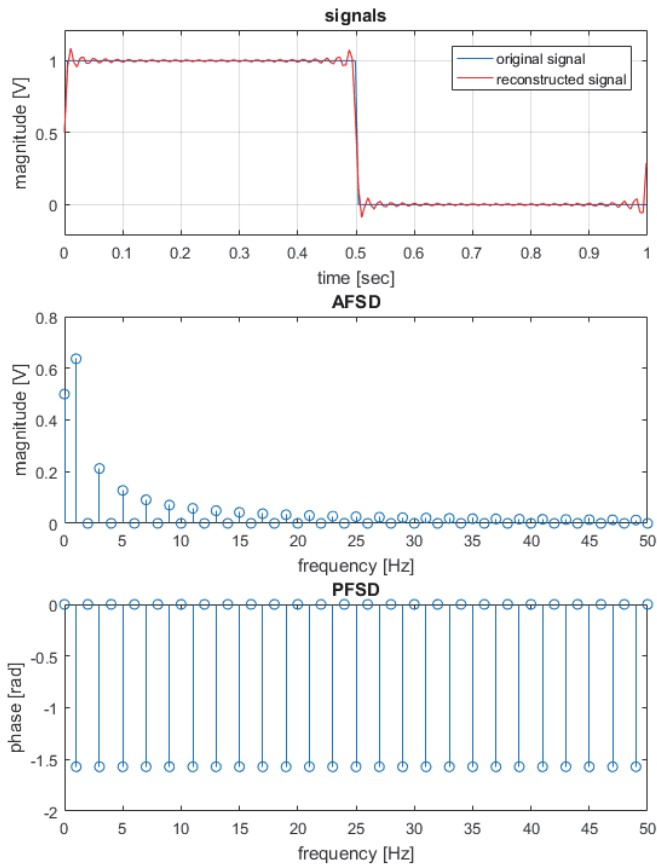


Fig. 2. Rectangular pulse train for one period and its spectrum

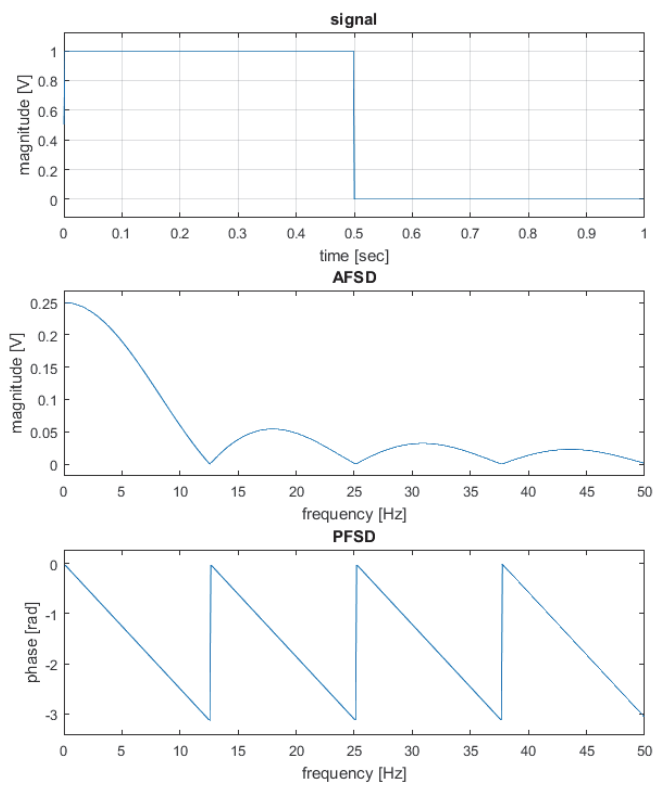


Fig. 3. Aperiodic rectangular pulse and its spectrum

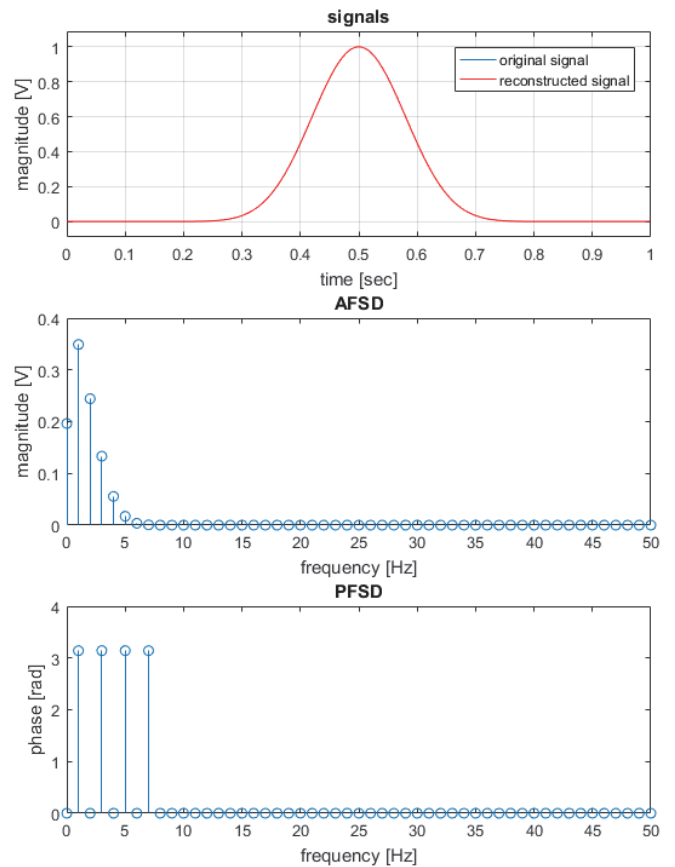


Fig. 4. Gaussian pulse train for one period and its spectrum

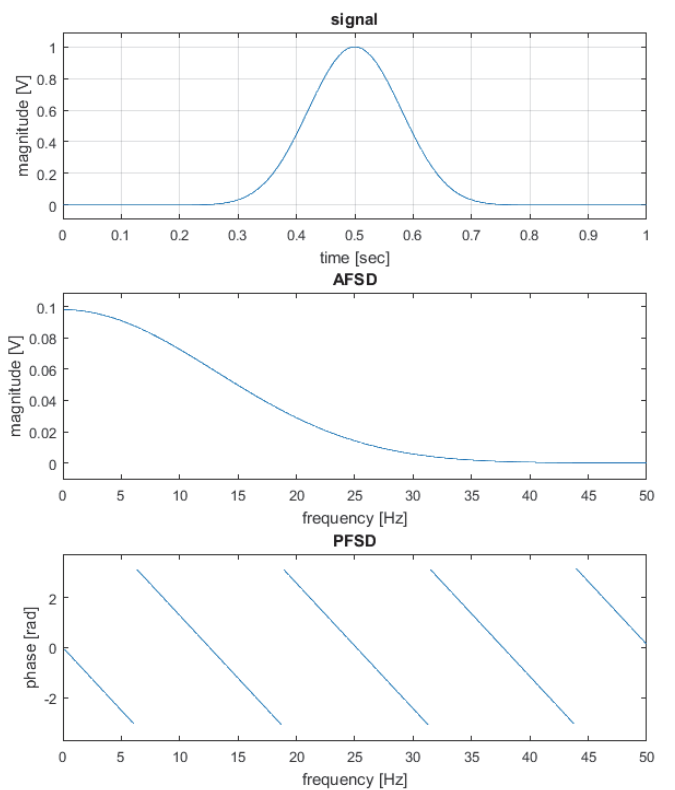


Fig. 5. Aperiodic Gaussian pulse and its spectrum

In Figs. 6-8 are shown the original and the reconstructed signal in three different stages of reconstruction (represented as a sum of sinusoids with the appropriate amplitude, frequency and initial phase offset). The mathematical description of the reconstructed signals shown on Fig. 6 and Fig. 7 are given in Eq. 9 and Eq. 10 respectively. The mathematical description of the original signal is given in Eq. (8). On each subsequent step (usually by 0,1 seconds) the next harmonic is added to the current state of the reconstructed signal. This continually changing figure shows how with the increase of the number of harmonics that are summed, the reconstructed signal becomes closer to the original.

$$S(t) = \frac{a_0}{2} + A_1 \cos(\omega_0 t + \phi_1) = 0.5 + 0.637 \cdot \cos(2\pi t - 1.571) \quad (9)$$

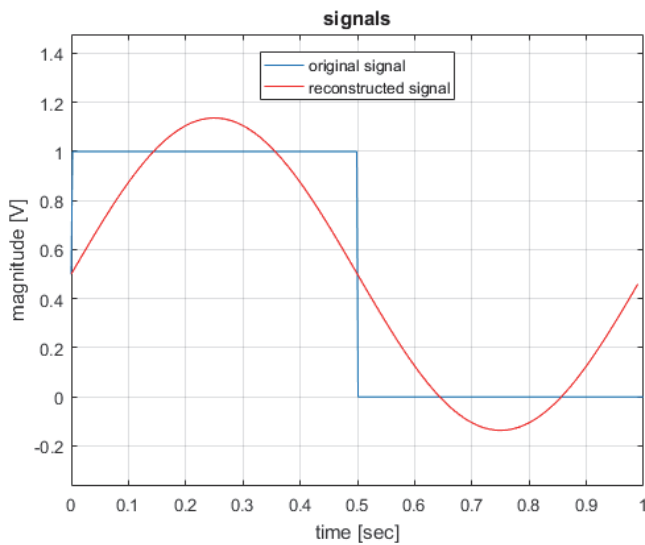


Fig. 6. The original and reconstructed signal by 1 harmonic

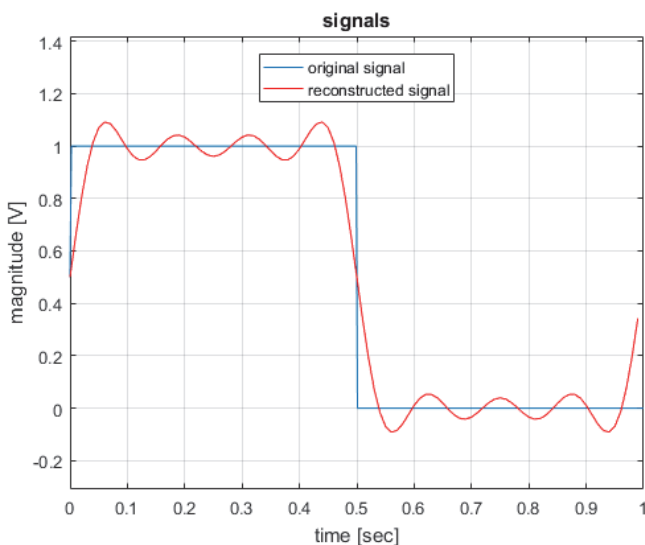


Fig. 7. The original and reconstructed signal by 7 harmonics

$$S(t) = \frac{a_0}{2} + \sum_{n=1}^7 A_n \cos(n\omega_0 t + \phi_n) = 0.5 + 0.637 \cdot \cos(2\pi t - 1.571) + 0.212 \cdot \cos(3\pi t - 1.571) + 0.127 \cdot \cos(5\pi t - 1.571) + 0.091 \cdot \cos(7\pi t - 1.571) \quad (10)$$

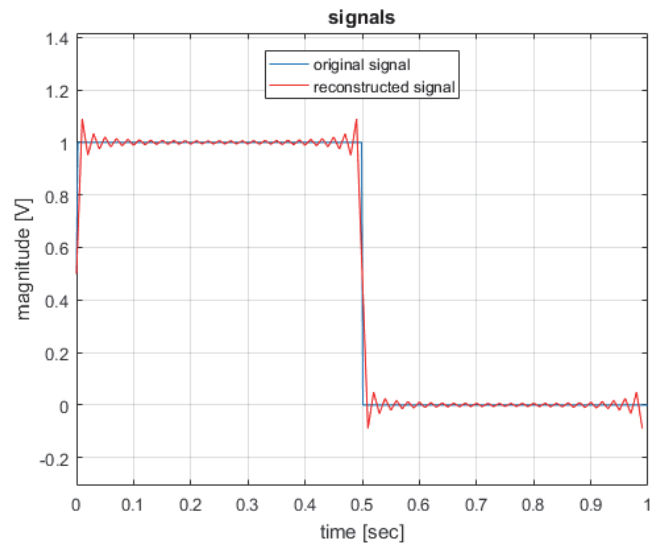


Fig. 8. The original and reconstructed signal by 50 harmonics

V. CONCLUSION

In this paper an approach for studying on spectrum of periodic and aperiodic analog signals has been proposed. An algorithm and its implementation in MATLAB environment have been developed. The algorithm provides the solution of the inverse task, associated with reconstruction of the signal. The students can create software models of signals with different forms. They will be able to change the signal parameters and analyze their influence on the signal in time and frequency domain.

The proposed approach can be developed for making exercises, which will be used for web based distance education.

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