Identification Method for Objects by High Order Models in Closed Loop

Jordan Badev[1], Angel Lichev[2] and Georgi Terziyski[3]

Abstract – **This paper presents a method for conducting an experiment with controllable object to acquire data, through which is achieved the identification of the high order linear models. The experiment is conducted in the closed loop with a controller. Frequency characteristics are used for obtaining the parameters of the model, i. e. the critical operating mode values.**

Keywords – **Frequency response function, Frequency characteristics, Critical operation mode, Stability margin.**

I. Introduction

During the identification of the controllable objects, there is a search for an analytical model, based on experimental readings. The analytical models are suitable for easily and repeatedly reproduction of experiments with the object. The results obtained from the models about the properties of the object, are with reasonable accuracy. Analytical models can also be used for experimental setup of the elements of the control device or for producing scaled (reduced or enlarged) physical models.

Collecting informative data is a very important stage of the identification, by which an adequate analytical model of the physical processes can be evaluated. The proposed method for obtaining experimental data does not require specialized equipment. For this purpose a classical linear or state controller is needed. The circuit of the experimental arrangement is the possible wiring diagram for constant operation between the control device and a controllable object, or the so called circuit for operation in a closed loop [1, 2].

II. Theoretical Setup of the Identification Method

The proposed identification method uses the analytical expressions for: amplitude frequency characteristics (AFC) and phase-frequency characteristics (PFC) for the alleged model and the experimental results for the critical frequency.

[1]Jordan Badev is with the Department of Atomation, information and control systems, Technical Faculty, University of Food Technologies, 26 Maritza Blvd., 4002 Plovdiv, Bulgaria, E-mail: jobadev@abv.bg

[2]Angel Lichev is with the Department of Electrical Engineering and Electronics, Technical Faculty, University of Food Technologies, 26 Maritza Blvd., 4002 Plovdiv, Bulgaria, E-mail: angel_lichev@abv.bg

[3]Georgi Terziyski is with the Department of Atomation, information and control systems, Technical Faculty, University of Food Technologies, 26 Maritza Blvd., 4002 Plovdiv, Bulgaria, E-mail: g_terziyski@uft-plovdiv.bg

The analytical expressions for AFC and PFC are known from the control theory (CT) [3] and contain the unknown parameters of the model. The experimental results for the amplitude and the phase are measured from the critical operation mode of the object in the closed loop. Eventually, the task becomes solving a system of non-linear equations, according to the unknown parameters.

Fig. 1 shows, in general, the circuit of the closed loop with elements and indications of the signals.

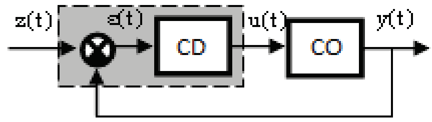

Fig. 1. General structure and indications in the closed loop

It is indicated in the figure:
CO – Controllable object;
CD – Control device (controller);
z(t) –Reference value (desired set point);
u(t) – Control signal sent to the system;
y(t) – The measured output of the system;
y(t)-z(t) = ε(t) – error value.

What is the critical operating mode of the object, and why it is used? The well-known Nyquist stability criterion for stability of linear systems presents best the critical mode. Amplitude-phase-frequency characteristics of the open structure $W_{oc}(j\omega)$ for three values of the static coefficient of amplification $K_{p1} < K_{p2} < K_{p3}$ are presented in Fig. 2. The coefficient is denoted as K_{p1}, because it can be considered as the coefficient of the controller.

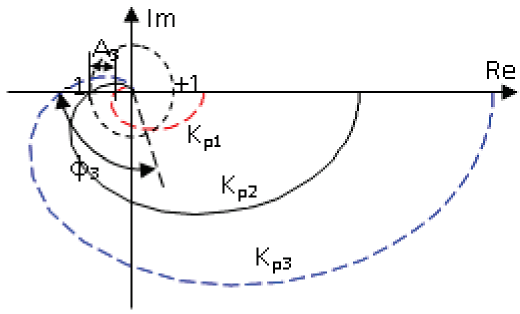

Fig. 2. APFC characteristic of the open structure

According to the Nyquist stability criterion [3], APFC for K_{p1} represents a stable closed loop system, and with A3 and Φ3 are marked the corresponding stability margins. APFC for K_{p2} represents a closed loop system, which is on its limit of stability, and this operating mode is known as critical operating mode. It is known by the theory, that the crossing of the negative part of the real axis of APFC is only possible in case of third and higher order open loop system without transportation lag. The critical operating mode is possible only for first and second order systems.

The parameters of this mode are widely used: for setting the controllers in the loop; as asymptotic quality parameters, which keep performance unchanged; at the design stage and in particular to the identification of the systems. The critical operating mode is characterized by the following parameters:

Critical frequency – $\omega_{кр}$, this is the frequency at which:

the module of the | $W_{oc}(j\ \omega_{cr})$ | = $A(\omega_{кр})$ =1 and the phase arg($W_{oc}(j\ \omega_{кр})$) = $\Phi(\omega_{кр})$ = -π.

The last of this also implies that the process (for K_{p2} and $\omega_{кр}$) at the outcome y(t) will oscillate with constant amplitude, and will be in anti-phase with the input signal u(t). The last fact is used for simple experimental achieving of the critical operation mode, by soft settings variation of the controllers (linear and state).

III. A Setting of the Closed Loop for Critical Operation Mode

The critical operating mode in the closed loop can be realized by proportional (linear) or two state (on/off) controller.

In case of operation with classical PID controller, the critical operating mode shall be set up as follows:

1. It is set for operation as P controller, and the integral and the derivative terms are excluded, by setting the appropriate values of the parameters for setting ($T_и \rightarrow \infty$, $T_д \rightarrow 0$).
2. An average value is set in the adjustment interval – the reference value
3. A small value is defined of the coefficient of proportionality $K_п$, for which the process of the output y(t) decreases.
4. The coefficient Kп is increased smoothly, and the process of the output y(t) is monitored. The process is supposed to change to a slower attenuation and to an increase of the oscillation. Kп continues to increase until fluctuations with constant amplitude Δy_{kp} and constant frequency $\omega_{кр}$ are established. These are the parameters which are necessary for the further identification and they are as accurately as possible defined from the diagram of the established fluctuations.

In case of operation with classic two state controller, the critical operation mode is set as follows:

1. A two state controller is set without non unique function (hysteresis→0),

$$y_{cp} = y_з \tag{1}$$

2. A set point is set z(t) = constant, by absolute value in the middle of the adjustment interval, corresponding to a set point in relative units z% = 50%, where

$$z\% = \frac{y_{cp} - y_{(\infty)\min}}{y_{(\infty)\max} - y_{(\infty)\min}} 100\% \tag{2}$$

The variables in the last formula are:

$$y_{(\infty)\max} = K_{об} * u_{\max} \tag{3}$$

– steady state value, which is achieved by maximum control effect in Fig.3

$$y_{(\infty)\min} = K_{об} * u_{\min} \tag{4}$$

– steady state value, which is achieved by minimum control effect in Fig.3

y_{av} – average value of the steady state oscillations – Fig. 3.

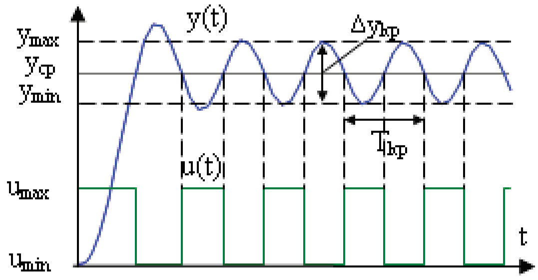

Fig. 3. Operation of a two state controller with a high order object

The completion of the critical process is easier with the two state controller, because significant state interference by the operator is needed in the first case.

The following parameters are needed for the identification which is calculated with the experimental diagram of the critical process – Fig. 3:

- Critical frequency

$$\omega_{кр} = \frac{2\pi}{T_{кр}} \tag{5}$$

- Fluctuation range of the controllable (output) variable

$$\Delta y_{кр} = y_{\max} - y_{\min} \tag{6}$$

- Variation range of the control impact for:
  sinusoidal impact

$$\Delta u = u_{\max} - u_{\min} \tag{7}$$

  rectangular pulses

$$\Delta u = \frac{4}{\pi}(u_{\max} - u_{\min}) \tag{8}$$

- Average value of the controllable variable

$$y_{cp} = \frac{y_{\max} - y_{\min}}{2} \tag{9}$$

- Average value of the control impact

$$u_{cp} = \frac{\Delta u}{2} \tag{10}$$

IV. AN ALGORITHM FOR THE CALCULATION METHOD OF THE PARAMETERS OF MODELS

TF, AFC and PFC of the models, which are defined by the method are shown in Table 1.

The identification method can be depicted with the following algorithm:

1. Preparation and realization of the experiment and construction of the process in Fig. 3.
2. Calculation of the static coefficient of the object by the formula:

$$K_{об} = \frac{y_{cp}}{u_{cp}} \tag{11}$$

3. Calculation of the critical frequency by the formula:

$$\omega_{кр} = \frac{2\pi}{T_{кр}} \tag{12}$$

4. Calculation of the critical module by the formula:

$$A_{кр} = \frac{y_{кр}}{\Delta u} \tag{13}$$

5. For the models 1 and 2 in Table 1, it is easy T and τ to be defined – first T from AFC and after that τ from PFC
6. For the models 3, the system is nonlinear according to T and n. The second equation can be simplified, i. e.

$$A_{кр} = K_{об}\left(\cos\frac{\pi}{n}\right)^{n} \tag{14}$$

– in this form, it is suitable for nomograms.

Methods for solving non-linear equations systems are known from theory.

- It is cleared that the unknown n is number of first order lags in the model and can accept equivalent and positive units only, which are greater than or equal to 3, i. e. ($n \geq 3$)
- The unknown T is meant as a time constant and thus it can accept positive values only, i. e. $T \geq 0$.

The system in the Table.1 (for the third kind of models) can be solved by building the graphics of the functions: $n = f_1(T)$, $n = f_2(T)$ and defining the coordinates of the intersection of the graphics. The co-ordinate on the axis (n) is approximated to the nearest equivalent and positive unit. It is accepted as an order of TF.

The right kind of the functions is:

$$n = \frac{\pi}{arctg(\omega_{кр}T)} = f_1(T) \tag{15}$$

$$n = \frac{\lg(\frac{K_{об}}{A_{кр}})}{\lg(\sqrt{1+(\omega_{кр}T)^2})} = f_2(T) \tag{16}$$

TABLE 1
MODELS OF APFC AND PFC [3]

№	TF of the model	System of PFC and AFC
1.	$\frac{K_{об}}{Ts+1} * e^{-\tau s}$	$\pi = \omega_{кр}\tau + arctg(\omega_{кр}T)$ $A_{cr} = \frac{K_{об}}{\sqrt{1+(\omega_{кр}T)^2}} = \frac{\Delta y_{кр}}{\Delta u}$
2.	$\frac{K_{об}}{(Ts+1)^2} * e^{-\tau}$	$\pi = \omega_{кр}\tau + 2 * arctg(\omega_{кр}T)$ $A_{cr} = \frac{K_{об}}{1+(\omega_{кр}T)^2} = \frac{\Delta y_{кр}}{\Delta u}$
3.	$\frac{K_{об}}{(Ts+1)^n}$	$\pi = n * arctg(\omega_{кр}T)$ $A_{cr} = \frac{K_{об}}{(\sqrt{1+(\omega_{кр}T)^2})^n} = \frac{\Delta y_{cr}}{\Delta u}$

V. VALIDATION OF THE METHOD, RESULTS AND CONCLUSIONS

The presented method is validated in the computing environment of MATLAB [1, 2]. Numbers of closed loops are simulated with the two kinds of controllers (S and two state). Each of the controllers control objects with different models, from the presented in Table 1. In all cases, satisfactory results are reached (considering carefully the graphics). There are no doubts that the method is correct.

For approbation of the method, an example of fourth order model is proposed with simulated data. The reader can verify the results, in order to convince himself in the merits of the method. The two variants for achieving a critical operating mode are shown – by the circuit in Fig. 1.

TF of the controllable object is:

$$W_{oy(s)} = \frac{50}{(10s+1)^4} \tag{17}$$

and the desired set point is:

$$z(t) = 25 * 1(t) \tag{18}$$

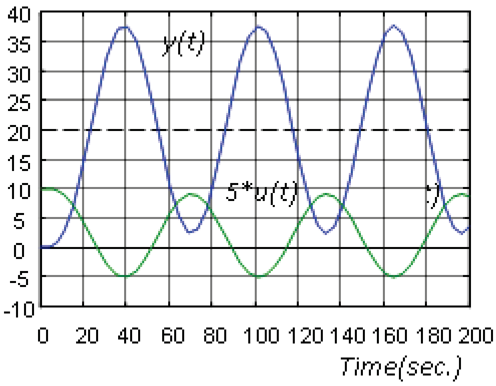

Fig. 4. (a)

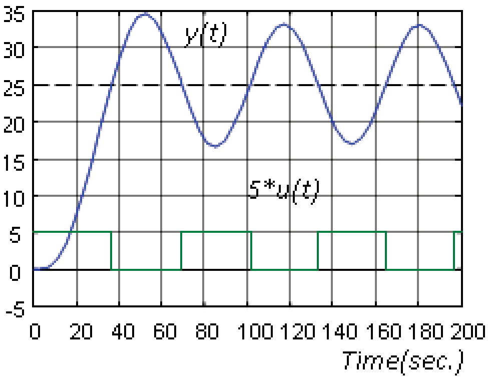

Fig. 4. (b)

Variant: (a) – a loop with linear P controller c $K_p = 0{,}08$

Variant: (b) – a loop with two state controller, without hysteresis and $u_{min} = 0$ and $u_{max} = 1$

The following quantities are reported from the last graphics of the Fig. 4*a* and 4*b*: $\Delta y_{кр}$, $T_{кр}$, y_{av}, Δu, u_{cp}

It is calculated from the reported values in Table 2:

$\omega_{cp} = 2\pi/T_{кр}$; $K_{об} = y_{cp}/u_{cp}$; $A_{кр} = \Delta y_{кр}/\Delta u$

TABLE 2

	$\Omega_{кр}$	$K_{об}$	$A_{кр}$
(a)	0.0967	40.8831	12.2807
(b)	0.0977	50.00	12.5984

The system of equations (1) and (2) is solved graphically by the obtained values in Table 2. As a result, the graphics in Fig. 5 are obtained.

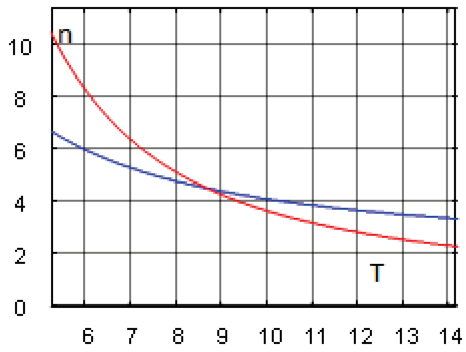

Fig. 5. (a)

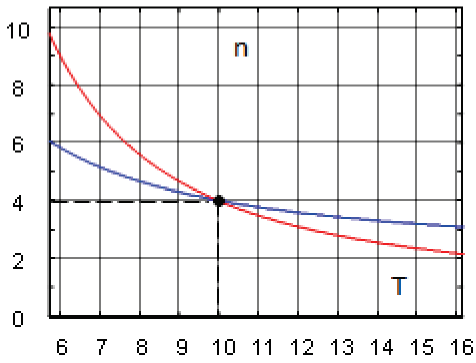

Fig. 5. (b)

On the basis of the results which are obtained (the coordinates of the intersections), the following conclusions can be drawn:

1. The case a) – the results for T and n are with unacceptable accuracy. That is seen in the graphic an can be calculated by the formula [4]:

$$\varepsilon = \frac{1}{K_{oc}} Z = \frac{1}{K_p * K_{об}} Z = \frac{1}{0.08 * 50} 25 = 5 \qquad (19)$$

If the value of $K_{об}$ in the table is being corrected with this value, the system of equations (1) and (2) are going to be changed and the inaccuracy of the result is going to decrease.

2. The case b) – from the graphic b), the results are: $T \approx 10$s and $n \approx 4$, which is acceptable accuracy.
3. It appeared preferable to realize the experiment in loop with two state controller. This is due to the easier set-up of the critical operating mode, and the lack of the steady state error, by which the experimental results are corrected.

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