Examination of the polar response of a circular piston acoustic transducer with a cone form of the membrane

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Abstract – **In this paper the problem about polar field of a circular piston transducer with a cone form of the membrane is considered. Theoretical expression is proposed for calculating sound pressure level (SPL) and plot polar diagram of a circular piston transducer with a cone form of the membrane. Conclusions with practical value have been made. Experimental measurements of the polar response diagram have been made.**

Keywords – **polar response, polar diagram, sound pressure level, circular piston transducer, conical membrane.**

I. Introduction

The acoustic transducer's radiation pattern is of great interest recently due to the increased demand of quality radiators for echography, geology and architectural acoustics[1]. Furthermore requirements for the design and modeling of loudspeakers and microphones for audio systems are increased[2]. Therefore, a theoretical analysis of the expressions for calculating the directional diagram of a circular piston acoustic transducer with conical form is made. Analytical expression for calculating polar diagram of circular piston with conical shape radiator is proposed. An experiment has been conducted to measure a cone shape loudspeaker polar diagram to demonstrate the practicability of the proposed expression.

II. Solution of Research Problem

A. Theoretical background

The purpose of this study is a circular piston transducer with a conical shape of the membrane. Because of its symmetry with respect to the z axis (fig. 1) it can be assumed that conical piston polar pattern will be one with rotation symmetry. In order to determine this symmetry, it is sufficient to determine the sound pressure level at a distance *d* in a plane whose norm is perpendicular to the z-axis.

B. Aanalytical presentation of the problem

The acoustic pressure generated by an emitter in the environment can be calculated by assuming that its surface consists of a plurality of elementary sections *dS* that can be obtained by the solution of a differential equation (fig.1).

$$dS = \sqrt{E \cdot G - F^2}\, d\alpha d\rho \qquad (1)$$

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where

$$E = \left(\frac{\partial x}{\partial \alpha}\right)^2 + \left(\frac{\partial y}{\partial \alpha}\right)^2 + \left(\frac{\partial z}{\partial \alpha}\right)^2$$

$$G = \left(\frac{\partial x}{\partial \rho}\right)^2 + \left(\frac{\partial y}{\partial \rho}\right)^2 + \left(\frac{\partial z}{\partial \rho}\right)^2$$

$$F = \frac{\partial^2 x}{\partial \alpha\, \partial \rho} + \frac{\partial^2 y}{\partial \alpha\, \partial \rho} + \frac{\partial^2 z}{\partial \alpha\, \partial \rho}$$

where

$x = r \cdot \rho \cdot \cos\alpha$
$y = r \cdot \rho \cdot \sin\alpha \quad 0 \le \alpha \le 2\pi \quad u \quad 0 \le \rho \le 1$
$z = \rho \cdot h$

which are the coordinates of the point of the elementary emitter E.

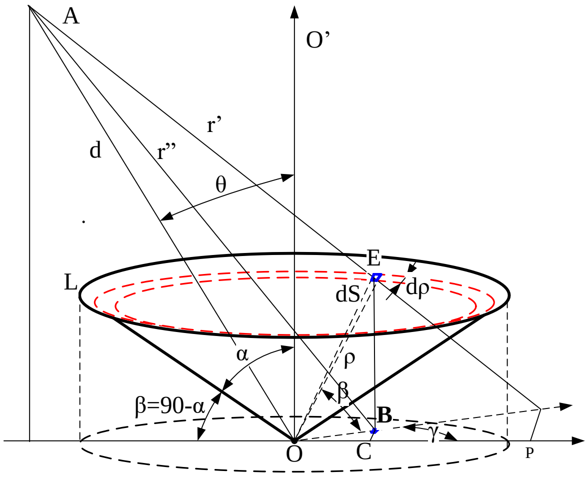

Fig. 1. Geometry of conical piston acoustic transducer.

As a result of the solution of the equations, the following result was obtained:

$$dS = \rho \cdot r\sqrt{r^2 + h^2}\, d\gamma d\rho \qquad (2)$$

where

- ρ and γ are current radius vector and angle
- *h* – height of the cone transducer
- *r* – radius of the piston

Each elementary section dS creates a sound pressure at the point of observer A. Point A is located at a distance r' from the point of elementary radiator E and angle θ relative to axis z and has the following coordinates:

$x_1 = d \cdot \sin\theta \cdot \cos\varphi$

$y_1=d \cdot sin\theta \cdot sin\varphi$, $-\pi/2 \leq \theta \leq \pi/2$ $0 \leq \varphi \leq \pi$
$z_1=d \cdot cos\theta$

where

- d – distance from the center of the cone to A.
- θ and φ current angles on the z and x axes.

C. Aanalytical solution

From analytical geometry the distance r' (from E to A) is determined by the expression:

$$r' = \sqrt{(d \cdot sin\theta \cdot cos\varphi - r \cdot \rho \cdot cos\gamma)^2 + (d \cdot sin\theta \cdot sin\varphi - r \cdot \rho \cdot sin\gamma)^2 + (d \cdot cos\theta - \rho \cdot h)^2} \quad (3)$$

Due to the rotational symmetry, r' is not dependent on the angle φ along the x - axis and it can be assumed that φ = 0 ° and for r' is obtained:

$$r' = \sqrt{(d \cdot sin\theta - r \cdot \rho \cdot cos\gamma)^2 - (r \cdot \rho \cdot sin\gamma)^2 + (d \cdot cos\theta - \rho \cdot h)^2} \quad (4)$$

The elementary section dS creates a sound pressure at a point A which is at a distance r' and at an angle θ relative to the OZ axis. This elementary sound pressure is [3]:

$$dp_\theta = \frac{r_0 \cdot p_m}{r'} e^{j(\omega t - kr')} \quad (5)$$

where

- r' is the distance from the elementary emitter to the point of observer A.
- p_m - the amplitude of the sound pressure
- r_0 distance OE which is distance from the center of the cone to emitting section dS.

$$p_m = \rho_s \cdot c_0 \cdot k \cdot r_0 \cdot v_m \quad (6)$$

where

- c_0 - the speed of the sound
- $k = \frac{2\pi}{\lambda}$ - a wave number
- ρ_s - density of the environment
- λ - wavelength

After multiplying both sides of equation (6) with and also multiplying the right side with $\frac{2\pi}{2\pi}$, taking into account that the elementary area of the emitter is $dS = 2\pi r_0^2$ therefore:

$$r_0 \cdot p_m = \frac{\rho_s \cdot c_0 \cdot k \cdot v_m}{2\pi} dS \quad (7)$$

If the expressions (2) and (7) are replaced in (5) the elemental sound pressure in the direction determined by the angle θ will be:

$$dp_\theta = \frac{\rho_s \cdot c_0 \cdot k \cdot v_m}{2\pi \cdot r'} e^{j(\omega t - kr')} \rho . r \sqrt{r^2 + h^2} d\gamma d\rho \quad (8)$$

After replacing an expression (4) with an expression (8) and integrating the piston area, an expression proposed by the author is obtained for sound pressure level (SPL) determination of a circular piston acoustic transducer with conical shape of membrane:

$$p_\theta = \frac{\rho_s \cdot c_0 \cdot k \cdot v_m}{2\pi \cdot r'} e^{j\omega t} \int_0^r \int_0^{2\pi} \frac{e^{-jk\sqrt{(d \cdot sin\theta - r \cdot \rho \cdot cos\gamma)^2 - (r \cdot \rho \cdot sin\lambda)^2 + (d \cdot cos\theta - \rho h)^2}}}{\sqrt{(d \cdot sin\theta - r \cdot \rho \cdot cos\gamma)^2 - (r \cdot \rho \cdot sin\lambda)^2 + (d \cdot cos\theta - \rho \cdot h)^2}} \rho . r \sqrt{r^2 + h^2} d\gamma d\rho \quad (9)$$

Radiation pattern G(θ) is derived from the ratio[3,4]:

$$G(\theta) = \frac{p_\theta}{p_{0^\circ}} \quad (10)$$

where

- p_θ is the sound pressure amplitude in the direction determined by the angle θ;
- p_{0° is the sound pressure on the acoustic axis.

Therefore, in order to obtain the radiation pattern, it is necessary to determine the SPL for θ = 0 °[5,6]. Due to the zero value of the angle θ the expression (9) yields the type:

$$p_{0^\circ} = \frac{\rho_s \cdot c_0 \cdot k \cdot v_m}{2\pi \cdot r'} e^{j\omega t} \int_0^1 2\pi \cdot r \cdot \sqrt{r^2 + h^2} \cdot \frac{e^{jk\sqrt{(\rho \cdot r)^2 + (d - \rho \cdot h)^2}}}{\sqrt{(\rho \cdot r)^2 + (d - \rho \cdot h)^2}} d\rho \quad (11)$$

In general, after replacing an expression (9) and an expression (11) an expression (10) produces [7]:

$$G(\theta)=\frac{\int_0^r\int_0^{2\pi}\frac{e^{-jk\sqrt{(d\cdot sin\theta-r\cdot\rho\cdot cos\gamma)^2-(r\cdot\rho\cdot sin\lambda)^2+(d\cdot cos\theta-\rho\cdot h)^2}}}{\sqrt{(d\cdot sin\theta-r\cdot\rho\cdot cos\gamma)^2-(r\cdot\rho\cdot sin\lambda)^2+(d\cdot cos\theta-\rho\cdot h)^2}}\rho.r\sqrt{r^2+h^2}\,d\gamma\, d\rho}{\int_0^1 2\pi\cdot r\sqrt{r^2+h^2}\cdot\frac{e^{jk\sqrt{(\rho r)^2+(d-\rho h)^2}}}{\sqrt{(\rho\cdot r)^2+(d-\rho\cdot h)^2}}d\rho} \tag{12}$$

Expression (12) is proposed by the author for calculating a radiation pattern of a circular cone acoustic transducer.

In the case of a distant zone, the distance is much larger than the diameter of the radiator (d >> 2r), it can be assumed that with respect to the amplitudes, the distances are commensurate with $d = r' = r''$[3,8,9], and we can assume that a known expression is applied to the distances for a flat piston which is a projection of the cone on the xOy plane [10] fig. 2:

$$G_f(\theta)=2\cdot\frac{I_1(k\cdot r\sin\theta)}{k\cdot r\cdot\sin\theta}$$ [10](13)

where I_1 is Bessel's function in first-order about argument ($krsin\theta$).

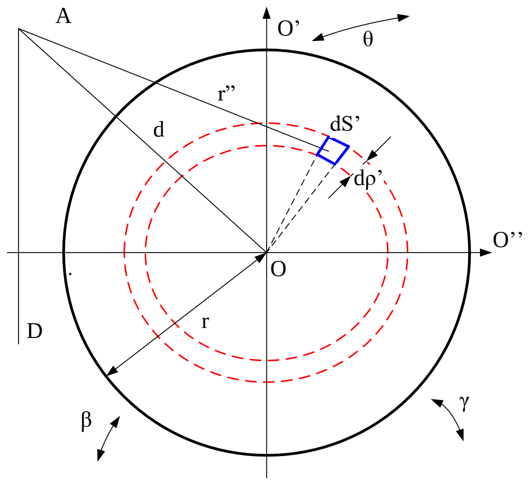

Fig.2. Flat piston transducer geometry [9,10] which is a projection of cone in xOy plane.

D. Experimental solution

To demonstrate the applicability in practice of the proposed expression, an experiment has been performed with measurement an radiation pattern of Selenium 10MB3P loudspeaker. Which is with the following geometric characteristics:

- diameter - 2r = 229 mm = 0.229 m
- height of conical membrane - h = 71 mm = 0.071 m.

The results of the measurement were compared with the analytical solutions of the known and proposed expression. Calculations and graphical depiction of analytical expressions and measurements were presented using Mathcad[11]. The measurement of the radiation pattern was performed in the TU-Varna anechoic chamber, using the Robotron Präzisions microphone MK301, the generator used is Realtek High Definition Audio and Matlab® sound test program [12] at frequency *f = 2000Hz* distance *d = 1m*. by using time scavenging [13] to obtain the following result on fig. 3:

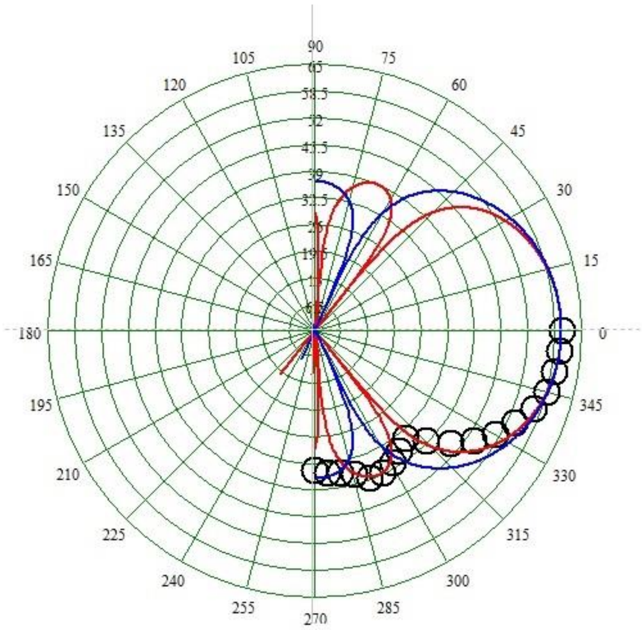

Fig. 3 Experimental result: Measured radiation pattern (black circles), known expression (13)(blue dense line) and proposed expression (12)(red dense line).

According to the result demonstrate in Fig. 3 the applicability and the higher precision of the proposed expression (12) were proved valid for the MidBass - type loudspeaker polar calculations with a pronounced cone shape of the acoustic transducer.

III. Additional Remarks

In addition the experiment was carried out at distances: d = 0.2m, 0.6m with the same frequency, no difference in the position of the SPL minimum in measured polar diagrams was notified.

IV. CONCLUSION

The object of this paper is the problem of calculating the polar diagram of a circular piston acoustic transducer with conical membrane shape after the analysis were reached the following results:

- an alytical expressions for determining the sound pressure on the axis of radiation expression (11) and in the area in front the acoustic transducer expression (9) are proposed.
- an analytical expression is proposed for determining the radiation characteristic applicable to cone-shaped circular piston acoustic transducers expression(12);
- experimental and analytical comparison of expressions (12) and (13) was carried out by demonstrating the practical applicability of the proposed expression (12) for cone-shape acoustic transducers.
- important conclusions were drawn for the dependence of the polar characteristic on the geometric characteristics of the transducer (diameter and height of the cone) and the frequency *f* which opened a field for further studies.
- there are practically no clear minimum values in the space in front of the loudspeaker along the radiating axis, but in the far field, minimum values that can be measured with the measuring system appear.

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