

Possibilities of Synthesized WDFs Models of Lowpass Filters Exhibiting Equiripple Characteristics

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Abstract – This paper introduces possibilities of synthesized lowpass wave digital filter models. The design process starts by calculations of element values of the reference lowpass structure for given specifications. Equations yield the network elements of the reference structure in order to simplify design procedure. Then, the design is solved by direct transformation of reference structure, i.e. its one-port elements and interconnections, into structure of wave digital filter (WDF). Different case studies of synthesized WDFs are presented and compared here.

Keywords - Digital signal processing; Wave digital filters; Lowpass ladder structures; MATLAB/Simulink; Chebyshev approximation, Direct transformation method.

I. INTRODUCTION

Digital filters are essential part of most digital communication systems. They have important role in digital signal processing (DSP) systems. The most attractive properties of the wave digital filters, short WDFs, include an excellent sensitivity and consequently the low accuracy requirements for the register wordlength, higher dynamic range, and automatic insurance of stability. The basic concept of WDFs has their origins in the field of filter design. It is developed by A. Fettweis [1-3]. The theory, principles and properties of wave digital filters are well explained in variety of publications delivered by several authors [1–10].

In the literature, variety of design methods and algorithms are available for WDFs. The most valuable papers are [6-8]. In the chapter [6], "Design of the wave digital filters", authors propose a very simple procedure for design, analysis and realization of low-pass, high-pass, band-pass and band-stop wave digital filters from reference LC filters given in the ladder configuration. The input data for the LC filters are obtained from introduced tables for different attenuations. MATLAB program for calculation of coefficients of the wave digital filters is obtained directly from the structure. The paper [7] presents a direct design method for highpass ladder wave digital filters.

WDFs are made up of individual modules that represent components of a system. In the case of model synthesis, this means that components of the system, such as resistors, capacitors, inductors and voltage sources are replaced with their wave digital counterpart. Each element contains a wave input, a wave output, and a value which determines port resistance. These basic building blocks are known as one-port elements. An adaptor is a memoryless interface which interconnects one-port wave digital elements.

This paper is subjected to design of lowpass WDFs. A

detailed description of different synthesis processes is given here. The contributions of this paper include an introduction to wave digital structures, as well as new aspects that relate to design procedure and WDF model possibilities, and finally use of MATLAB/Simulink to simulate filter characteristics. One can follow the described procedure and develop wave digital filter for any possible required filter specifications.

Different case studies of WDF design are presented and can be applied to any filter structure with even or odd order [5]. The presented case studies of this example of design, modeling and simulation of WDFs is very good for engineering education. A computer-aided design (CAD) software-based engineering drawing course had a positive effects on developing engineering students' spatial visualisation skills.

The multiport parallel and series adaptors and their WD models are described in [1-4,9]. A brief review of WDF design procedure is described in Section 2. Then, different forms of lowpass WDFs are synthesized and simulated here. They can be implemented directly in the Simulink toolbox of MATLAB environment. In this section, block-based wave digital models of these filters are described. Response in WDFs can be found by use of formed block-diagram networks and some basic MATLAB functions. Possibilities of the synthesized models in different case studies are described in Section 3. Conclusion of the paper is given in Section 4.

II. DESIGN PROCEDURE OF WAVE DIGITAL FILTERS

Special emphasis is put here on the synthesis procedure and simulation of lowpass filter. This section presents shortly efficient algorithms for the automatic generation of lumped elements of reference filter structure and further corresponding blocks of WDF structure.

At the beginning of the WDF design and synthesis processes one should specify some filter requirements.

Here, a design of a lowpass wave digital filter that has a passband attenuation upper edge frequency of $f_p = 1.5 \text{ KHz}$, and stopband attenuation lower edge frequency of $f_s = 2.5 \text{ KHz}$ is described. Suppose a Chebyshev prototype with passband ripple (maximum passband attenuation) of 0.5 dB and minimum stopband attenuation of 30 dB is to be used in the design.

A. Synthesis of WD Model

WDF is designed by transforming the network of reference low-pass filter structure. Let's calculate firstly the reference filter elements. The entered filter specifications include passband upper edge and stopband lower edge frequency. The first step is calculation of sampling frequency, and choosing

the most suitable electrical length $\theta_p = \pi/2$. The sampling frequency is calculated as $F_o = 4 \cdot f_p = 6 \text{ KHz}$. With determined sampling frequency, the normalized stopband edge frequency for the reference structure and the selective factor can be found as follows $\Omega_s = \tan(\pi \cdot f_s / F_o) = 3.73$, and $\gamma = \Omega_s / \Omega_p = 3.73$, where the normalized passband frequency is $\Omega_p = 1$. In order to simplify design of the WDFs, simple equations yield the network elements of the reference structure. They are given in [5]. The designed values of the reference structure elements are $g_1 = g_3 = 1.6$, and $g_2 = 1.1$.

The designed filter is of order 3, and in case of filter Type B that means that it has 2 series inductors separated by one shunt capacitor, Fig. 1 [5].

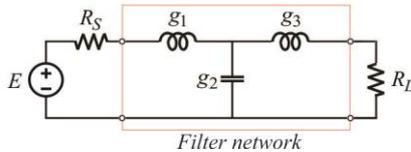


Fig. 1. The structure of reference filter - Type B.

The WDF structure is reached by applying direct transformation of its elements and their interconnections, i.e. by connections of one-port elements and two- and three-port parallel and series adaptors. Direct transformation of the reference structure into WD model happens within several steps, wherein some intermediate model is constructed. Starting point in the synthesis procedure is a structure model in the form of an electrical circuit (reference filter ladder structure [5]). First of all, identification of one-port elements and adaptor types in a structure model is required. After that, characterization of adaptors should be done (determination of port resistances and coefficient values). The network structure of a 3rd order Chebyshev lowpass filter and its adaptor connections and port resistances for the digital form transformation are depicted in Fig. 2.

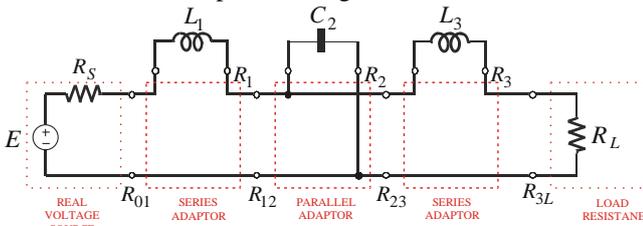


Fig. 2. A structure model of reference filter.

Starting with the K-domain representation of one-port element it is very simple to derive WD equivalent of the observed element. The one-port elements and their WD models are already presented and used in practice [1-5]. Some of them are shortly reported here once again because it could be useful for those readers start to deal with WDF theory. One-port elements identified in the reference structure are discretized by use of bilinear transform. WD representation of capacitor is simply a delay element, i.e. the reflected wave is simply a unit-delayed version of the incident wave ($B = z^{-1} \cdot A$). In case of inductors, the reflected wave is a unit-delayed incident wave with a change in phase

($B = -z^{-1} \cdot A$). These models are shown later in synthesized WDF models.

B. Different Case Studies of WDF Model Design

The building blocks of a structure model shown in Fig. 2 are resistive voltage source, inductors, capacitor and resistive load. A filter network driven by resistive voltage source indicates two case studies of filter design. That allows one to design wave digital filter in two possible ways: CASE A - Synthesis procedure starts from the driven side or CASE B - Synthesis starts from the load side.

CASE A: The first case study is the digital network synthesis **starting from the driven side**. In this case the first wave port resistance is selected as $R_{01} = R_S$. Port resistances corresponding to one-port inductors are set to $R_1 = L_1$ and $R_3 = L_3$. Port resistance corresponding to one-port capacitor is set to $R_2 = 1/C_2$.

Resistive voltage source with independent variables U and I in K-domain, and A_S and B_S , and port resistance R_{01} in WD-domain is shown in Fig. 3a. Its wave digital representation for case $R_{01} = R_S$ is depicted in Fig. 3b.

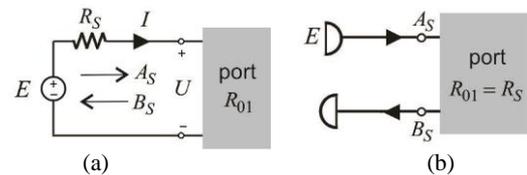


Fig. 3. Resistive voltage source: (a) one-port analog network, and (b) wave digital model for port resistance chosen to be $R_{01} = R_S$.

For network given in Fig. 3a, Kirchoff's voltage law gives $E = U + R_S \cdot I$, and using the relations for wave variables in the frequency domain $A_S = U + R_{01} \cdot I$ and $B_S = U - R_{01} \cdot I$,

one can obtain relation $E = \frac{A_S + B_S}{2} + R_S \cdot \frac{A_S - B_S}{2R_{01}}$. In

case of $R_{01} = R_S$, the relation is $A_S = E$, i.e. the incident wave is equal to voltage of the resistive voltage source. WD model of this resistive voltage source is shown in Fig. 3b.

One-port network of the resistive load R_L , and its WD representations for different cases $R_{3L} = R_L$ and $R_{3L} \neq R_L$ are described further. The following part of paper presents two different forms of WD-representation of the resistive load (Form I and Form II).

Form I: The first WD model of resistive load for port resistance chosen to be $R_{3L} \neq R_L$ is represented as cascade of two-port adaptor and wave digital model of resistor for case $R_{3L} = R_L$, Fig. 4a. Two-port adaptor coefficient is

$$\alpha = \frac{R_{3L} - R_L}{R_{3L} + R_L}$$

In the case of one-port resistor with port resistance of $R_{3L} = R_L$, the reflection coefficient is equal to zero, $\frac{B_L}{A_L} = \frac{Z_L - R_{3L}}{Z_L + R_{3L}} = 0$, because of $U_L = R_L \cdot I_L$ and

$R_{3L} = R_L$. This results in $B_L = 0$, i.e. the reflected wave is

zero. The incident wave is $A_L = 2 \cdot U_L$. This WD-model of resistive load for case of $R_{3L} = R_L$ is shaded part in Fig. 4a.

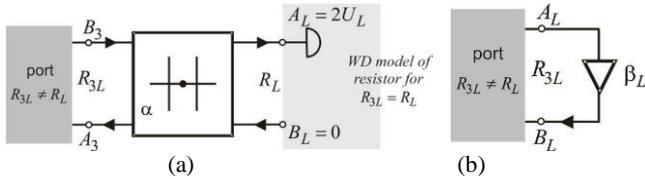


Fig. 4. Symbolic models of wave digital network for resistance in case of $R_{3L} \neq R_L$.

Symbolic representation of the synthesized WDF structure for a given electrical circuit is shown in Fig. 5 and obtained by direct transformation of network from Fig. 2 by use of wave digital models of real voltage source and load resistance. Wave digital formulation uses matched (reflection-free) ports to eliminate reflections and avoid delay-free loops.

Response in the formed model can be easily found by use of block-diagram network drawn in Simulink toolbox, Fig. 6, and some basic MATLAB functions allowing for accurate and fast modeling and analyzing of circuits. The frequency responses in dBs (transmission and reflection parameters) obtained from this model are given in Fig. 7. The magnitude function S_{21} has equal ripple in the pass-band and decreases monotonically in the stop-band.

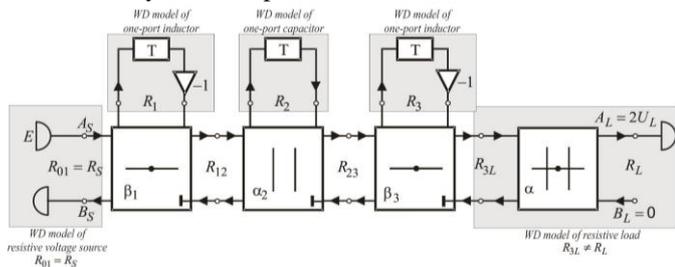


Fig. 5. Symbolic representation of WDF realized starting from the excitation side - Form I.

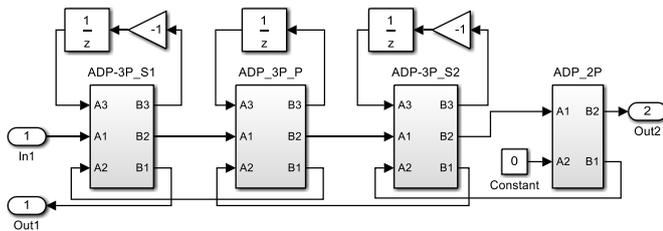


Fig. 6. MATLAB/Simulink model of WDF structure - Form I.

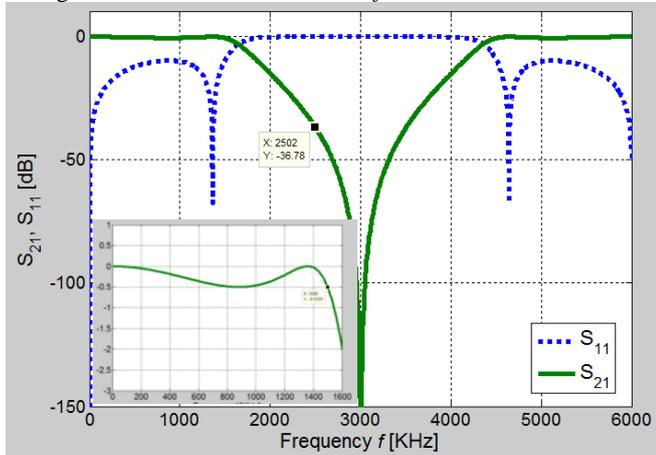


Fig. 7. Frequency responses in dBs.

Form II: WD model of resistive load is one multiplier shown in Fig. 4b. The reflection coefficient of one-port resistance network for case of $R_{3L} \neq R_L$ is $\frac{B_L}{A_L} = \frac{R_L - R_{3L}}{R_L + R_{3L}} = \beta_L$, and

this results in $B_L = \beta_L \cdot A_L$. The output is $U_L = \frac{(1 + \beta_L) \cdot A_L}{2}$.

Symbolic representation of the synthesized wave digital structure for a given electrical circuit is shown in Fig. 8 and obtained by direct transformation of network from Fig. 2 by use of wave digital models described and Figs. 3b and 4b. Model has one input and one output port.

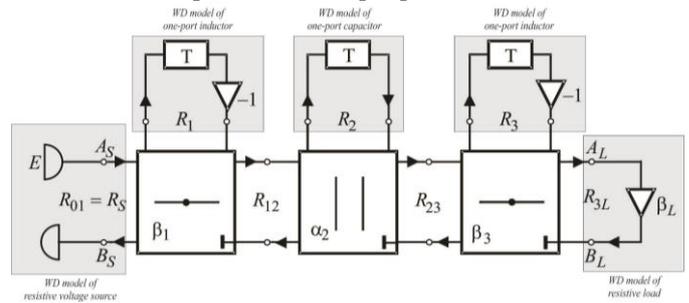


Fig. 8. Symbolic representation of WDF realized starting from the excitation side - Form II.

The model for the analysis of the wave digital filter drawn in MATLAB/Simulink follows here, Fig. 9. Sub-block models of three-port series and parallel adaptors are given in [1-4]. The frequency response in dBs obtained from this model is just a reflection coefficient (S_{11} parameter), the same as the one given in Fig. 7.

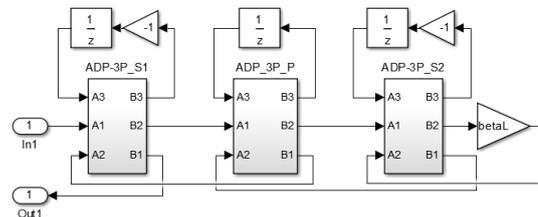


Fig. 9. MATLAB/Simulink model for WDF structure - Form II.

The wave digital network model is completely characterized by its parameters: port resistances and adaptor coefficients. These coefficients are explicitly found from the defined port resistances. As can be seen from [1-4], in case of series adaptors one have to sum up the port resistances, and for parallel adaptors port conductances. The calculation begins with the first unknown port resistance at the beginning of the structure, R_{12} , and goes further to the port resistance at the structure end, R_{3L} .

CASE B: In the second case study the digital network is formed **starting from the load side** and the last wave port resistance is selected as $R_{3L} = R_L$. Port resistances corresponding to inductors are set to $R_1 = L_1$ and $R_3 = L_3$. Port resistance corresponding to capacitor is set to $R_2 = 1/C_2$. For the network of real voltage source, in the case of $R_{01} \neq R_S$, and coefficient elected as $\beta_S = \frac{R_S - R_{01}}{R_S + R_{01}}$, it is obtained the relation for reflected wave as

$A_S = \beta_S \cdot (B_S - E) + E$. Model of one-port resistive load shown in Fig. 10 for case of $R_{3L} = R_L$ is the part of the network model shown in Fig. 4a (shaded part with excluded two-port adaptor).

Symbolic representation of the synthesized wave digital structure for a given electrical circuit is shown in Fig. 10. It is obtained by direct transformation of network from Fig. 2 by use of wave digital models previously described here.

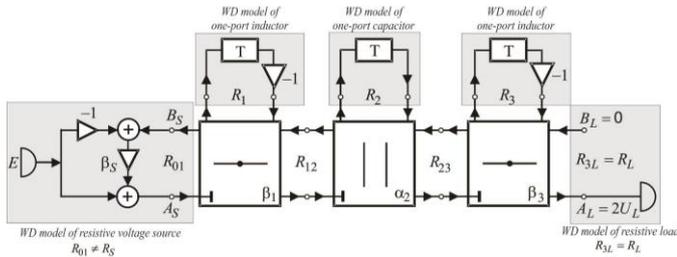


Fig. 10. Symbolic representation of WDF designed from the load side.

The model for the analysis of the wave digital filter drawn in MATLAB/Simulink, Fig. 11, can be loaded into MATLAB script in order to calculate the frequency response in dBs. The frequency response in dBs obtained from this model is just a transmission coefficient (S_{21} parameter), the same as the one given in Fig. 7. The port resistance calculation begins with the unknown port resistance at the structure end, R_{23} , and goes further to the port resistance at the structure beginning, R_{01} .

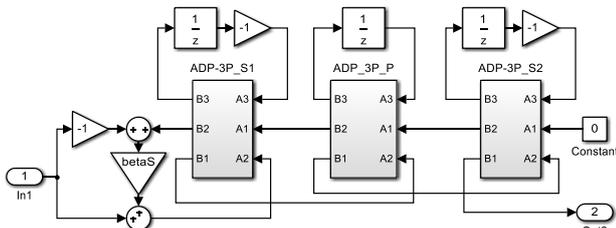


Fig. 11. MATLAB/Simulink model of WDF designed from load side.

III. POSSIBILITIES OF SYNTHESIZED MODELS

This paper presents an approach for the systematic synthesis of lowpass wave digital filters for given specifications. The synthesis of WDF is realized from the designed reference circuit with elements calculated with explicit formulas for filters exhibiting equiripple characteristics. The concept presented here is exemplified for one lowpass filter and different case studies of its synthesis.

Models for different case studies can be synthesized for any required specifications. Explicit formulas allows simple and easy calculation of filter order and structure elements. Synthesized models can be also used for other values of structure elements, but for the same filter order as here. Synthesis of wave digital model of structure with more or less elements is easy task with simple addition or subtraction of existing blocks. The calculated frequency response are the same and do not depend of the chosen case study. Chosen case study just allows calculation of different responses.

In this paper an essential example is presented. In this way, students and engineers can understand basic principles of wave digital filter design and its software implementation before moving to complicated hardware-based design projects and its related issues. Once students have a good foundation in the theory, they are then ready to move on and begin design projects using hardware. Examples on how to implement developed WD equivalent structures are given in MATLAB which not require any specific hardware. The reason of choosing it is its simplicity which allows one to concentrate on the WD concept rather than programming itself. Explicit formulas for the calculation of the reference structure elements are used which makes design for any possible required specifications very simple and not time consuming. This is also important because it allows each user to select his own filter for design and analysis with desired specifications. In that manner, a lot of variances among the resulting filters depending of the specifications can be seen.

In general, the influence of CAD software-based instruction on the spatial visualisation skills of freshman engineering students in computer-aided engineering drawing course is very positive.

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