An Expert System for Uncertain, Inconsistent, and Paracomplete Data Decision-Making

Jair M. Abe[1], Kazumi Nakamatsu[2], Seiki Akama[3], and Ari Ahrary[4]

Abstract – **Nowadays deal with big data or lack of data are an ordinary matter. We need more formal and generic tools to deal with. In this review paper, we discuss a skeleton of an expert system which can deal with such concepts, uncertain, inconsistent and paracomplete data without the danger of trivialization. Such skeleton leans on paraconsistent annotated evidential logic Eτ. More specifically, in the underlying of the lattice of truth-values and an algorithm called para-analyzer.**

Keywords – **Expert systems; Inconsistency; Paracompleteness; Uncertainty; Decision-Making.**

I. Introduction

The contradictions or inconsistencies, as well as uncertainties, are common when we make the description of parts of the real world. The control systems used in Automation and Robotics and the Expert Systems used in AI perform this description, in general, with base in the classical logic. The classical logic makes this description considering only two states. These common binary systems cannot manipulate the contradictory situations appropriately. Paraconsistent logic was born of the need to find means for giving a non-trivial treatment to the contradictory situations. The Paraconsistent logics have presented results that make possible to consider the inconsistencies in its structure in a non-trivial way. Abe and collaborators, has shown some applications of the concept of paraconsistency in areas mentioned above [2], [4], [5].

II. The Paraconsistent Annotated Logic Eτ

The atomic formulas of the paraconsistent annotated logic $E\tau$ is of the type $p_{(\mu,\lambda)}$, where $(\mu, \lambda) \in [0, 1]^2$ and [0, 1] is the real unitary interval (p denotes a propositional variable). There is an order relation defined on $[0, 1]^2$: $(\mu_1, \lambda_1) \leq (\mu_2, \lambda_2) \Leftrightarrow \mu_1 \leq \mu_2$ and $\lambda_1 \leq \lambda_2$. Such ordered system constitutes a lattice that will be symbolized by τ. A detailed account is to be found in [1], [3].

[1]Jair M. Abe - Graduate Program in Production Engineering, Paulista University, R. Dr. Bacelar, 1212, CEP 04026-002 São Paulo – SP, Brazil, E-mail: jairabe@uol.com.br.
[2] Kazumi Nakamatsu - School of Human Science and Environment/H.S.E. – University of Hyogo, Japan, nakamatu@shse.u-hyogo.ac.jp
[3] Seiki Akama, C-Republic, Tokyo, Japan, E-mail: akama@jcom.home.ne.jp
[4] Alireza Ahrary - Faculty of Computer and Information Sciences - Sojo University, Kumamoto, Japan, E-mail: ahrary@ieee.org

$p_{(\mu,\lambda)}$ can be intuitively read: "It is believed that p's belief degree (or favorable evidence) is μ and disbelief degree (or contrary evidence) is λ." [1]

So, we have some interesting examples:

- $p_{(1.0,\,0.0)}$ can be read as a true proposition.
- $p_{(0.0,\,1.0)}$ can be read as a false proposition.
- $p_{(1.0,\,1.0)}$ can be read as an inconsistent proposition.
- $p_{(0.0,\,0.0)}$ can be read as a paracomplete (unknown) proposition.
- $p_{(0.5,\,0.5)}$ can be read as an indefinite proposition.

Note. The concept of paracompleteness is the "dual" of the concept of inconsistency.

The consideration of the values other than evidence, such as belief degree and disbelief degree is made, for example, by experts that use heuristics knowledge, probability [12] or statistics [13].

The output can be of two types: situations of *extreme states* that are, False, True, Inconsistent and Paracomplete, and the situations of *non-extreme states*, all these situations are represented in the lattice presented in the next figure:

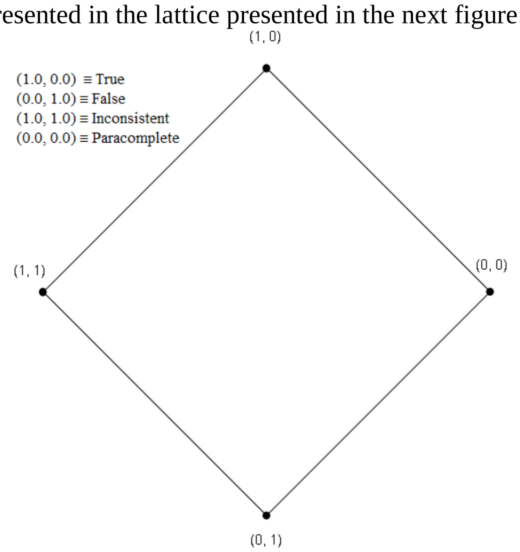

Fig. 1. Lattice τ

There is a natural operator in the lattice $\sim : |\tau| \rightarrow |\tau|$ is defined as $\sim [(\mu, \lambda)] = (\lambda, \mu)$. Such operator works as the "meaning" of the logical negation of the logic $E\tau$.

Also we have the operations OR and AND:

(μ_1, λ_1) OR $(\mu_2, \lambda_2) = (\text{Max}\{\mu_1, \mu_2\}, \text{Min}\{\lambda_1, \lambda_2\})$

(μ_1, λ_1) AND $(\mu_2, \lambda_2) = (\text{Min}\{\mu_1, \mu_2\}, \text{Max}\{\lambda_1, \lambda_2\})$

Where Max and Min are the usual maximization and minimization operations on real numbers with usual order.

The usual cartesian system can represent the lattice τ.

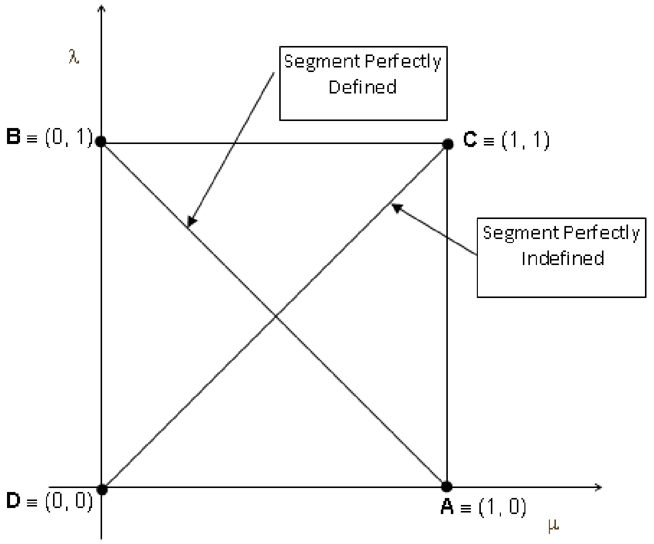

Fig. 2. Segments Perfectly Defined and Undefined

We can consider several important segments:
Segment *DB* - segment perfectly defined: $\mu + \lambda - 1 = 0$
Segment *AC* - segment perfectly undefined: $\mu - \lambda = 0$
Segment *DB* - segment perfectly defined: $\mu + \lambda - 1 = 0$
Segment *AC* - segment perfectly undefined: $\mu - \lambda = 0$
Uncertainty Degree: $G_{un}(\mu, \lambda) = \mu + \lambda - 1$; Certainty Degree: $G_{ce}(\mu, \lambda) = \mu - \lambda$;

To fix ideas, with the uncertainty and certainty degrees we can get the following 12 regions of output: *extreme states* that are, False, True, Inconsistent and Paracomplete, and *non-extreme states*. All the states are represented in the lattice of the next figure: such lattice τ can be represented by the usual Cartesian system (Figure 4).

These states can be described with the values of the certainty degree and uncertainty degree using suitable equations. In this work, we have chosen the resolution 12 (number of the regions considered according to the Figure 1), but the resolution is entirely dependent on the precision of the analysis required in the output, and it can be externally adapted according to the applications.

So, such limit values called Control Values are:

V_{cic} = maximum value of uncertainty control = C_3
V_{cve} = maximum value of certainty control = C_1
V_{cpa} = minimum value of uncertainty control = C_4
V_{cfa} = minimum value of certainty control = C_2
In this paper we have used: $C_1 = C_3 = ½$ and
$C_2 = C_4 = -½$.

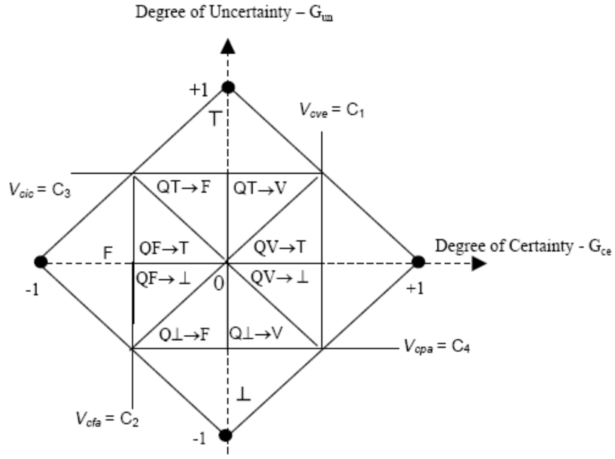

Fig. 3. Representation of the certainty degrees and uncertainty degrees.

With the values in the lattice, some regions can be considered in the unitary square of the Cartesian plan that will define the *outputs resulting states*.

These states can be described with the values of the certainty degree and contradiction degree using the equations.

In this work, we have chosen the resolution 12 (number of the regions (states) considered according to in Figure 4), but the resolution is entirely dependent on the precision of the analysis required in the output. Also, the resolution (states) can be easily modified according to each application and accuracy requested.

TABLE I
EXTREME AND NON-EXTREME STATES

Extreme States	**Symbol**
True	V
False	F
Inconsistent	T
Paracomplete	⊥
Non-extreme states	**Symbol**
Quasi-true tending to Inconsistent	QV→T
Quasi-true tending to Paracomplete	QV→⊥
Quasi-false tending to Inconsistent	QF→T
Quasi-false tending to Paracomplete	QF→⊥
Quasi-inconsistent tending to True	QT→V
Quasi-inconsistent tending to False	QT→F
Quasi-paracomplete tending to True	Q⊥→V
Quasi-paracomplete tending to False	Q⊥→F

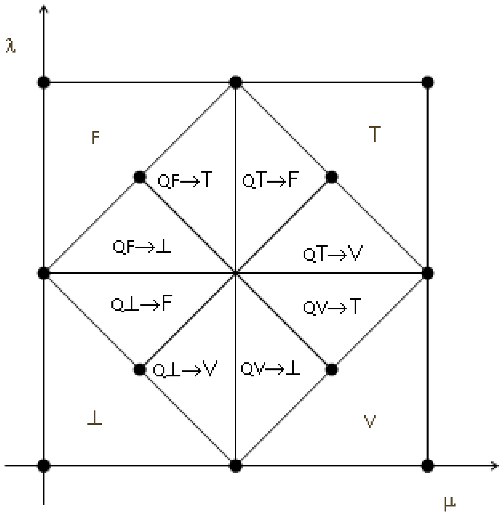

Fig. 4. Representation of the *extreme* and *non-extreme* state's regions

To make easier the recognition of each region, each one received a denomination in agreement with its proximity to the extreme states points of the lattice.

The algorithm that expresses the calculations of the inputs μ and λ is:

III. Algorithm Para-analyzer

In what follows, we present the algorithm para-analyzer [10]. The primary concern in any analysis is to know how to measure or to determine the certainty degree regarding a proposition if it is False or True. Therefore, for this, we take into account only the certainty degree G_{ce}. The uncertainty degree G_{un} indicates the measure of the inconsistency or paracompleteness. If the certainty degree is low or the uncertainty degree is high, it generates an indefinite.

The resulting certainty degree G_{ce} is obtained as follows:

If: $V_{cfa} \le G_{un} \le V_{cve}$ or $V_{cic} \le G_{un} \le V_{cpa}$ $\Rightarrow$ G_{ce} = Indefinite

For: $V_{cpa} \le G_{un} \le V_{cic}$

If: $G_{un} \le V_{cfa} \Rightarrow$ G_{ce} = False with degree G_{un}

$V_{cic} \le G_{un} \Rightarrow$ G_{ce} = True with degree G_{un}

The algorithm Para-analyzer is as follows:

*/ Definitions of the values */

$Max_{vcc} = C_1$ */ maximum value of certainty Control*/

$Max_{vctc} = C_3$ */ maximum value of uncertainty control*/

$Min_{vcc} = C_2$ */ minimum value of certainty Control */

$Min_{vctc} = C_4$ */ minimum value of uncertainty control*/

*/ Input Variables */

μ

λ

*/ Output Variables */

digital output = S1

Analogical output = S2a

Analogical output = S2b

* / Mathematical expressions * /

being:

$0 \le \mu \le 1$ and $0 \le \lambda \le 1$

$G_{un}(\mu; \lambda) = \mu + \lambda - 1;$

$G_{ce}(\mu; \lambda) = \mu - \lambda$

* / determination of the extreme states * /

if $G_{ce}(\mu; \lambda) \ge C_1$ then $S_1 = V$

if $G_{ce}(\mu; \lambda) \ge C_2$ then $S_1 = T$

if $G_{un}(\mu; \lambda) \ge C_3$ then $S_1 = F$

if $G_{un}(\mu; \lambda) \le C_4$ then $S_1 = \perp$

*/ determination of the non-extreme states * /

for $0 \le G_{ce} < C_1$ and $0 \le G_{un} < C_2$

if $G_{ce} \ge G_{un}$ then $S_1 = QV \to T$

else $S_1 = QT \to V$

for $0 \le G_{ce} < C_1$ and $C_4 < G_{un} \le 0$

if $G_{ce} \ge |G_{un}|$ then $S_1 = QV \to \perp$

else $S_1 = Q\perp \to V$

for $C_3 < G_{ce} \le 0$ and $C_4 < G_{un} \le 0$

if $|G_{ce}| \ge |G_{un}|$ then $S_1 = QF \to \perp$

else $S_1 = Q\perp \to F$

for $C_3 < G_{ce} \le 0$ and $0 \le G_{un} < C_2$

If $|G_{ce}| \ge G_{un}$ then $S_1 = QF \to T$

else $S_1 = QT \to F$

$G_{ct} = S_{2a}$

$G_{ce} = S_2$

*/ END */

In this way, contradictory, paracomplete, and uncertainty information can be treated in a close approach to the reality, through combinations of evidence.

The external adjust permitted in the defined regions of the unitary square of degrees, make the applications of the "Para-analyzer" more natural and more faithful when in the elaboration of control systems for Automation areas, Artificial Intelligence, and Robotics. The Para-analyzer also allows optimization and offers good controllability of operations, including crucial situations of the real world. The visualization through the Hasse's diagram of the lattice, with the axis of values of certainty degrees and the contradiction degrees, gives a more realistic vision of the situations through sensor information of the environment at any moment, portraying several conditions more entirely and faithfully. Therefore, the fundamental importance of the algorithm presented is to show that the Paraconsistent logic is applicable in real systems.

Some examples of applications are in Expert Systems, Neural Networks, Robotics and Artificial Intelligence [2], [8].

IV. Application to Increase Industrial Equipment Availability through Maintenance

Industrial equipment needs a permanent availability through conservation. Information about the material is captured by many devices such as sensors, visual cameras, etc. Such information almost always contains uncertainties and conflicts. We show how the Para-analyzer can be useful to increase equipment availability through maintenance.

Suppose that three experts are making the maintenance of some equipment. The chief engineer receives an amount of information, each of them is a proposition with a certainty degree and uncertainty degree (or if it is the case, favorable evidence and contrary evidence degrees): for instance, last maintenance, past recordings, etc.

The information, whatever they come, can have agreements, disagreements or even uncertainties. The Para-analyser proposed can perform paraconsistent reasoning that will analyze each evidence for the favorable and contrary evidence. A suggested form for this implementation is the use of a maximization analysis with the connective OR and minimization with the connective AND among the three experts' information. The figure 5 display this implementation.

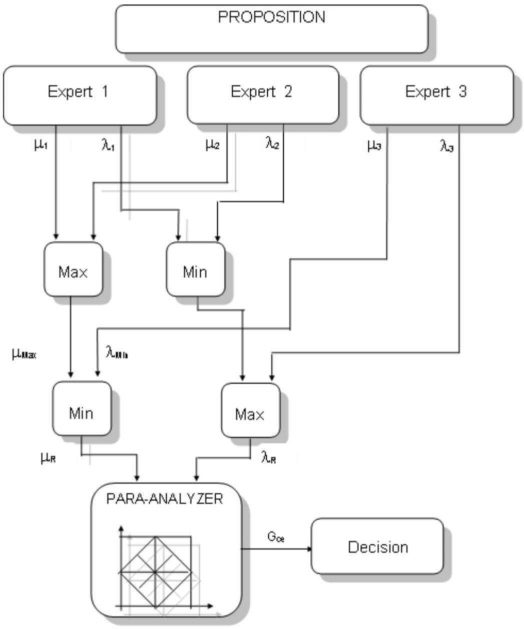

Fig. 5. Symbolic net of the information given by three experts.

The next table II displays the net with more details, where it stands out the action Para-analyzer in the information brought by the three experts.

TABLE II
RESULTING ANALYSIS BY THE THREE EXPERTS

E1	E2	OR	E3	AND	RESULTING STATE
⊤	⊤	⊤	⊤	⊤	A
⊤	V	⊤	V	V	B
⊤	F	⊤	F	F	C
⊤	⊥	⊤	⊥	⊥	D
V	⊤	⊤	⊤	⊤	E
V	V	V	V	V	F
V	F	⊤	F	F	G
V	⊥	V	⊥	⊥	H
F	⊤	⊤	⊤	⊤	I
F	V	⊥	V	V	J
F	F	F	F	F	K
F	⊥	F	⊥	⊥	L
⊥	⊤	⊤	⊤	⊤	M
⊥	V	V	V	V	N
⊥	F	F	F	F	O
⊥	⊥	⊥	⊥	⊥	P

-

E1 = Expert 1 OR = maximization
E2 = Expert 2 AND = minimization
E3 = Expert 3

Description of each resulting State of the table:

State A - All the three experts are presenting inconsistent values. Therefore the system should go back requesting new information.

State B - Two of the three experts are presenting true values. Therefore the system concludes that the proposition is true.

State C - Two of the three experts are presenting false values. Therefore the system concludes that the proposition is false.

State D - Two of the three experts are presenting indefinite values. Therefore the system should go back requesting more information for the two experts that present amount of insufficient data.

State E - Two of the three experts are presenting inconsistent values. Therefore the system should come back requesting new information for the two experts that present contradictions.

State F - All the three experts are presenting true values. Therefore the system concludes that the proposition is true.

State G - Two of the three experts are presenting false values. Therefore the system concludes that the proposition is false.

State H - Two of the three experts are presenting indefinite values. Therefore the system should come back requesting more information for the two experts that present amount of insufficient data.

State I - Two of the three experts are presenting inconsistent values. Therefore the system should go back requesting new information for the two experts that present contradictions.

State J - All the three experts are presenting true values. Therefore the system concludes that the proposition is true.

State K - Two of the three experts are presenting false values. Therefore the system concludes that the proposition is false.

State L - Two of the three experts are presenting indefinite values. Therefore the system should come back requesting more information for the two experts that present amount insufficient of information.

State M - A specialist is presenting indefinite values and the other ones two they are presenting inconsistent values. Therefore the system should come back requesting more information for the specialist than he/she has the little amount and new information for the two that present contradictions.

State N - A specialist is presenting indefinite values and the other ones two they are presenting true values. Therefore the system concludes that the proposition is true.

State O - A specialist is presenting indefinite values and the other ones two they are showing false values. Therefore the system concludes that the proposition is false.

State P - All the three experts are presenting indefinite values. Therefore the system should come back requesting more information because all are coming in insufficient amounts for analyses.

The return conditions described in the analysis of table II they do not necessarily force that the system specialist paraconsistent should make new searches of evidence, but, to the opposite, until they can be conclusive depending on the resulting values of the certainty degrees and uncertainty.

The values of certainty degrees and of uncertainty they can decide the conclusion in the cases in that, after the analysis, are obtained resulting states of inconsistencies or inconclusive. The certainty degree Gce and the one of uncertainty degree Gun are values that can be compared with the resultants of other analyses and the system can then, to decide for that that presents a smaller uncertainty degree or greater certainty degree, depending on the application of the project.

In this example, the paraconsistent expert system that analyses the proposition "the equipment need to make maintenance," examines various information. The result brings as answer a been resulting suitable for two values of certainty degree Gce and of uncertainty Gun that gives full conditions to the control System to make a decision and to choose when is the instant to stop.

V. Conclusion

In this paper, we have sketched an expert system based on Paraconsistent Annotated Evidential Logic Eτ. Due to the properties of logic Eτ, such expert systems can deal directly with imprecise, inconsistent, and paracomplete data, analyzing more reliable and realistic data, without the need to use extra-logical devices.

A suitable combination of the para-analyzer algorithm can be used as a tool in a flexible ways, can be applied in problems of greater complexity.

Acknowledgment

Thanks to J.I. da Silva Filho for useful comments.

References

[1] J.M. Abe, “Foundations of Annotated Logics”, Doctor thesis (in Portuguese) University of São Paulo, Brazil, 1992.

[2] J.M. Abe, *Paraconsistent Intelligent Based-Systems: New Trends in the Applications of Paraconsistency*, editor, Book Series: “Intelligent Systems Reference Library,” Springer-Verlag, Vol. 94, ISBN:978-3-319-19721-0, 306 pages, Germany, 2015.

[3] J.M. Abe, S. Akama, K. Nakamatsu, *Introduction to Annotated Logics - Foundations for Paracomplete and Paraconsistent Reasoning*, Series Title Intelligent Systems Reference Library, Volume 88, Springer International Publishing, Copyright Holder Springer International Publishing Switzerland, ISBN 978-3-319-17911-7, Edition Number 1, 190 pages, 2015.

[4] J.M. Abe, “Some Aspects of Paraconsistent Systems and Applications”, Logique et Analyse, 157, 83-96, 1997.

[5] S. Akama, *Towards Paraconsistent Engineering*, Intelligent Systems Reference Library, Volume 110, 234 pages, 2016, ISBN: 978-3-319-40417-2 (Print) 978-3-319-40418-9 (Online), Series ISSN 1868-4394, Publisher Springer International Publishing, DOI: 10.1007/978-3-319-40418-9

[6] J.I. da Silva Filho, G.L Torres & J.M. Abe, *Uncertainty Treatment Using Paraconsistent Logic - Introducing Paraconsistent Artificial Neural Networks*, IOS Press, Netherlands, Vol. 211, ISBN 978-1-60750-557-0, doi: 10.3233/978-1-60750-558-7-I, 328pp., 2010.

[7] J.I. da Silva Filho, “Métodos de interpretação da Lógica Paraconsistent Anotada com anotação com dois valores LPA2v com construção de Algoritmo e implementação de Circuitos Eletrônicos”, Ph.D. Thesis (in Portuguese), University of São Paulo, São Paulo, 1999.

[8] N.F. Reis, “Método Paraconsistente de Cenários Prospectivos”, PhD thesis, in Portuguese, Paulista University, 2014.

[9] J.P.A. Prado, “Uma Arquitetura em IA Baseada em Lógica Paraconsistente”, PhD Thesis (in Portuguese), University of São Paulo, 2006.

[10] J.I. da Silva Filho, J. M. Abe, “Paraconsistent analyzer module”, International Journal of Computing Anticipatory Systems, vol. 9, ISSN 1373-5411, ISBN 2-9600262-1-7, pp.346-352, 2001.

[11] P. Jackson, *Introduction To Expert Systems* (3 ed.), Addison Wesley, p. 2, ISBN 978-0-201-87686-4, 1998.

[12] A.P. Dempster, “Generalization of Bayesian inference”, Journal of the Royal Statistical Society, Séries B-30, pp.205-247, 1968.

[13] R.O. Duda, P.E. Hart, K. Konolid & R.Reboh, *A computer-based Consultant for Mineral Exploration*, TR, SRI International, 1979.