Correlation Analysis of Analog and Digital Signals in MATLAB Environment

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Abstract – **This paper presents an approach for studying the correlation analysis of analog and digital signals with various shape forms. An algorithm and its implementation in MATLAB environment have been developed. The implemented software allows studying the correlation of different analog and digital signals and the influence of signals parameters on its correlation functions. The presented approach can be used in engineering education for studying this process.**

Keywords – **signal processing, correlation analyses, auto correlation function, cross correlation function, computer simulation, MATLAB.**

I. Introduction

The correlation analysis of the signals is a basic part in the theory of the telecommunications. The correlation function quantifies the relation between two signals at different time points. Two signals are uncorrelated if their scalar product (their common energy) is zero. Depending on the signals, there are two types of correlation function: auto-correlation function and cross-correlation function. Auto-correlation function gives the relation of one signal at different moments of time. The cross-correlation function gives the relation between two different signals at different moments of time. Correlation analysis is widely used to detect signals in high level of noise or by low power signals. Another application of correlation analysis is packet synchronization, when the specific combination of bits is searched, used as marker for packet beginning.

The correlation analysis is studying in the course "Signals and Systems" at the undergraduate course in specialties "Telecommunications", "Computer and Software Engineering" and "Electronics" at Technical University of Sofia. During the seminar exercises, the students will have the opportunity to study the auto-correlation function (ACF) of different periodic and aperiodic analog and digital signals, as well as cross-correlation function (CCF) between some of them. Students will explore the influence of various parameters of the signals on their ACF and CCF.

We propose to use a program code in the MATLAB environment in order to generate signals with maximum number of different shape forms and to change as much as possible signal parameters [1]. The MATLAB environment has a definite advantage over other software products and other programming languages, because of built-in functions [2], [3]. This would considerably facilitate students in their work. Writing source code in MATLAB is preferable to the MATLAB Simulink because of the additional features that provide students with ability to change signals in a way that MATLAB Simulink does not provide [3], [4]. Writing the source code also leads to the requirement that the students will understand the functions describing the calculation of the correlation functions of signals.

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II. Problem Formulation

The main tasks, solved in the correlation analysis of signals are associated with the variety of signals as types (analog and discrete, periodic and aperiodic) and shapes (sinusoidal, rectangular, triangular, sawtooth, Gaussian, etc.).

Auto-correlation function (ACF) shows the relation between a signal and its time-shifted copy. In the case of periodic analog signals it is defined as follows:

$$\psi(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} S(t)S(t-\tau)dt \qquad (1)$$

where $S(t)$ is the signal, and τ is a parameter showing the time slot on which the copy of the signal has been moved $S(t-\tau)$. For aperiodic analog signals, the expression for ACF has the following form:

$$\psi(\tau) = \int_{-\infty}^{\infty} S(t)S(t-\tau)dt \qquad (2)$$

In the cross-correlation function (CCF), the two signals, which are compared, are different, thus the CCF shows the relation between one signal $S_1(t)$ and time-shifted copy of another signal $S_2(t-\tau)$, where τ is a time shift interval. Cross-correlation function of periodic analog signals is defined as follows:

$$\psi_{1,2}(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} S_1(t)S_2(t-\tau)dt \qquad (3)$$

where $S_1(t)$ and $S_2(t)$ are signals whose relation is quantified. For aperiodic analog signal, CCF is described by the following expression:

$$\psi_{1,2}(\tau) = \int_{-\infty}^{\infty} S_1(t)S_2(t-\tau)dt \qquad (4)$$

If the signals are discrete, it is necessary to modify the above given expressions, by replacing the integral with summation and "τ" with "nT". The expression for the Discrete Auto-Correlation Function (DACF) for periodic signals is following:

$$\psi(nT) = \frac{1}{NT} \sum_{k=-\frac{NT}{2}}^{\frac{NT}{2}} S(kT) S\left[(k-n)T\right] \qquad (5)$$

where $S(kT)$ is the signal, n is the number of samples, on which the copy of the signal has been shifted $S[(k-n)T]$, and N is number of samples for one period of the signal. For aperiodic signals, the expression for DACF has the following form:

$$\psi(nT) = \sum_{k=-\infty}^{\infty} S(kT) S\left[(k-n)T\right] \qquad (6)$$

Discrete Cross-Correlation Function (DCCF) for periodic discrete signals is defined by:

$$\psi_{1,2}(nT) = \frac{1}{NT} \sum_{k=-\frac{NT}{2}}^{\frac{NT}{2}} S_1(kT) S_2\left[(k-n)T\right] \qquad (7)$$

where $S_1(kT)$ and $S_2((k-n)T)$ are signals whose relation is quantified. For aperiodic signals, DCCF is defined as follows:

$$\psi_{1,2}(nT) = \sum_{k=-\infty}^{\infty} S_1(kT) S_2\left[(k-n)T\right] \qquad (8)$$

Often, instead of ACF and CCF, their normalized values are used. They represent the ratio of the current value of the ACF or CCF and its maximum value. Their advantage is that they reduce the impact of impulse noise on signal recognition.

The main tasks, which can be solved by computer simulations, are following:

1. Create an appropriate model for a presentation of the signals.
2. Create algorithms for calculation of the analog and digital ACF and CCF.
3. Investigate the influence of the waveforms and signal parameters on the ACF and CCF.
4. Present the obtained results graphically.

III. Basic Algorithm for Correlation Analysis

The block diagram of the proposed algorithm for correlation analysis of the analog and discrete signals is shown in Fig. 1.

Initially the global parameters and variables are set. Then it is necessary to select the type (analog or discrete) of the signals, whose ACF and CCF will be calculated. Signals with different shape forms have different specific parameters (basic parameters such as period, amplitude, etc. as well as specific parameters for the particular signal). Depending on the signals type, there are specific parameters to be set (for analog ones - the variables in which the signals expressions will be stored, and for digital ones - the number of samples, etc.). In addition to the type of signal, it is necessary to know whether it is periodic or not. Depending on this, different models of signals will be created and different expressions for determining ACF and CCF will be used.

Considering all the parameters of the signals, students must create a mathematical description of the researched signals (the studies of the following analog and discrete, periodic and aperiodic signals are envisaged: sinusoidal, rectangular, triangular, sawtooth, and Gaussian, as well as digital sequences).

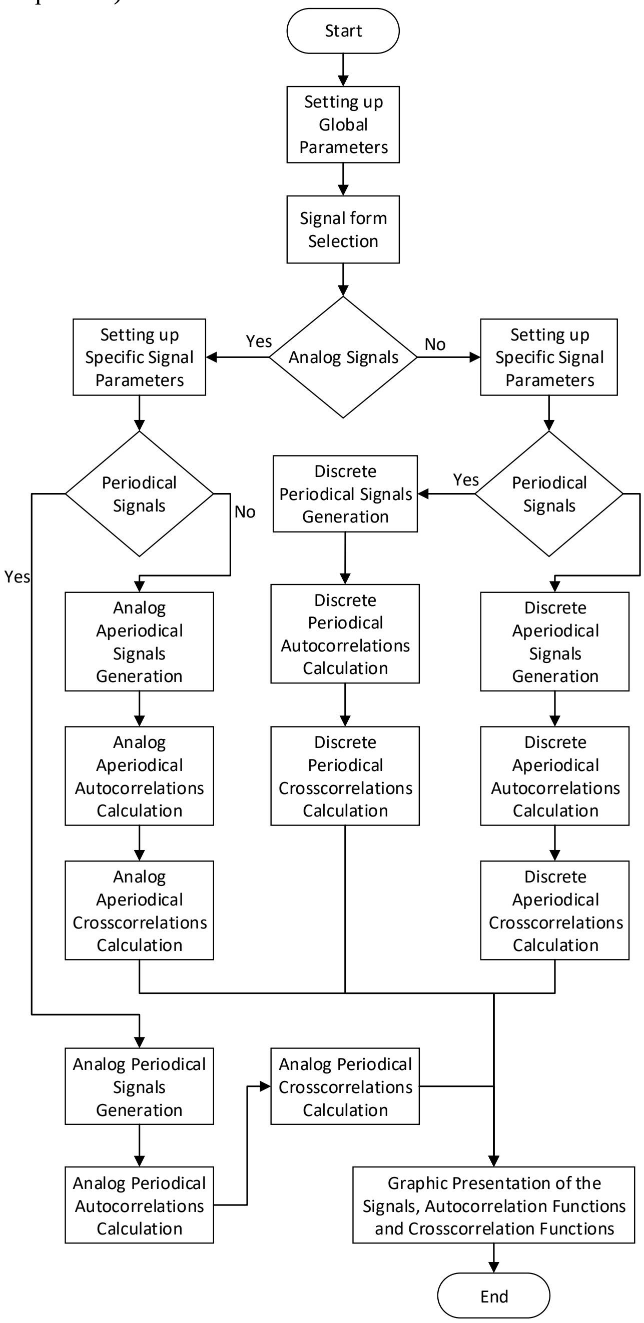

Fig. 1. Block diagram of the basic algorithm

Once the signal models are created, it is necessary to create an algorithm, which calculates the auto-correlation functions of both signals as well as the cross-correlation functions between them. For the different types of signals, the mathematical expressions describing ACF and CCF are different, so the used mathematical models will differ. The discrete part of the algorithm allows to be calculated ACF and CCF, not only for discrete signals but also for digital sequences.

The implemented software modules for calculation of the correlation functions of periodic and aperiodic signals allow creating a graphical representation of the signals, their auto-correlation and cross-correlation functions. This gives possibilities for investigation and visualization of the impact of certain signal parameters of the signal and its correlation functions.

IV. EXPERIMENTAL PART

The formulated problems are solved by computer simulation in MATLAB environment.

Some results from simulations of non-periodic analog, periodic discrete and periodic digital signals are shown in next figures bellow. The single analog rectangular impulse and its auto-correlation functions are presented in Fig. 2. In the first row of Fig. 3, the two mentioned above signals are shown, assuming that the signal displayed in red is shifted over time. The cross-correlation functions, which is obtained by offsetting the corresponding signal is shown in the second row of Fig. 3.

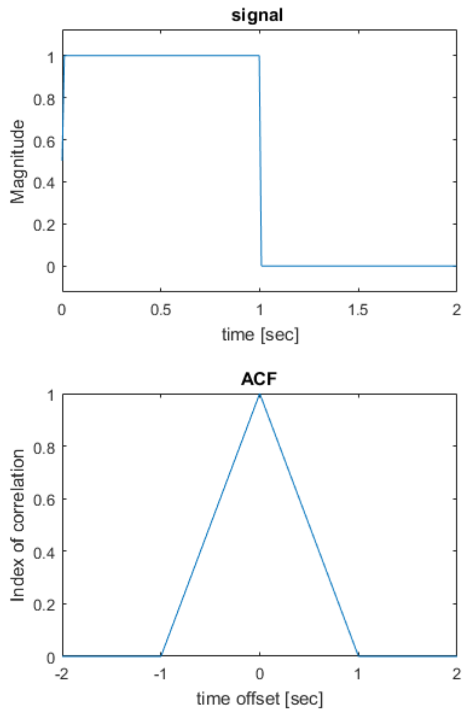

Fig. 2. Analog rectangular pulse and its ACF

The discrete sequence of rectangular impulses and its auto-correlation function are shown in Fig 4. Two discrete periodical signals (rectangular impulses and sawtooth impulses), where with red is marked the signal that is shifting in time are presented in the first row of Fig. 5. The second row of Fig. 6 shows the cross-correlation function when the corresponding signal is time shifted.

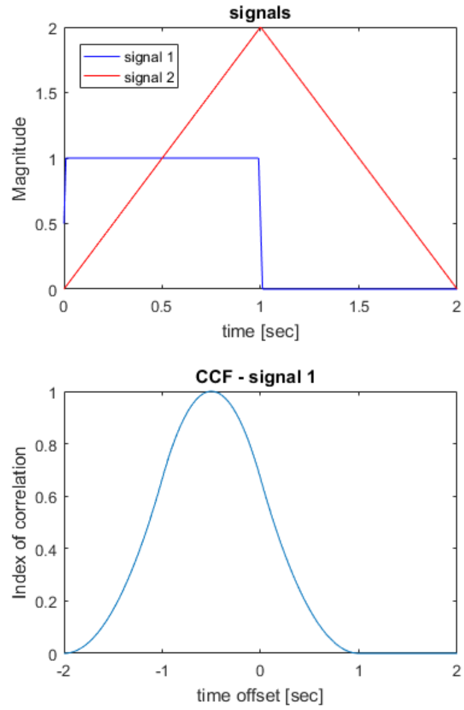

Fig. 3. Analog rectangular pulse and triangle pulse and their CCF

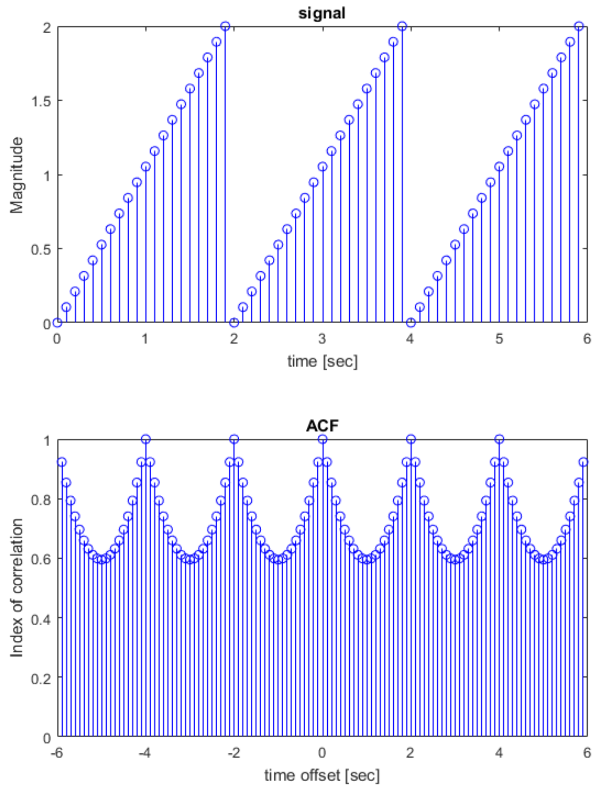

Fig. 4. Discrete sawtooth pulse train and its ACF

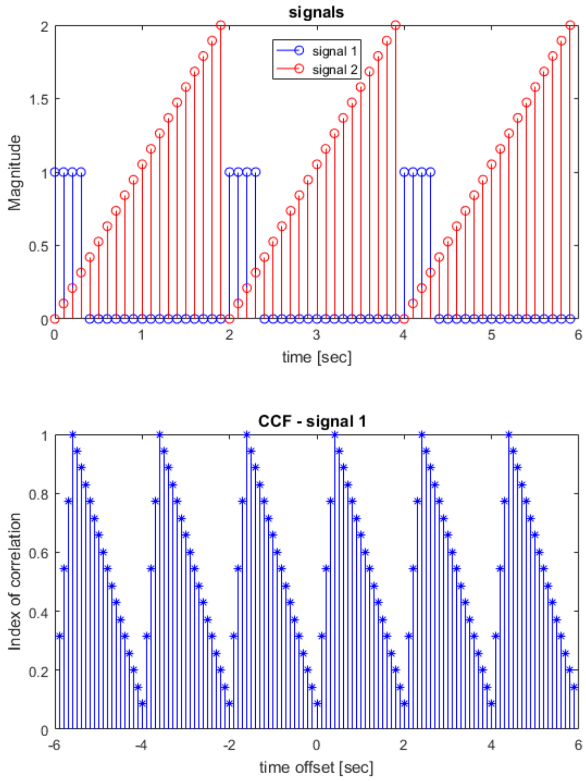

Fig. 5. Discrete rectangular pulse train and sawtooth pulse train and their CCF

In Fig. 6, a period of periodic digital sequence 10101111 and its auto-correlation function are shown. In the first row of Fig. 7, a period of the following two 8 bit digital sequences is shown: 10101111 and 01000011. The second row of this figure shows the cross-correlation function between the selected digital sequences, when shifted in time is second sequences (marked in red on the first row).

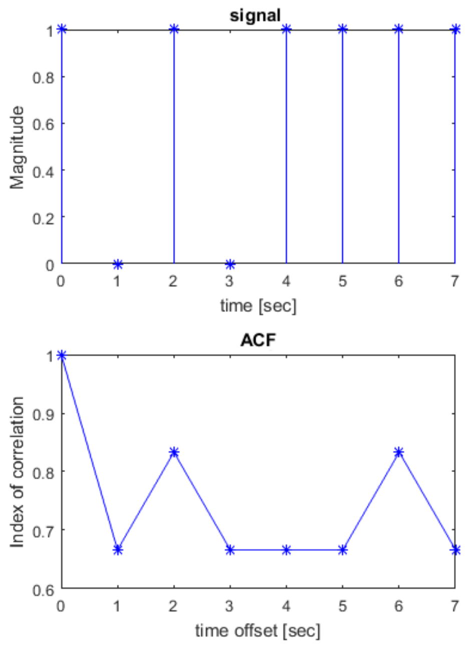

Fig. 6. Digital Auto-Correlation Function for digital sequence

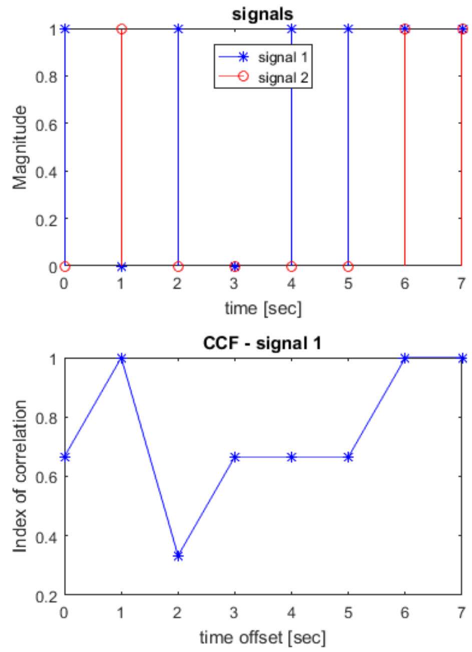

Fig. 7. Two digital sequences and their CCF

V. Conclusion

In this paper, we propose an approach for studying on correlation functions of periodic and aperiodic analog and digital signals. An algorithm and its implementation in MATLAB environment have been developed. The students can create models of signals with different type, shape forms and different parameters. They will be able to analyze influence of different signal parameters on the corresponding auto-correlation and cross-correlation functions. Our next work will be focused on further developing the proposed approach to conducting web distance learning exercises.

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