VHF Chebyshev Low Pass Filter with Lumped Elements

Veljko Crnadak[1] and Siniša Tasić[2]

Abstract – **In this paper, the practical design of the VHF Chebyshev low pass filter with lumped elements is presented. The magnitude response of the filter is approximated with the Chebyshev polynomial of the 9th order, resulting in a ladder network, consisting of five series inductors and four shunt capacitors. The passband ripple of the filter is 0.02 dB and the cutoff frequency is 220 MHz. The ladder network, with the ideal reactive elements, is then converted to the real low pass filter with the real lossy reactive elements, which is then simulated in 3D EM simulator. The values of the S_{11} and S_{21} parameters of the simulated filter are then being compared, to the values of the same S-parameters of the produced filter, in order to test the design process. The low pass filter is designed to have minimal insertion loss and maximal return loss, for the frequency range from 150 MHz to 200 MHz. Purpose of the filter is to reduce or eliminate harmonics, at the output of a high-power amplifier**

Keywords – **capacitor, Chebyshev, coil, filter, inductor, low pass, lumped, VHF.**

I. Introduction

The magnitude square of the Chebyshev low pass filter transfer function of the *N*th order is,

$$|H(j\omega)|^2 = \frac{H_0}{1+\varepsilon^2 T_N^2(\frac{\omega}{\omega_c})}, \tag{1}$$

where H_0 is the dc attenuation, ε is the ripple magnitude, ω_c is the cutoff frequency and $T_N(\frac{\omega}{\omega_c})$ is the Chebyshev polynomial of the *N*th order,

$$T_N\left(\frac{\omega}{\omega_c}\right) = \begin{cases} cos[N\, cos^{-1}\left(\frac{\omega}{\omega_c}\right)],\, 0 \le \omega \le \omega_c \\ cosh[N\, cosh^{-1}\left(\frac{\omega}{\omega_c}\right)],\, \omega > \omega_c \end{cases}. \tag{2}$$

The Chebyshev response, shown in Fig. 1, oscillates in the passband between the two values, H_0 and $\frac{H_0}{1+\varepsilon^2}$, while in the stopband it approaches zero at infinity. The Chebyshev low pass filter is also known as all pole filter, because the transfer function has zeros only at infinity [1]. The poles of the transfer function are all on the left half of the s-plane, and are given with the following expression,

$$s_k = \omega_c(\sigma_k + j\omega_k)\;,\; k = 1, 2, \dots, N, \tag{3}$$

where

$$\sigma_k = -\sinh[\tfrac{1}{N}\sinh^{-1}\left(\tfrac{1}{\varepsilon}\right)]\sin\tfrac{(2k-1)\pi}{2N}, \tag{4}$$

and

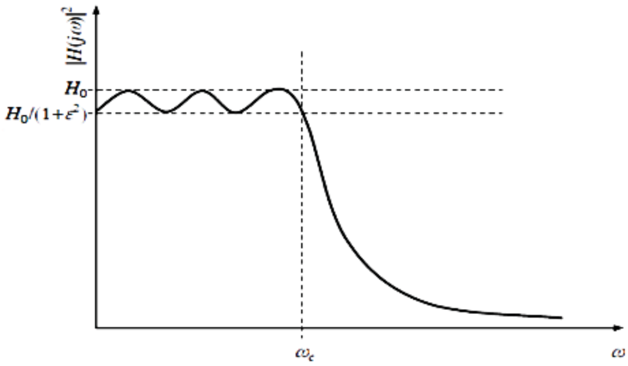

Fig. 1. The Chebyshev low pass response of the 6th order [1].

$$\omega_k = cosh[\tfrac{1}{N} sinh^{-1}\left(\tfrac{1}{\varepsilon}\right)]\, cos\tfrac{(2k-1)\pi}{2N}. \tag{5}$$

II. Design Theory

Our objective is to design a low pass Chebyshev filter that has a passband ripple of $A = 10\log(1+\varepsilon^2) = 0.02$ dB, the cutoff frequency of $f_c = 220$ MHz and an insertion loss at $f = 300$ MHz of at least $L = 35$ dB. Number of reactive elements in the filter is determined from the following inequality,

$$N \ge \frac{cosh^{-1}\sqrt{\frac{10^{0.1L}-1}{10^{0.1A}-1}}}{cosh^{-1}(\frac{f}{fc})}. \tag{6}$$

From the inequality (6), we learn that the filter has to be of the 9th order at least. Because the source and the load resistance are of 50 Ω each, Fig. 2, the dc attenuation of the filter is $H_0 = 1$. Before determining the values of the ideal reactive elements of the circuit in Fig. 2, we must obtain the values of the elements of the Chebyshev low pass prototype. The values of the prototype inductances are, $g_1 = g_9 = 0.9021$, $g_3 = g_7 = 1.88$ and $g_5 = 1.972$, the values of the

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prototype capacitances are, $g_2 = g_8 = 1.4518$ and $g_4 = g_6 = 1.7053$, and the values of the prototype resistances are, $g_0 = g_{10} = 1$. The values of the prototype elements were obtained using the recursion formulas [2].

The actual values of the filter inductances are,

$$L_k = \frac{R_0 g_{(2k-1)}}{\omega_c}, k = 1, 2, 3, \tag{7}$$

and the actual values of the filter capacitances are,

$$C_k = \frac{g_{2k}}{R_0 \omega_c}, k = 1, 2, \tag{8}$$

where $R_0 = 50\ \Omega$ and $\omega_c = 2\pi f_c$.

We see from the formula (7), that there are three different inductances in the filter, as it is shown in Fig. 2, $L_1 = 32.63$ nH, $L_2 = 68.0026$ nH and $L_3 = 71.3123$ nH.

We see from the expression (8), that there are two different capacitances in the filter, as it is shown in Fig. 2, $C_1 = 21.0056$ pF and $C_2 = 24.6733$ pF.

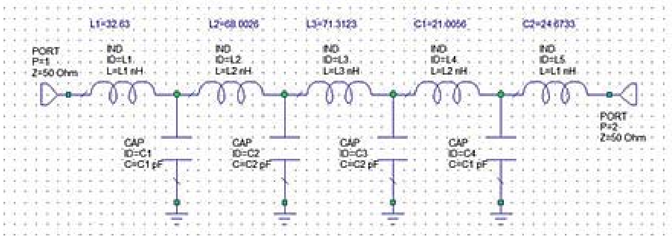

Fig. 2. The schematic of the low pass filter circuit.

During the filter realization, all the inductors were designed with the help of formula [3],

$$L = \frac{0.394 r^2 N^2}{9r + 10l}, \tag{9}$$

where r is the coil radius in cm, l is the coil length in cm, N is the number of turns and L is the inductance in µH.

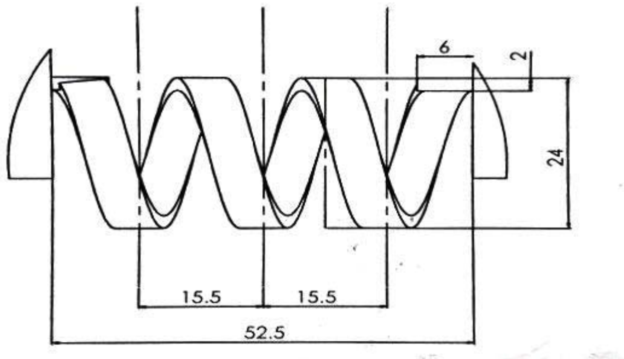

Fig. 3. The schematic of a coil, with the inductance L_A.

Every ideal inductor in Fig. 2, with the inductance L_2, was substituted with the coil in Fig. 3, with the inductance L_A, in order to realize the filter. The coil in Fig. 3, has three turns and was designed as a copper strip of thickness $t = 2$ mm, of radius $r = 10$ mm, of length $l = 52.5$ mm and of width $w = 6$ mm. The inductance of a coil in Fig. 3, according to formula (9), is $L_A = 57.6585$ nH.

The ideal inductor in Fig. 2, with the inductance L_3, was substituted with the coil in Fig. 4, with the inductance L_B, in order to realize the filter. The coil in Fig. 4, has three turns and was designed as a copper strip of thickness $t = 2$ mm, of radius $r = 10$ mm, of length $l = 44.3$ mm and of width $w = 5$ mm. The inductance of a coil in Fig. 4, according to formula (9), is $L_B = 66.5291$ nH.

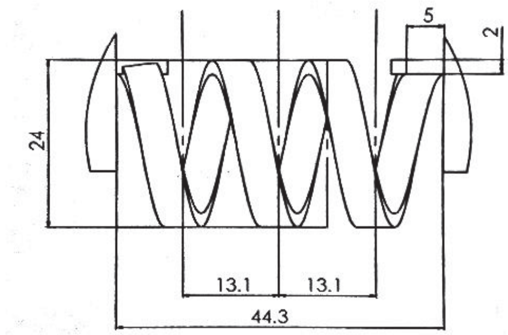

Fig. 4. The schematic of a coil, with the inductance L_B.

Every ideal inductor in Fig. 2, with the inductance L_1, was substituted with the coil in Fig. 5, with the inductance L_C, in order to realize the filter. The coil in Fig. 5, has two turns and was designed as a copper strip of thickness $t = 2$ mm, of radius $r = 9$ mm, of length $l = 30.45$ mm and of width $w = 6$ mm. The inductance of a coil in Fig. 5, according to formula (9), is $L_C = 33.1144$ nH. All the dimensions in Figs. 3, 4 and 5 are in mm.

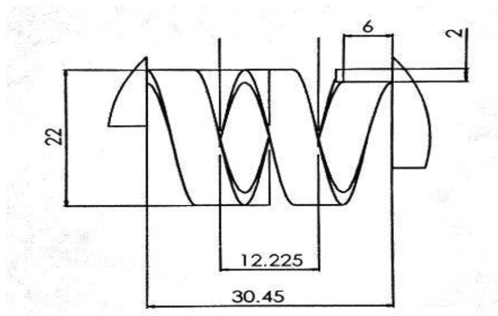

Fig. 5. The schematic of a coil, with the inductance L_C.

In Fig. 6, looking from left to right, we can see the substitutions for the ideal capacitors C_1 and C_2 from Fig. 2. Every ideal capacitor C_1, in Fig. 2, was substituted with the parallel connection of two capacitors, Fig. 6, with the equivalent capacitance of $C_1' = 19.2281$ pF. Every ideal capacitor C_2, in Fig. 2, was substituted with the parallel connection of two capacitors, Fig. 6, with the equivalent capacitance of $C_2' = 20.9374$ pF.

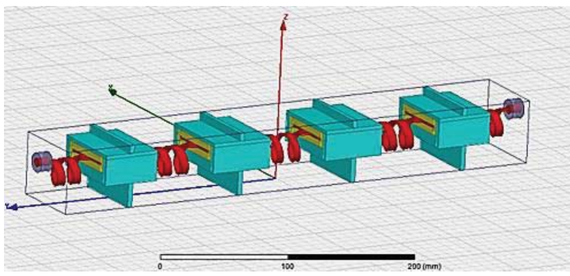

Fig. 6. The isometric view of the model of the VHF low pass filter in 3D EM simulator

III. Simulation And Measurement Results

A. Simulation

Being aware of the criteria that our filter has to meet, the return loss, $-20\log|S_{11}|$, larger than 20 dB, and the insertion loss, $-20\log|S_{21}|$, smaller than 0.1 dB, for the frequency range from 150 MHz to 200 MHz, we were able to direct the process of 3D EM simulation towards satisfying the above-mentioned conditions and obtaining the final dimensions of the filter, shown in Fig. 6. Frequencies below 150 MHz are of no interest to us because they were reduced or eliminated by previous filtering stages of the system. The inductors dimensions were firstly determined through the application of formula (9) and were later corrected through 3D EM simulation. The capacitors dimensions were firstly obtained through the application of a well-known formula for the capacitance of parallel-plate capacitors:

$$C = \varepsilon_0 \varepsilon_r \frac{S}{d} \tag{10}$$

where ε_0 is the vacuum permittivity, ε_r is the relative permittivity of the dielectric, S is the area of the plates, and d is the distance between the plates. In our case dielectric is Teflon, so $\varepsilon_r = 2.1$. The capacitors dimensions were later corrected through 3D EM simulation. As we can see from Fig. 7, capacitors in a parallel connection that form equivalent capacitor C_1', share the same copper plate (colored in red), of the dimensions 45 mm × 46.2 mm × 2 mm, that is surrounded by Teflon in an aluminum housing. In Fig. 7, Teflon is colored in yellow and aluminum is colored in blue.

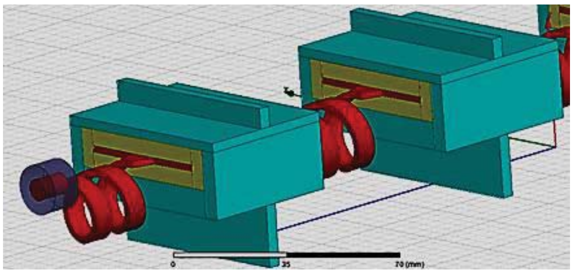

Fig. 7. The model of the capacitors C_1' and C_2' in 3D EM simulator.

As we can see from Fig. 7, capacitors in a parallel connection that form equivalent capacitor C_2', share the same copper plate (colored in red), of the dimensions 45 mm × 50.8 mm × 2 mm, that is surrounded by Teflon in an aluminum housing. The distance d between the plates, for capacitors in a parallel connection that form both C_1' and C_2', is the same and equal to 5.45 mm. The photograph of the produced filter is given in Fig. 8.

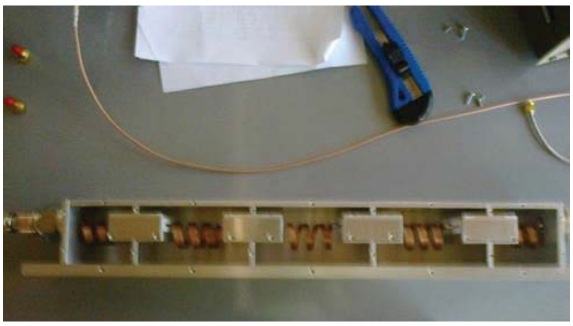

Fig. 8. The produced VHF low pass filter.

Because of the edge effect, the capacitances C_1' and C_2' were calculated using the Method of Moments. The inductors and the capacitors are placed in the aluminum casing, filled with air. The length of the casing is 404.2 mm, the height of the casing is 51.4 mm and the width of the casing is 65 mm. All the dimensions in Fig. 9 are in mm.

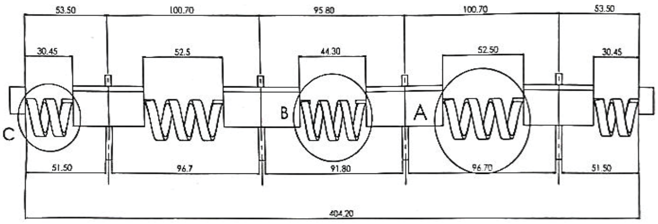

Fig. 9. The schematic of the lateral dimensions of the VHF low pass filter.

The filter uses two 7/16-connectors.

B. Simulated and measured results

As we can see from Fig. 10, the return loss is larger than 21 dB in the frequency range from 150 MHz to 200 MHz, which is satisfactory. Unfortunately, the insertion loss exceeds 0.1 dB, in the same frequency range, due to losses in conductors and dielectric. The highest insertion loss in the frequency range of interest is 0.16 dB.

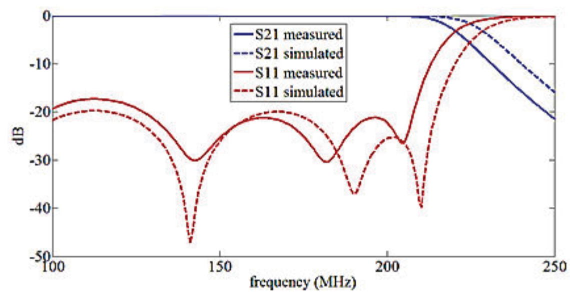

Fig. 10. Comparison of simulated and measured magnitudes of S_{21} and S_{11}.

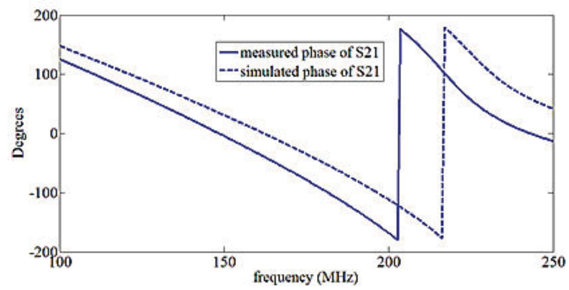

Fig. 11. Comparison of simulated and measured phase angle of S_{21}.

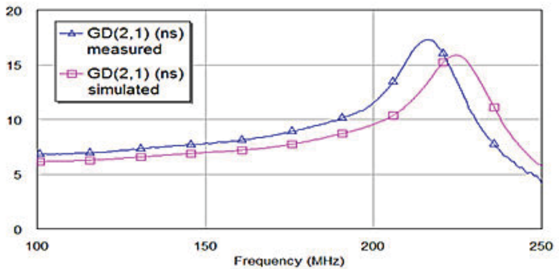

Fig. 12. Comparison of simulated and measured group delay.

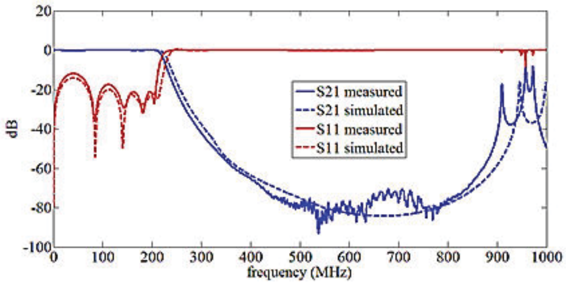

Fig. 13. Comparison of simulated and measured magnitudes of S_{21} and S_{11}.

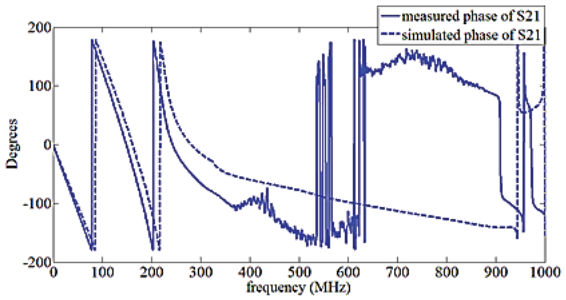

Fig. 14. Comparison of simulated and measured phase angle of S_{21}

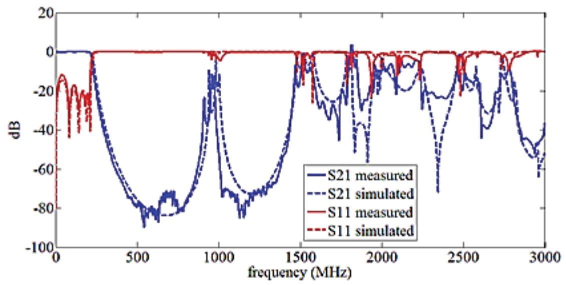

Fig. 15. Comparison of simulated and measured magnitudes of S_{21} and S_{11}.

It can be seen from Fig. 11, that the phase response of the filter is nonlinear, which has for a consequence a nonconstant group delay, as it is shown in Fig. 12.

These are all the typical traits of a Chebyshev low pass filter. Ideally, the filter is a lossless two-port network, which means that the zeros of the reflection coefficient are located at the frequencies, $f_k = f_c \cos\frac{(2k-1)\pi}{2N}$, that is $f_k = 0$, 75.24 MHz, 141.41 MHz, 190.52 MHz, and 216.66 MHz. We can see from Fig. 13, that there are five zeros of the reflection

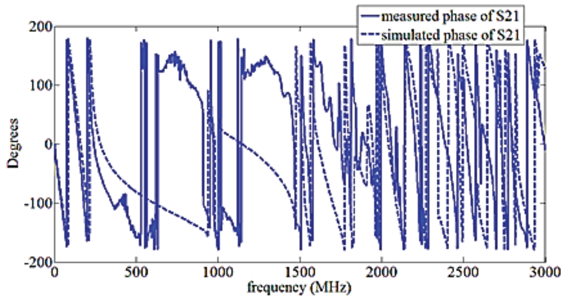

Fig. 16. Comparison of simulated and measured phase angle of S_{21}.

coefficient and they are located around above-mentioned frequencies, which is the sign of good filter design. The insertion loss at 300 MHz is 42.01 dB, as it is shown in Fig. 13. Nonlinearity of the filter phase response only increases with frequency, as can be seen in Figs. 14 and 16. Real filters have re-entry modes that limit the high-frequency capability of the filter [4]. It can be seen in Figs. 13 and 15, that higher frequency signals can appear at the output of the filter.

IV. Conclusion

In this paper, we have thoroughly explained the theory behind and the design of the VHF Chebyshev low pass filter with lumped elements. Our primary objective was to design the low pass filter that can be used for high power applications, more precisely at the output of a high power amplifier, where it would reduce or eliminate harmonics. In such an application, the nonlinearity of the phase response is not important, the only thing that matters is that the transition between passband and stopband be sharp enough, so for that reason, the Chebyshev low pass response was chosen. The bulky dimensions of the filter are a consequence of the purpose for which the filter was built. The goals regarding the level of the return loss in the passband and the level of the insertion loss in the stopband were met. Only the level of the insertion loss in the passband has exceeded the projected value by a small margin, at the first place because of the losses introduced by real inductors, and at the second place because of the losses introduced by Teflon.

Acknowledgement

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