Throughput Performance of MU-MIMO-OFDM with Optimal Pair-wise Algorithm under Imperfect CSI

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Abstract – **In this paper, a multiuser multiple-input multiple-output orthogonal frequency division multiplexing (MU-MIMO-OFDM) system with zero-forcing beamforming (ZFBF) precoding applying optimal pair-wise semi-orthogonal user selection (SUS) algorithm is considered. The knowledge of perfect channel state information at transmitter (CSIT) is required to exploit full benefit of that system, but it is the ideal case. We analyse real scenario in which an imperfect CSIT affects throughput performance of system compliant with IEEE 802.11ac. Extensive simulation results are presented to support our analysis.**

Keywords – **CSI, IEEE 802.11ac, MU-MIMO-OFDM, User scheduling algorithm, ZFBF.**

I. Introduction

To satisfy demands of broadband wireless communication market for channel capacity, higher and higher from day to day, Wi-Fi standard IEEE 802.11ac suggests multiple-input multiple-output orthogonal frequency division multiplexing (MU-MIMO-OFDM) as technique enabling speeds ranging from 500 Mbps up to several Gbps [1]. In order to achieve those speeds it is necessary to tackle with beamforming (precoding), user selection and power allocation.

Beamforming increases performance of wireless network focusing signal towards selected user and reduces in that way multi-access interferences. Capacity achieving precoding technique is dirty paper coding (DPC) [2]. Although optimal, DPC is impractical because of a tremendous computational complexity at both side (transmit and receive) even for moderate number of users. Therefore, more practical linear (zero-forcing beamforming (ZFBF) and block diagonalization (BD)) or nonlinear (Tomlinson-Harashima) precoding techniques should be applied [3, 4].

To materialize the huge potential that MU-MIMO brings, in addition to precoding, access point (AP) has to select a group of users which should be served in that time slot. The optimal schedule is found by exhaustive search, i.e. achieved sum rate is evaluated for all combination of users and the user combination providing the maximal sum rate is scheduled. However, in the case when number of the users, N_u, is large, exhaustive search cannot be used any longer, since the size of search space, $\sum_{i=1}^{N_T}\binom{N_u}{i}$, becomes prohibitively large. Therefore, design of suboptimal user selection algorithm is very important issue. Semi-orthogonal user selection (SUS) algorithm in combination with ZFBF gives performance reasonably close to that of DPC under practical value of N_u [5]. Modification of that algorithm to be applicable in IEEE 802.11ac system is presented in [6-8] as generalized multicarrier SUS (GMSUS) algorithm. In order to realize better performance than one achieved with GMSUS algorithm, in [9] authors propose the optimal pair-wise SUS algorithm.

To achieve benefit arising from precoding and user selection it is required to know channel state information at transmitter (CSIT). Unfortunately, in practice CSIT is imperfect. Its quality depends on quantization effects and/or delays. Imperfect CSIT, besides precoder design and selection of user group, also influences on link adaptation (transmission mode selection) causing throughput and error performance drops. In [10], authors present general framework to evaluate the performance of various linear multicarrier MU-MIMO schemas taking into account the accuracy of the channel information feedback to the AP.

In this paper we investigate how imperfect CSIT, expressed in number of bits using in quantization process of channel information, influences on MU-MIMO system performance. We suppose that ZFBF precoding technique is applied, since it cancels both inter- and intra-user interferences. According to results presented in [9], we choose the optimal pair-wise SUS as algorithm for user selection.

This introduction ends with notational remarks. Vector and matrices are denoted by lower- and upper-case bold letters, respectively, while scalars are represented with non-bold letters. $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote transpose and complex transpose, correspondingly, $D(x)$ is a (block) diagonal matrix with x at its main diagonal, $|Q|$ is the cardinality of subset Q, $\boldsymbol{I}_L$ is $L \times L$ identity matrix, $\|\boldsymbol{a}\|$ represents the Euclidian norm of a vector $\boldsymbol{a}$, and R and C are the set of real and complex numbers, respectively.

II. System model

We consider the downlink of MU-MIMO-OFDM system with N_T transmit antennas at AP and N_u users each equipped with N_R $(N_T \geq N_R)$ receive antennas. The system operates over N_c OFDM subcarriers, out of which N_d are used to transmit data, while rest of them correspond to pilots and guard band. In downlink scenario with large number of users, the AP serves users with favourable channel conditions. At the given scheduling period AP conveys information to a subset

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$Q=\{u_1,\ldots,u_{|Q|}\}$, $Q\subset N_u$, of selected physical users with receiving L_{u_i} spatial streams. Following inequality should be satisfied $L_Q \triangleq \sum_{i=1}^{|Q|} L_{u_i} \le N_T$. Let $\mathbf{H}_{u_i}[q]\in C^{N_R\times N_T}$, $i=\overline{1,N_u}$, be MIMO propagation channel between AP and u_i-th user over q-th subcarrier. Singular value decomposition (SVD) applied on $\mathbf{H}_{u_i}[q]$ results into

$$\mathbf{H}_{u_i}[q]=\mathbf{U}_{u_i}[q]\mathbf{\Sigma}_{u_i}[q]\mathbf{V}_{u_i}^H[q], \tag{1}$$

where $\mathbf{U}_{u_i}[q]=\left[\mathbf{u}_{u_i,1}[q],\ldots,\mathbf{u}_{u_i,N_R}[q]\right]\in C^{N_R\times N_R}$ and $\mathbf{V}_{u_i}[q]=\left[\mathbf{v}_{u_i,1}[q],\ldots,\mathbf{v}_{u_i,N_T}[q]\right]\in C^{N_T\times N_T}$ are unitary matrices containing the left and the right singular vectors of $\mathbf{H}_{u_i}[q]$ and $\mathbf{\Sigma}_{u_i}[q]=D\left(\sigma_{u_i,1}[q],\ldots,\sigma_{u_i,N_R}[q]\right)\in R^{N_R\times N_T}$ is a diagonal matrix which elements on the main diagonal are singular values of $\mathbf{H}_{u_i}[q]$. In order to eliminate inter-user interference, ZFBF post-processing is applied at receiver, so now the equivalent channel matrix corresponding to selected spatial streams of u_i-th user on q-th subcarrier is defined as

$$\tilde{\mathbf{H}}_{u_i}[q]=\tilde{\mathbf{U}}_{u_i}^H[q]\mathbf{U}_{u_i}[q]\mathbf{\Sigma}_{u_i}[q]\mathbf{V}_{u_i}^H[q]=\tilde{\mathbf{\Sigma}}_{u_i}[q]\tilde{\mathbf{V}}_{u_i}^H[q]. \tag{2}$$

where $\tilde{\mathbf{U}}_{u_i}[q]\in C^{N_T\times L_{u_i}}$ contains left singular vectors associated to L_{u_i} spatial streams. Similar, $\tilde{\mathbf{\Sigma}}_{u_i}[q]\in C^{N_T\times L_{u_i}}$ contains singular values, while $\tilde{\mathbf{V}}_{u_i}[q]\in C^{N_T\times L_{u_i}}$ contains right singular vectors associated to L_{u_i} spatial streams.

In order to eliminate intra-user interferences post-processing at transmitter have to be performed as

$$\mathbf{x}[q]=\mathbf{W}_Q[q]\mathbf{P}_Q^{1/2}[q]\mathbf{s}_Q[q], \tag{3}$$

where $\mathbf{W}_Q[q]=\left[\mathbf{w}_{u_1}[q]\ldots\mathbf{w}_{u_{|Q|}}[q]\right]\in C^{N_T\times L_Q}$ is ZFBF precoding matrix on subcarrier q, $\mathbf{P}_Q[q]=D\left(\mathbf{P}_{u_1}[q],\ldots,\mathbf{P}_{u_{|Q|}}[q]\right)\in R^{L_Q\times L_Q}$ is power allocation matrix and $\mathbf{s}_Q[q]=\left[\mathbf{s}_{u_1}^T[q]\ldots\mathbf{s}_{u_{|Q|}}^T[q]\right]^T\in C^{L_Q\times 1}$ being the vector which contains the information symbols sent to selected users.

Due to limited feedback, channel between AP and selected physical users can be modelled as $\hat{\mathbf{H}}_Q[q]=\tilde{\mathbf{H}}_Q[q]-\tilde{\mathbf{\Sigma}}_Q[q]\mathbf{E}_Q[q]$, where $\tilde{\mathbf{H}}_Q[q]\triangleq\left[\tilde{\mathbf{H}}_{u_1}^T[q]\cdots\tilde{\mathbf{H}}_{u_{|Q|}}^T[q]\right]^T$. Furthermore, $\hat{\mathbf{H}}_{u_i}[q]\triangleq\tilde{\mathbf{\Sigma}}_{u_i}[q]\hat{\mathbf{V}}_{u_i}^H[q]$, where $\hat{\mathbf{V}}_{u_i}^H[q]$ is quantized version of $\tilde{\mathbf{V}}_{u_i}[q]$, i.e. $\hat{\mathbf{V}}_{u_i}[q]=\tilde{\mathbf{V}}_{u_i}[q]-\mathbf{E}_{u_i}[q]$ and $\mathbf{E}_Q[q]=\left[\mathbf{E}_{u_1}^T[q]\ldots\mathbf{E}_{u_{|Q|}}^T[q]\right]^T\in C^{L_Q\times N_T}$ is global quantisation noise matrix. Now, for ZFBF precoding, precoding matrix is defined as $\mathbf{W}_Q[q]=\hat{\mathbf{H}}_Q^H[q]\left(\hat{\mathbf{H}}_Q[q]\hat{\mathbf{H}}_Q^H[q]\right)^{-1}$. It yields to following equation for the signal at the output (after postprocessing) of selected user

$$\begin{aligned}\mathbf{y}_{u_i}[q]&=\mathbf{P}_{u_i}^{1/2}[q]\mathbf{s}_{u_i}[q]+\tilde{\mathbf{\Sigma}}_{u_i}[q]\mathbf{E}_{u_i}[q]\\&\times\sum_{j=1}^{|Q|}\mathbf{W}_{u_i}[q]\mathbf{P}_{u_j}^{1/2}[q]\mathbf{s}_{u_j}[q]+\tilde{\boldsymbol{\eta}}_{u_i}[q],\end{aligned} \tag{4}$$

where $\tilde{\boldsymbol{\eta}}_{u_i}[q]\sim CN\left(0,\sigma_\eta^2\mathbf{I}_{L_{u_i}}\right)$. The second term in Eq. (4) represent interference leakage due to imperfect CSI.

III. Optimal pair-wise SUS algorithm

The optimal scheduled user group can be found through exhaustive search. Such approach is not acceptable when number of users is large, especially when mutliantenna users should be served. Therefore, design of user scheduling algorithm is important issue in MU-MIMO systems. In [9], authors present the optimal pair-wise SUS algorithm and show its advantage over other SUS-based algorithms for small and medium SNR. The steps of the optimal pair-wise SUS algorithm are:

Step 1: Initialization

$$Q_0=\{1,\ldots,K\}, \tag{5}$$

$$i=1. \tag{6}$$

Step 2: Determine the degrees of orthogonality, $\beta_{l,p}$, between all spatial stream pairs $l\neq p$:

$$\beta_{l,p}=\sum_{q=1}^{N_d}\frac{\left|\tilde{\mathbf{h}}_l[q]\tilde{\mathbf{h}}_p^*[p]\right|}{\left\|\tilde{\mathbf{h}}_l[q]\right\|\left\|\tilde{\mathbf{h}}_p[q]\right\|}. \tag{7}$$

where $\tilde{\mathbf{h}}_l[q]$ represents lth row of matrix $\tilde{\mathbf{H}}_Q[q]$.

Step 3: Find the pair P_i from Q_{i-1} with the smallest degree of orthogonality

$$P_i=\{l,p\}=\arg\max_{l,p\in Q_{i-1}}\beta_{l,p}. \tag{8}$$

Step 4: Select i-th spatial strem to be eliminated as follows

$$\pi(i)=\arg\min_{r\in P_i}\sum_{q=1}^{N_d}\left\|\tilde{\mathbf{h}}_r[q]\right\| \tag{9}$$

$$Q_i=\{m\in Q_{i-1}\,|\,m\neq\pi(i)\}, \tag{10}$$

$$i\leftarrow i+1. \tag{11}$$

If $\left|Q_{i-1}\right| > N_T$, go to Step 3.

Step 5: Apply exhaustive search, i.e. calculate the realized throughput for all combination of spatial streams selected in Step 4 and find the combination providing maximal throughput.

$$Q_{opt} = \underset{S \subset Q_{i-1}}{\arg\max}\, T(S). \tag{12}$$

In IEEE 802.11ac system, only finite set of transmission modes, which represents a combination of modulation type and coding rate (MCS), is available [8]. Subsequent to selection of the group of users, fast link adaptation (FLA) carries out a procedure to allocate MCS to users in order to maximize system throughput satisfying in the same time predetermined quality of service (QoS) constraints. Usually, QoS constraint is in the form of an outage probability of target packet error rate (PER_0). Since system packet error rate (PER) depends on many parameters (allocated MCS, the received SNR, packet length and channel realization), derivation and evaluation of analytical expression for PER is almost impossible. Therefore, there are a look-up table which maps all those parameters onto a single link quality metrics (LQM) which is then associated to PER value. In this paper, LQM known as effective SNR (SNR_{eff}) is used [11]. Namely, the optimal MCS for particular conditions in the channel is determined using SNR_{eff}.

Effective SNR is a function of received SNR of l-th spatial stream associated to physical user u_i which can be expressed as [10]

$$\gamma_{u_i,l}[q] = \frac{P_{u_i,l}[q]}{\left[\mathbf{R}_{u_i}[q]\right]_{l,l} + \left[\mathbf{R}_{\eta_{u_i}}[q]\right]_{l,l}}, \tag{13}$$

in environment with limited feedback, where

$$\begin{aligned}\mathbf{R}_{u_i}[q] = \tilde{\boldsymbol{\Sigma}}_{u_i}[q]\mathbf{E}_{u_i}[q]\left(\sum_{j=1}^{|Q|}\mathbf{W}_{u_j}[q]\mathbf{P}_{u_j}[q]\mathbf{W}_{u_j}^H[q]\right)\\ \times \mathbf{E}_{u_i}^H[q]\tilde{\boldsymbol{\Sigma}}_{u_i}^H[q]\end{aligned} \tag{14}$$

and $\mathbf{R}_{\eta_{u_i}}[q] = \sigma_n^2[q]\mathbf{I}_{Q_{u_i}}$. It is obvious that incomplete CSIT influence on MCS selection reflecting that influence also on throughput and error.

IV. Numerical Results

In order to show influence of imperfect CSIT on throughput performance of MU-MIMO-OFDM system with optimal pair-wise SUS scheduling algorithm, this section presents the simulation results obtained using parameters from IEEE 802.11ac standard. System operates at 5.25GHz carrier frequency with bandwidth of 20MHz that is divided into N_c = 64 subcarriers out of which N_d = 52 are used to carry data while the rest correspond to pilot signals and guard intervals. The AP has N_T = 4 transmit antennas, while all users are equipped with N_R = 2 receive antennas. Channel profile B and E from [12] is used in the simulations testbed to generate a space-time-frequency-selective fading channel. The values of parameters for FLA are taken from Table I in [9]. Homogenous scenario in which all users sitting on circumference centered at AP and all experiencing the same average SNR is supposed.

Figures 1 and 2 present the system throughput as a function of average received SNR for MU-MIMO-OFDM in B channel characterizing environment with little-to-moderate frequency selectivity. System throughput is evaluated for different degree of CSIT, i.e. for different number of bits used in quantization process of channel information [13]. That quantized value of estimated channel is fed back to transmitter and further used in beamforming, user selection and FLA. If we analyse results from these two figures, we can conclude that using seven bits in quantization provides enough quality of CSIT, so difference in throughput realized for perfect CSIT and one achieved for seven bits is negligible, even for high SNR. In addition, it is obvious that throughput performance for low SNR is independent on quality of CSIT. So, in that region small number of bits should be used in quantization process of channel information. Even results for 6 bits are not presented here, it is useful to emphasize that evaluated throughput results for that case is not close enough to throughput curve for perfect CSIT. Therefore, quantization with 7 bits remains as the best possible solution. Comparison these results with ones presented in [10] show that somehow optimal pair-wise SUS algorithm is a bit more resistant to imperfect CSIT than other SUS-based algorithms.

In addition, It can clearly be seen from those two figures that having more users in the system leads to higher throughput provoked by multiuser diversity that a larger number of users brings along.

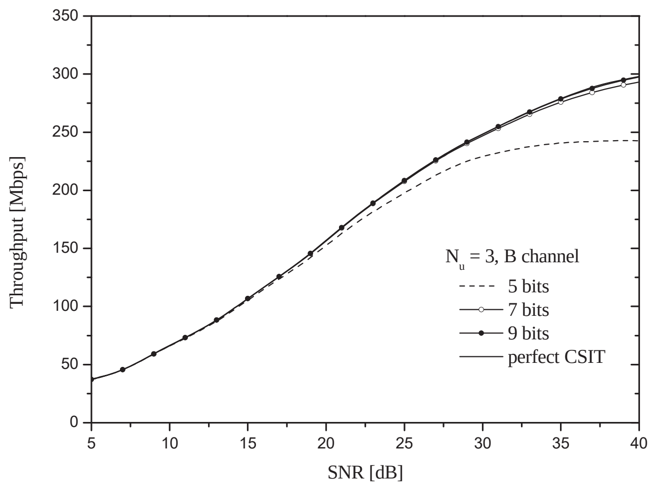

Fig. 1. Throughput for different degree of CSIT for N_u = 3 users and B channel

Figure 3 represents throughput of MU-MIMO-OFDM system with imperfect CSIT operating in E channel, i.e. environment with moderate-to-large frequency selectivity. System in such environment is sensitive to imperfect CSIT, especially for high SNR, when throughput starts to decrease for small number of bits used in quantization process.

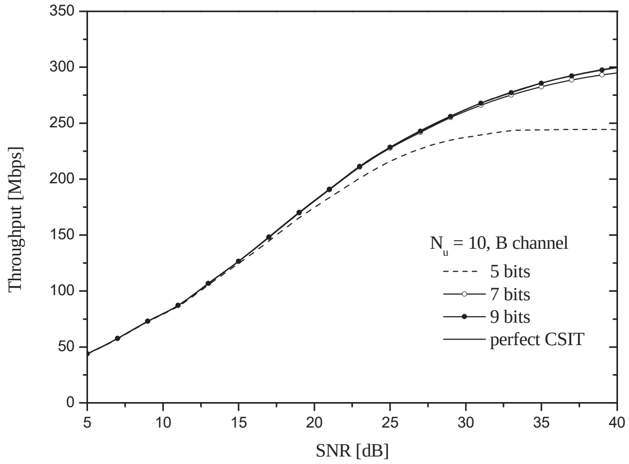

Fig. 2. Throughput for different degree of CSIT for N_u =10 users and B channel

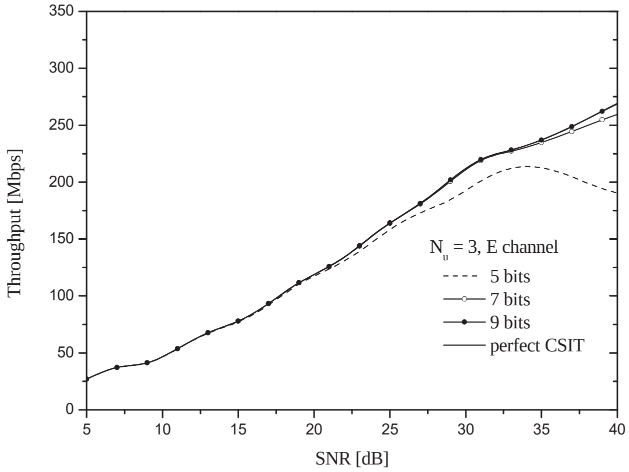

Fig. 3. Throughput for different degree of CSIT for N_u =3 users and E channel

V. Conclusion

This paper has presented study of the influence of limited feedback on throughput performance of MU-MIMO-OFDM system complaint with IEEE 802.11ac standard. According to the previously published results which pointed out advantage of optimal pair-wise SUS algorithm over other SUS-based counterparts, that scheduling algorithm has been used.

Numerical results have shown that the most accurate quantisation level which hardly affects system performance with respect to that achieved under a perfect CSIT is given in the form of using 7 bits for quantization. Noticeable throughput degradation has been evident for using 5 bits to quantize channel information, especially for system operating in E channel.

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