Brain Rhythms, Pascal Triangle, and Brain-Computer Interface

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Abstract – **The ranges of brain rhythms are empirically defined and a theoretical process is needed to offer definition of those ranges. The paper makes an observation that the central frequencies of brain rhythms can be defined with a integer sequence from the Pascal triangle. That way the paper defines a theoretical process underlying the ranges of brain rhythms.**

Keywords – **brain rhythms, central frequencies Pascal Triangle, brain-robot interface**

I. Introduction

At the beginning of 20th century two works pointed toward electrical activity of the animal and human brain. In 1913 Pravdich-Nemensky [1] showed electrical brain activity an animal and Berger showed a human brain activity [2]. Berger named it Elektrenkephalogramm which today in English literature is translated as electroencephalogram (EEG). He also mentioned that in an EEG a certain frequency rhythm is dominant under certain conditions, and named it alpha rhythm. Since then various other rhythms were observed, and named delta theta, beta, and gamma, with additional variants.

Brain rhythms are often used in Brain-Computer Interface (BCI), i.e. in controlling external objects using EEG signals. The Brain-Computer Interface challenge was stated by Vidal in 1973 [3]. Vidal proposed the use of EEG rhythms as well as evoked brain potentials as features of EEG that will be used to control external devices. He suggested that the Contingent Negative Variation (CNV) potential be used in a BCI. After Vidal's challenge in 1973, the first five applications of BCI were: (1) control of a cursor-like object on a screen using the Visual Evoked Potential (VEP) [4]; (2) control of a buzzer using the CNV potential [5-9]; (3) control of a physical object (a robot) using the alpha rhythm [10-15]; and (4) writing text on a screen using the P300 potential [16].

In this paper we will consider brain rhythms and focus on their ranges and central frequencies.

II. Brain Rhythms

Brain rhythms are signals obtained from a human electroencephalogram (EEG) after filtering certain frequencies with band pass filters.

Figure 1 shows filtering an EEG with band pass filters and obtaining brain rhythms. Figure 1 shows 6 signals.

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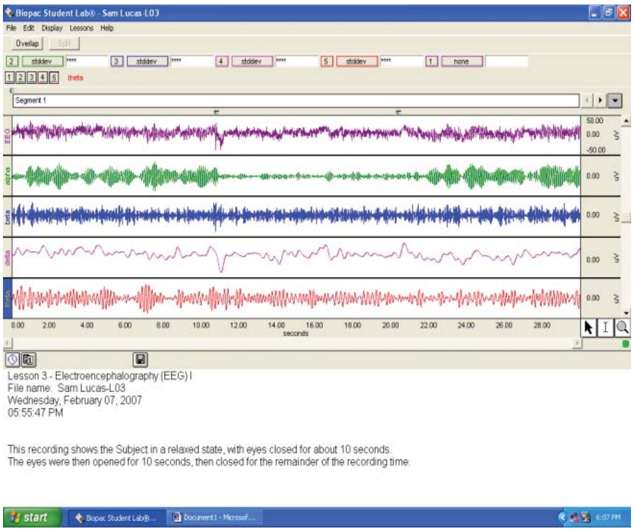

Fig. 1. EEG rhythms obtained in our lab

The signal on the top of the Figure 1 is the EEG signal obtained from a subject. It is the unprocessed (i.e., “raw”) EEG, whereas the signals below it are the signals obtained when the “raw” EEG is band pass filtered in the alpha, beta, delta, and theta frequencies, respectively. The definitions of the bands may vary, and in Figure 1 the definitions are: delta: 1-5Hz, theta: 4-8 Hz, alpha: 8-13 Hz, beta: 13-30Hz [17].

Various interpretations are given for mental states at which a brain rhythm is dominant. The most well known interpretation is that the alpha rhythm appears in a wake brain but relaxed from doing a particular task. For example, when the eyes are closed and the visual cortex is not active, this rhythm is dominant at the back of the head. Placing electrodes in the occipital and parietal area might record an EEG where the alpha rhythm is visually seen from the raw EEG, without any signal processing. A variant of the alpha rhythm is the mu rhythm, which has the same frequency band as the alpha rhythm but is recorded from the brain sensorimotor area. It is dominant when there is no movement (imagined or actual) of the limbs such as hands and legs. The beta rhythm is dominant when the brain is awake and engaged. The border between the alpha and beta can be used as a switch for application of a BCI. For example, the first control of a physical object (a robot) using BCI was based on the alpha/beta switch [9-11, 18].

In this paper we will not discuss interpretations of other brain rhythms. However, we will make an observation that the delta rhythm is simply a signal obtained by a low pass filter from an EEG. The obtained signal might be used as an average activity of an EEG. It gives the slow brain potential around which the EEG signal stochastically oscillates.

III. Defining brain rhythms. Our contribution

The brain rhythms are defined by their frequency bands. The frequency ranges are defined differently depending on the author and application. Here is an example of definition of the frequency bands of the brain rhythms [19]:

delta:	0-4Hz
theta	4-8 Hz
alpha	8-13 Hz
beta	13-30 Hz
gamma	30-100 Hz

So a theoretical process defining frequency ranges is needed. In this paper we propose a theoretical integer sequence which defines the central frequencies and ranges of brain rhythms. Some previous works[20][21] on integer sequences by the second author are registered in Online Encyclopedia of Integer Sequences (OEIS). Here we point out toward a integer sequence derived from the Pascal triangle.

The Pascal triangle is a pattern of generating integers. It starts with number 1 at level 0. Level 1 contains the sequence (1,1). The process is shown in Figure 2.

n = 0 1
n = 1 1 1
n = 2 1 2 1
n = 3 1 3 3 1
n = 4 1 4 6 4 1
n = 5 1 5 10 10 5 1
n = 6 1 6 15 20 15 6 1
n = 7 1 7 21 35 35 21 7 1
n = 8 1 8 28 56 70 56 28 8 1
n = 9 1 9 36 84 126 126 84 36 9 1
etc.

Fig. 2. The Pascal triangle

The Pascal triangle can be constructed using the rule that each number in a level is a sum of the two number above it , and each level starts and ends with the number 1. The Pascal triangle generates various well known integer sequences, for example positive integers $\{n\}_1^{\infty}$, triangular numbers $\{n(n+1)/2\}_1^{\infty}$ (e.g. [22]) etc.

Well known interpretations of the rows of a Pascal triangle is that they represent coefficients of the expansion of $(a+b)^n$ for n=0,1, etc. .

Considering the Pascal triangle, we made an observation that the central frequencies of the brain rhythms can be defined by the middle values of the Pascal triangle. Figure 3 shows our definition of central frequencies of brain rhythms.

n = 0 1
n = 1 1 1
n = 2 1 2 1
n = 3 1 3 3 1
n = 4 1 4 6 4 1
n = 5 1 5 10 10 5 1
n = 6 1 6 15 20 15 6 1
n = 7 1 7 21 35 35 21 7 1
n = 8 1 8 28 56 70 56 28 8 1
n = 9 1 9 36 84 126 126 84 36 9 1

Fig. 3. Our definition of central frequencies and ranges of brain rhythms

So we define brain rhythms by their central frequencies and deviations. Each central frequency with its deviation can be considered as a fuzzy number. The following is our definition of brain rhythms obtained from Figure 3.

rhythm	central frequency $\pm$ variation	frequency range
delta	3 ± 1 Hz	2-4 Hz
theta	6 ± 2 Hz	4-8 Hz
alpha:	10 ± 3 Hz	7-13 Hz
beta	20 ± 6 Hz	14-26 Hz
gamma1	35 ± 10 Hz	25-45 Hz
gamma2	70 ± 20 Hz	50-90 Hz

The central frequency for each band is taken from the middle values of the Pascal triangle. That defines the sequence 3,6,10,20,35,70. This proposes a mathematical phenomenon which defines the central frequencies of brain rhythms.

Table I shows comparison between our theoretically obtained ranges and other authors' empirical ranges of brain rhythms.

Table I. Comparison of our theoretical ranges with empirical ranges used by other authors.

author	year	delta	theta	alpha	beta	gamma	Lgamma	Hgamma
Kelly et al.[23]	2005			8-14	18-26			
Jia et al.[24]	2011	0-4	4-8	8-12	12-30		30-80	>80
Kumar [19].	2012	0-4	4-8	8-13	13-30	30-100		
Biopac[17]	2013	1-5	4-8	8-13	13-30			
Wiki.alpha [25]	2019	0.5-3	4-7	8-12	16-31	32-100		
Wiki.eeg [26]	2019	0-4	4 - 8	8 -14	> 14			
Wiki..eeg [27]	2019	0-4	4-7	7-13	14-30	30-100		
Wiki.neur.osc[28]	2019	1-4	4-8	8-12	13-30		30-70	70-150
Perotti [29]	2019	4-12					25-45	
This paper.	2019	2-4	4-8	7-13	14-26		25-45	50-90

As can be seen from Table I, our theory about a sequence from Pascal triangle defining central frequencies of brain rhythm, fits well into the empirical definitions. Also our definition states clearly that there are two gamma bands, as some authors defined empirically.

According to our theory, the deviations around the central frequency of a band are also defined with integers from Pascal triangle. The definition does not insist that deviations are fixed. There is a possibility that deviations are defined with different values from Pascal triangle. For example the delta band can be defined as 3±2 Hz (range 1-5Hz) and Upper Gamma band can be defined as 70+35Hz (range 35-105Hz). However the central frequencies are defined from middle values of rows of the Pascal triangle.

In addition to having mathematical background, this definition has a cognitive background. It is related to a very important *cognitive operation of choice*. Indeed the numbers inside the Pascal triangle represent number of combinations (choices) when one chooses k elements from a set of n elements, or subsets from a set. Most often used notation is "n over k" which is read "n choose k". We can use a function named *choices(n, k)*. For example if the set has 4 elements and we need to chose 2 elements, the Pascal triangle gives the value *choices(4,2)* = 4!/(2!2!) = 6. The n-th row of the Pascal triangle gives number of choices for each k from 0 to n.

IV. Conclusion

This paper discovers a relation between central frequencies of brain rhythms and a sequence obtained from the middle values of the Pascal triangle. The sequence is 3,6,10,20,35,70. The deviations from central frequencies are also defined from the numbers of the Pascal triangle and they define ranges of brain rhythms. This work makes a contribution toward finding a theoretical process underlying brain rhythms. The knowledge can be used in brain-computer interface and in other applications of EEG.

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