

# Power Network Reliability Estimation Using Fuzzy Set Theory

Natasha Dimishkovska<sup>1</sup> and Atanas Iliev<sup>2</sup>

**Abstract**—The use of fuzzy logic for network reliability estimation is justified by its simplicity which enables its use in complex systems and creation of models that are easily applied. In this paper an upgraded method for power network reliability estimation using fuzzy logic considering the daily power load is presented. Also, a case study of two parallel transformers is reviewed. The network reliability is calculated as a complex system consisting of substation and network components each with different unavailability.

**Keywords:** Network reliability, fuzzy logic, switchgear, uncertainty

## I. INTRODUCTION

A strong power network is defined by its reliability and quality of energy supplied. Building a reliable and secure network is of technical and economic importance for the power distribution companies. Switchgear is part of the power system that consists of electrical equipment for protection, control and transmission of electrical energy. This equipment is directly linked to the electrical energy supply.

Network reliability is evaluated by its ability to maintain continuous service to the customers [1]. Each interruption in supply or fault decreases the reliability of the network. Since there are no absolutely reliable networks in practice, the problem arises as how to measure and quantify the reliability and to determine the fault as fast as possible.

The equipment in switchgears is usually from different manufacturers, which leads to big insecurity and uncertainty during the component reliability estimation process [2]. Therefore, finding a proper method for calculating the unavailability is a big challenge. Fuzzy logic is an approach of understanding the world based on degrees of truth. According to fuzzy logic, the world is not only “true or false”, as the modern technology works, but there is something in between. Fuzzy logic works much likely as the human brain works.

From the presented, it can be concluded that the fuzzy logic is an appropriate mathematical basis for network reliability evaluation. The mathematical expressions are simple to use and can be easily applied in the modern technology.

In the following will be explained the theory of fuzzy logic and fuzzy sets. Also an approach for calculating substations’ reliability based on a daily power load data presented as a triangle fuzzy number will be demonstrated.

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The case study of two parallel transformers in 400 kV and 110 kV network will be reviewed, considering the unavailability of the components in the switchgears.

## II. FUZZY SET THEORY

The fuzzy set theory was based in 1965 by Lotfi A. Zadeh. A fuzzy set is a mathematical tool for determining whether the statement is part of the set or not. In traditional binary logic, for instance, a statement can be true or false (1 or 0) and nothing in between. In set theory, an element can either belong to a set or not, or in optimization a solution can be feasible or not. It is determined by the degree of membership.

A fuzzy set is a set which has no crisp, clearly defined boundary. It helps coping with unclear boundaries, and it is considered to be an alternative way to deal with uncertainties. Fuzzy set is a set of information which consists of membership functions, and ordered pairs. Its elements only have partial degree of membership, using numbers from 0 to 1.

### a. Membership functions

A membership function for a fuzzy set  $A$  on the universe of discourse  $U$  is defined as  $\mu_A: X \rightarrow [0, 1]$ , where each element of  $U$  is mapped to a value between 0 and 1. This value, called membership value or degree of membership, quantifies the grade of membership of the element in  $U$  to the fuzzy set  $A$  [3].

If  $U$  is a collection of objects denoted generically by  $x$ , then a fuzzy set  $A$  in  $U$  is a set of ordered pairs:

$$A = \{(x, \mu_A(x)) \mid x \in U\} \quad (1)$$

### b. Real numbers presented as fuzzy numbers

A fuzzy number  $M$  is a convex normalized fuzzy set  $M$  of the real line  $\mathbb{R}$  such that

1. It exists exactly one  $x_0 \in \mathbb{R}$  with  $\mu_M(x_0) = 1$  ( $x_0$  is called the mean value of  $M$ ).
2.  $\mu_M(x)$  is piecewise continuous.

Fuzzy numbers basic mathematical operations include the standard algebraic operations addition (+), subtraction (-), multiplication ( $\cdot$ ), division ( $:$ ), except that they have different form  $\oplus, \ominus, \otimes, \oslash$  [3].

Real numbers can be presented as a fuzzy number of LR-type, meaning that the numbers left and right from the presented number are determined with a certain expression,

and the presented number is in that range. A fuzzy number  $A$  is of LR-type if there are reference functions  $L$  (for left),  $R$  (for right), scalars  $\alpha > 0, \beta > 0$  (left and right spreads) and  $m$ , the mean value of  $A$ . The number is defined as:

$$\mu_A(x) \begin{cases} L\left(\frac{m-x}{\alpha}\right) \text{ for } x \leq m \\ R\left(\frac{x-m}{\beta}\right) \text{ for } x \geq m \end{cases} \quad (2)$$

### c. Uncertainty modelling

Uncertainty is a feature of the real system caused by unpredicted events, such as lack of information, complexity of the system and human factor [3]. For instance, if a fault occurs in switchgear or in some point of the network, there is electricity not supplied, which leads to decrease of the reliability. It is unable to predict when and where exactly the fault will occur, but it can be taken in consideration that it will occur in some point of the time. Fuzzy logic provides analysis of uncertain events even if the probability is in a certain interval.

For instance, if the variable value is in the following range  $[a_1; a_2]$ , the membership function of the variable  $x$  is defined as:

$$\mu_A(x) = \begin{cases} 1 \text{ for } a_1 \leq x \leq a_2 \\ 0 \text{ for } x < a_1 \wedge x > a_2 \end{cases} \quad (3)$$

Generally the fuzzy interval is represented with two end points  $a_1$  and  $a_3$ , and a peak point  $a_2$ . If there is an  $\alpha$ -cut included (in this case it is a safety level), then the fuzzy interval is presented as in eq. (9).

$$A = [a_1; a_2; a_3] \quad (4)$$

$$A_\alpha = [a_1^{(\alpha)}; a_2^{(\alpha)}; a_3^{(\alpha)}] \quad (5)$$

The mathematical operations applied for fuzzy numbers can also be applied for fuzzy intervals.

Graphically, the presentation of fuzzy intervals is shown on Fig 1.

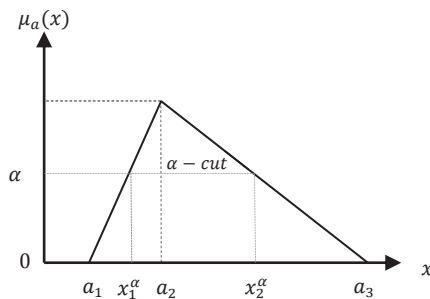


Fig 1 Triangular fuzzy number

## II. NETWORK RELIABILITY ESTIMATION USING FUZZY LOGIC

Substations are considered to be one of the most reliable parts of the power system, although they consist of many other components. The components in the substations can be connected in serial, in parallel or combined manner.

If  $K$  is the number of components connected in a series, as shown on Fig 2, and if  $n_i(x, \mu_i(x))$  is the unavailability, then the equivalent unavailability of a serial union of components for a given  $\alpha$ -cut (safety level) is defined as [2]:

$$n_e^{(\alpha)} = \sum_{i=1}^K n_i^{(\alpha)} - \sum_{i=1}^{K-1} (n_i^{(\alpha)} \cdot \sum_{j=i+1}^K n_j^{(\alpha)}) + \dots \cong \sum_{i=1}^K n_i^{(\alpha)} \quad (6)$$

If the unavailability is given with an interval fuzzy set  $n_i = [n_{1i}; n_{2i}; n_{3i}]$ , then the equivalent unavailability will also be a fuzzy set [2].

$$n_e = [n_{e1}; n_{e2}; n_{e3}] = [\sum_{i=1}^K n_{1i}; \sum_{i=1}^K n_{2i}; \sum_{i=1}^K n_{3i}] \quad (7)$$

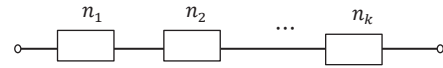


Fig 2 Serial connection of the components

In case when the components are connected in parallel, as shown on Fig 3, then the equivalent unavailability of the system is defined as:

$$n_e^{(\alpha)} = \prod_{i=1}^K n_i^{(\alpha)} \quad (8)$$

And when the unavailability of the components is given as a fuzzy set interval, then the equivalent unavailability is not a fuzzy number, but a fuzzy polynomial.

$$n_e^{(\alpha)} = [n_{1e}^{(\alpha)}, n_{2e}^{(\alpha)}] = [\prod_{i=1}^K [n_{1i} + (n_{2i} - n_{1i})\alpha]; \prod_{i=1}^K [n_{3i} + (n_{2i} - n_{3i})\alpha]] \quad (9)$$

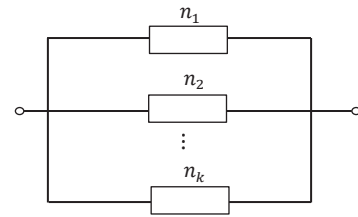


Fig 3 Parallel connection of the components

If the components in the system are connected both in serial and in parallel connection, then the equivalent unavailability is calculated with the combination of the equations (6-9).

## III. THE PROPOSED APPROACH

Using triangular fuzzy numbers and membership functions to represent the daily power load provides simpler energy loss analysis. In [4] a method for reliability evaluation of composite power systems is presented, using load flow results in different conditions. Based on that, first the

maximum, medium and minimum load points are determined (Fig 4). The load membership functions' borders are defined as in Eq. (10) and Fig 5.

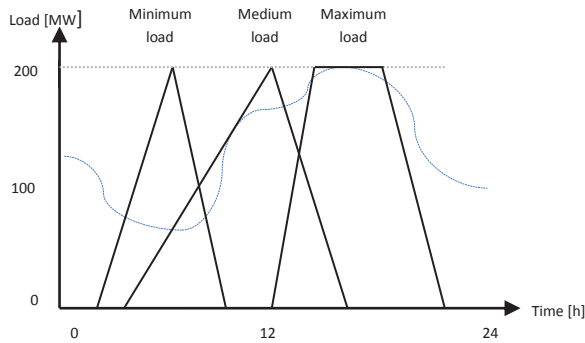


Fig 4 Load membership functions

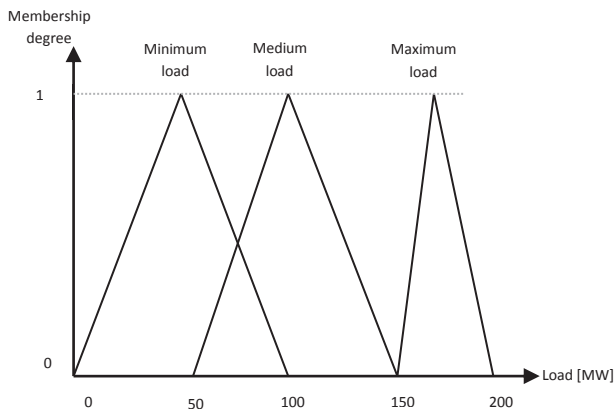


Fig 5 Definition of the power load membership functions

$$\begin{aligned} p^{max}: & \quad 150 \leq P \leq 200 \\ p^{mdl}: & \quad 50 \leq P < 150 \\ p^{min}: & \quad 0 < P \leq 100 \wedge p^{min} < p^{med} \end{aligned} \quad (10)$$

where,  $p^{max}$  is the maximum daily power load,  $p^{med}$  is the medium value the daily power load,  $p^{min}$  is the minimum daily load and  $P$  is daily load in the  $i^{th}$  hour.

The membership function of the daily load consists of the minimum, medium and maximum load point, Eq. (11). Time duration function consists of the time duration of each of the load points, Eq. (12).

$$P_{DL} = [p^{min}; p^{mdl}; p^{max}] \quad (11)$$

$$t_{DL} = [t^{min}; t^{mdl}; t^{max}] \quad (12)$$

where,  $t^{min}, t^{mdl}, t^{max}$  are the time durations of the minimum, medium and maximum load.

Total energy not supplied (ENS) is calculated in the following way described.

a) First the daily power loss is determined, which is a number between zero and the power of the failed transformer. If

the power load is less than or equal to the power of the failed transformer, and it can be supplied by other transformers in the network, the subtraction is quantified as zero. And, it is multiplied with its time duration.

$$\Delta P_{dl} = \begin{cases} P_{DL} - P_{TRO}, & \text{if } (P_{DL} - P_{TRO}) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

$$W_{dl} = \sum_{i=1}^{365} \sum_j^{min,mdl,max} \Delta P_{dl,i}^j \cdot t_{DL,i}^j \quad [MWh] \quad (14)$$

where,  $\Delta P_{dl}$  is the daily power loss,  $P_{TRO}$ , is the power of the failed transformer, and  $W_{dl}$  is the total energy loss.

b) Next, the total energy loss is multiplied with the equivalent unavailability, i.e. probability of component failure in the system.

$$ENS = W_{dl} \cdot N_{eqv} \quad [MWh] \quad (15)$$

c) Last, total energy not supplied is sum of the energy not supplied of all possible events' combinations.

#### IV. TEST EXAMPLE

In [2] a method for network reliability estimation and energy not supplied rate calculation is presented using triangular fuzzy numbers. In this paper that method for network reliability is used and upgraded for daily load data processing. The case study of two transformers connected in parallel in 400/110 kV/kV substation, is reviewed. The system is shown on Fig 6.

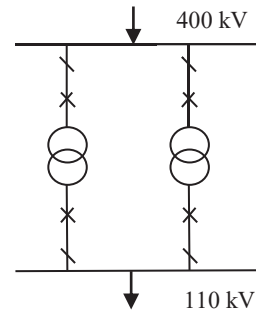


Fig 6 A two parallel transformers system

Each of the branches is equipped with two circuit-breakers (one on 400 kV side, and the other on 110 kV side), two disconnectors (one on 400 kV side, and the other on 110 kV side) and one transformer. Each of the components in the substations has a certain rate of unavailability, outage duration, depending on the voltage rate, as shown in the Table 1 (data from [5]).

TABLE 1  
SUBSTATION COMPONENTS' UNAVAILABILITY

Components	Voltage (kV)	Unavailability
Circuit breaker	400	$[2,02; 2,32; 2,62] \cdot 10^{-5}$
Disconnector	400	$[4,01; 4,51; 5,01] \cdot 10^{-6}$
Circuit breaker	110	$[3,21; 3,60; 4,09] \cdot 10^{-5}$
Disconnector	110	$[1,06; 1,20; 1,33] \cdot 10^{-6}$
Power transformer	400/110	$[3,50; 3,70; 4,05] \cdot 10^{-4}$

The calculated value of the equivalent unavailability of each of the branches is the following fuzzy set interval:

$$N_{eqv1} = N_{eqv2} = [3,78; 4,03; 4,42] \cdot 10^{-4}$$

The equivalent unavailability of the system calculated with the fuzzy logic is  $N_{eqv} = [1,43; 1,62; 1,95] \cdot 10^{-7}$ .

Another way to calculate the equivalent unavailability is with Eq. (9). For achieving higher accuracy, it is assumed that the  $\alpha$ -cut is  $\alpha = 0,9$ . The equivalent unavailability in that case is  $N_{eqv} = [1,60; 1,65] \cdot 10^{-7}$ . For higher safety level, in this case, the results are more accurate than those using fuzzy numbers.

In this paper, it is assumed that the power of each of the transformers is  $P = 100$  MW. The first case reviewed is failure of one branch. That means that the consumption greater than 100 MW won't be supplied. The other case reviewed is failure of two branches. In this case, no power is supplied. The power load and the energy not supplied due to one branch failure are presented in Fig 7. For more accurate computation of the energy not supplied, the power not supplied from each day is multiplied with the duration of the power load and the possibility that event occurs, as in Eq. (14) and Eq. (15).

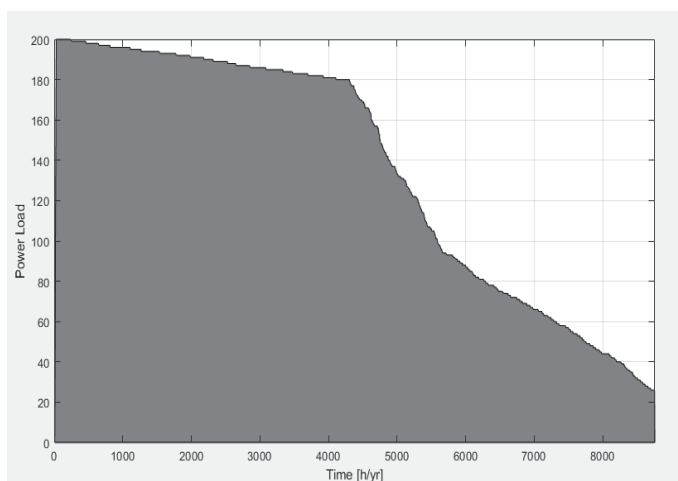


Fig 7 Year power load curve

In Table 2 the equivalent unavailability of the system of all possible combinations of branch failures and the energy not supplied from all the possible combinations is shown.

The total energy not supplied in one year due to one and

TABLE 2  
RELIABILITY ANALYSIS

#	1 <sup>st</sup> branch	2 <sup>nd</sup> branch	Probability	Power not supplied [MW]	ENS [MWh]
1	1	1	/	0	0
2	1	0	$[3,78; 4,03; 4,42] \cdot 10^{-4}$	0-100	[166,13; 176,68; 193,86]
3	0	1	$[3,78; 4,03; 4,42] \cdot 10^{-4}$	0-100	[166,13; 176,68; 193,86]
4	0	0	$[1,43; 1,62; 1,95] \cdot 10^{-7}$	0-200	[0,17; 0,19; 0,23]

two branches failure using the proposed method is:

$$ENS = [332,43; 353,55; 387,95] \text{MWh}$$

## V. CONCLUSION

Building a secure and a hundred percent reliable network is still a challenge for researches and power engineers. Creating a highly accurate method for ENS estimation is important from both technical and economical point of view.

From the presented, it can be concluded that the fuzzy logic based methods can be easily implemented. Therefore, the proposed approach is an expansion of the classical and fuzzy logic based methods for reliability estimation. Considering the fact that power load data is measured daily, it wouldn't be difficult for this method to be applied. Accuracy of the results depends on the power load measured data and the measured time duration of the power load.

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