Optimal Power Injection Placement in Radial Distribution Systems using Mixed Integer Second Order Cone Programming

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Abstract – **Nowadays power injections that originate from distributed energy resources play an important role in every power distribution system. Their proper location, sizing and operation contribute to power systems loss reduction, improved voltage profile, increased reliability and deferral of system upgrades. Many methods deal with the complex combinatorial and non-linear problem of distributed generation optimal placement and sizing. Most of these methods present acceptable and near optimal results but at the expense of using pre-processing procedures based on complex mathematical modelling techniques. This paper presents an alternative solution to that problem. Instead of constructing an algorithm that hopefully, deals best with the issue at hand, the proposed approach places its focus on proper problem shaping, formulation and solution utilizing the powerful benefits of today's solvers. The proposed approach is tested on a 69-bus distribution system and it yields far better results. It also outperforms other methods in terms of simplicity and easy implementation.**

Keywords – **Distributed generation, Power injection, Losses, Voltage profile, Distribution system.**

I. Introduction

Distributed energy resources play an important role in today's power systems. Their optimal placement and sizing contributes to several beneficial aspects such as decrease in power system losses, voltage profile improvement, increased reliability and ultimately deferral of system upgrades. The trade-off for utilizing these benefits is their optimal placement and sizing. [1]

Many technologies harness the energy from distributed resources. All of them possess some specifics in terms of power conversion, efficiency, operation etc. However, from power system point of view, all of these can be regarded as some sort of (active, reactive and/or apparent) power injection (PI), depending on the technology. In order to utilize the benefits these technologies provide, their proper sizing, location and operation regime is of utmost importance [2].

A plethora of methods exists that deal with the combinatorial and non-linear problem of optimal PI placement and sizing. They can be categorized in several groups, i.e. analytical [4],[5], heuristic and meta-heuristic [6],[7] and mathematical programing algorithms [8],[9]. They all possess some method specific advantages and disadvantages. They treat the aforementioned problem in ways that introduce simplifications/complications, linearization, natural process imitation's, coding, decoding etc. The primary focus in these approaches is the method itself and its proper shaping in order to address the problem at hand. This paper proposes a different approach. Rather than spending time on proper method shaping, a suitable reallocation in problem shaping is presented instead. Utilizing the benefits of today's powerful optimization solvers, i.e. YALMIP [11] and CPLEX [12], the problem is solved in a way that disburdens the user from complex mathematical formulations. The obtained results show superiority compared to other methods and approaches that deal with the highly complex non-linear problem of optimal placement and sizing of PI's.

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II. Problem Formulation

The problem of optimal placement and sizing of PI's is quantified through an objective function that minimizes power system losses, i.e. Eq. **Error! Reference source not found.**:

$$\min F = \sum_{N\text{branch}} R_{\text{branch}} \cdot I^2_{\text{branch}} \tag{1}$$

This objective function is subject to two subsets of constraints. The first subset C_{system} refers to a set of conditions that describe power system's performance. Instead of using conventional and distribution system appropriate load flow techniques [13]-[15], the power system here is described with set of equations that model and quantify its behavior, i.e. branch power flows and bus voltage profile [14], i.e. Eqs. (2) - (8):

$$V_1^2 = V_{\text{slack}}^2 \tag{2}$$

$$\begin{aligned} V_{to}^2 = V_{from}^2 - 2\cdot\left(R_{\text{branch}} \circ P_{\text{branch}}^{from} + X_{\text{branch}} \circ Q_{\text{branch}}^{from}\right) \\ + \left(R_{\text{branch}}^2 + X_{\text{branch}}^2\right) \circ I_{\text{branch}}^2 \end{aligned} \tag{3}$$

$$P_{\text{branch},from} - P_{\text{branch},to} = R_{\text{branch}} \circ I_{\text{branch}}^2 \tag{4}$$

$$Q_{\text{branch},from} - Q_{\text{branch},to} = X_{\text{branch}} \circ I_{\text{branch}}^2 \tag{5}$$

$$I_{\text{branch}}^2 \circ V_{from}^2 \geq \left(P_{\text{branch},from}^2 + Q_{\text{branch},from}^2\right) \tag{6}$$

$$I_{\text{branch}}^2 \geq 0 \tag{7}$$

$$V^2 \geq 0 \tag{8}$$

Eq. (2) strictly defines the slack bus voltage. Eqs. (3) - (5) describe the voltage profile and branch power flow of the power system [14]. Eqs. (6) - (8) define non-negativity conditions since those are required from the optimization solver in an explicit form [12]. Constraint (6) is written in a relaxed form and it is converted from an equality constraint, which it really is, to an inequality constraint. In such a way, with (2)-(8) we have defined second order cone programing problem, which is easily solved by state of the arts solver such as CPLEX. Inequality (6) is proven to converge to an equality in any optimal solution [10], so that the final solution will satisfy all network constraints in their original form. The vectors *from* and *to* refer to branch's sending and receiving end indices accordingly and the operator "∘" denotes a Hadamard's product or element wise multiplication of vectors.

The second subset of constraints C_{PI} refers to placement and sizing of different types of PI's, i.e. Eqs. (9) - (16):

$$P_{\text{slack}} + \mathbf{C}_{\text{P}} \cdot P_{\text{PI}} - P_{\text{demand}} = \mathbf{A}_{from} \cdot P_{\text{branch},from} + \mathbf{A}_{to} \cdot P_{\text{branch},to} \tag{9}$$

$$Q_{\text{slack}} + \mathbf{C}_{\text{Q}} \cdot Q_{\text{PI}} - Q_{\text{demand}} = \mathbf{A}_{from} \cdot Q_{\text{branch},from} + \mathbf{A}_{to} \cdot Q_{\text{branch},to} \tag{10}$$

$$0 \le P_{\text{PI},i} \le a_{\text{P}i} \cdot P_{\text{PI,max@bus}} \tag{11}$$

$$\sum_{i=1}^{N_{\text{P}}} a_{\text{P}i} \le N_{\text{P}} \tag{12}$$

$$\sum_{i=1}^{N_{\text{P}}} P_{\text{PI},i} \le P_{\text{PI,total}} \tag{13}$$

$$0 \le Q_{\text{PI},i} \le a_{\text{Q}i} \cdot Q_{\text{PI,max@bus}} \tag{14}$$

$$\sum_{i=1}^{N_{\text{Q}}} a_{\text{Q}i} \le N_{\text{Q}} \tag{15}$$

$$\sum_{i=1}^{N_{\text{Q}}} Q_{\text{PI},i} \le Q_{\text{PI,total}} \tag{16}$$

Eqs. (9) and (10) define power balance for active and reactive power. $\mathbf{C}_{\text{P}}$, $\mathbf{C}_{\text{Q}}$, $\mathbf{A}_{from}$ and $\mathbf{A}_{to}$ are generator-bus and branch-bus connection sub-matrices. Sub-matrices are derived from the appropriate connection/incidence matrices for the power system.

Eqs. (11) - (13) introduce limitations on active PI's size per bus and total system PI. Variable $a_{\text{P}i}$ is a binary variable that describes whether there will be an active PI at a certain bus. The binary variable is an element from a vector of length N_{P} where N_{P} is a maximum number of locations for active PI placement. $P_{\text{PI,max@bus}}$ and $P_{\text{PI,total}}$ are maximum and total active PI per bus and in system accordingly.

Similar to the previous triplet of limitations, Eqs. (14) - (16) introduce the same but for reactive PI's. Variables refer to the same quantities using appropriate indices for reactive PI's in this case.

Subset's C_{PI} number of constraints is variable depending on the type of PI placed:

- In case of apparent PI, all Eqs. (9) - (16) apply.
- In case of purely active PI, Eqs. (14) - (16) are omitted from the subset and the second term on the left hand side in Eq. (10) is zero.
- For purely reactive PI, Eqs. (11) - (13) are omitted from the subset and the second term on the left hand side in Eq. (9) is zero.

It is worth mentioning that using YALMIP, the problem can be described with symbolic equations just as the reader can see them in this paper. The latter makes the aforementioned optimization toolbox extremely suitable for problem shaping and optimization. YALMIP also offers possibilities for using other more efficient solvers for various types of problems [10].

III. Convergence Properties and Robustness

The proposed approach has no convergence and robustness issues as the utilization of solvers inherently eliminates the infeasible aspects of the optimization process. Computation time mostly depends on the set of possible locations for PI placement. Worst-case scenario suggests a set of locations that contains all buses apart from the slack. It is a user's choice to relax the optimization process by introducing restrictions to this set, i.e. reducing the number of candidate locations.

IV. Case Studies

The proposed approach is applied to the well-known 12.66 kV 69-bus distribution system [16]. Three scenarios are developed and analyzed, i.e. placing of purely active, reactive and apparent PI's at one to three locations accordingly. Obtained results are compared to those from recent studies.

Base case values for this distribution system are ΔP_0 = 225.00 kW and minimum voltage $U_{\text{min@65}}$ = 0.9092 pu.

For optimization purposes, the following input variables and values are initialized:

- All buses apart from the slack bus (index 1) are potential candidates for PI placement of any type, i.e. set of 68 buses indexed from 2÷69 for potential placement of one to three PI's of the same type, depending on the scenario, i.e. $L_{\text{P}}=L_{\text{Q}}=2\div69$.
- Maximum active and reactive power per bus is set to three MW/MVAr accordingly, i.e. $P_{\text{PI,max@bus}}$ = 3 MW, $Q_{\text{PI,max@bus}}$ = 3 MVAr.
- Maximum total active and reactive power injection in the system is set to five MW/MVAr accordingly, i.e. $P_{\text{PI,total}}$ = 5 MW, $Q_{\text{PI,total}}$ = 5 MVAr.
- Number of locations for PI placement varies from one to three locations depending on the scenario, i.e. $N_{\text{P}} = 1\div3$, $N_{\text{Q}}=1\div3$.
- Vector $a_{\text{P}}/a_{\text{Q}}$ of length $L_{\text{P}}/L_{\text{Q}}$ with binary variables $a_{\text{P}i}/a_{\text{Q}i}$ accordingly, that defines whether there will be a PI of active/reactive type in some bus. Maximum number of ones in these vectors corresponds to number of locations for PI placement, i.e. combination of N_{P} ones in a pool of L_{P} positions for a_{P} and N_{Q} ones in a pool of L_{Q} positions for a_{Q}.

TABLE I
COMPARISON OF RESULTS FOR ACTIVE PI PLACEMENT

Location №	NH [6]	HPSO [7]	Proposed method
	Size$^{@bus}$ (kW)	Size$^{@bus}$ (kW)	Size$^{@bus}$ (kW)
1	$1823^{@61}$	$1810^{@61}$	$1872.7^{@61}$
ΔP (kW)	83.30	83.40	83.22
2	$1733^{@61}$	$1733^{@61}$	$1781.4^{@61}$
	$520^{@17}$	$520^{@17}$	$532.3^{@18}$
ΔP (kW)	71.80	71.80	71.68
3	$1689^{@61}$	$1670^{@61}$	$1719.0^{@61}$
	$312^{@21}$	$380^{@17}$	$381.1^{@18}$
	$471^{@12}$	$510^{@11}$	$526.5^{@11}$
ΔP (kW)	69.70	69.60	69.43

Table I presents comparison of results for active PI placement with two other methods, Novel Heuristic (NH) [6] and Hybrid Particle Swarm Optimization (HPSO) [7]. Results show that for all three considered scenarios, the proposed approach presents better results in terms of system power losses. PI's are of same magnitude order for all considered scenarios and in all three approaches. Locations differ in all three methods when placing three PI's.

TABLE II
COMPARISON OF RESULTS FOR REACTIVE PI PLACEMENT

Location №	NH [6]	HPSO [7]	Proposed method
	Size$^{@bus}$ (kVAr)	Size$^{@bus}$ (kVAr)	Size$^{@bus}$ (kVAr)
1	$1310^{@61}$	$1290^{@61}$	$1330.0^{@61}$
ΔP (kW)	152.10	152.10	152.04
2	$1224^{@61}$	$1240^{@61}$	$1275.1^{@61}$
	$356^{@17}$	$350^{@18}$	$361.2^{@17}$
ΔP (kW)	146.50	146.50	146.44
3	$1210^{@61}$	$1190^{@61}$	$1232.5^{@61}$
	$226^{@21}$	$250^{@18}$	$231.4^{@21}$
	$320^{@12}$	$330^{@11}$	$412.6^{@11}$
ΔP (kW)	145.30	145.20	145.12

Table II presents comparison of results for reactive PI placement. Comparisons are made to the same methods referenced in Table I. PI size again slightly differs between methods. Proposed approach offers better results and slightly different set of locations when placing three PI's.

Table III compares results from the proposed approach to Improved Analytical (IA) [4]. Apart from the case of single PI placement where IA outperforms the proposed approach, in all other cases it is vice versa. PI's differ in terms of active and reactive PI size. Set of locations is different between methods for three locations. IA presents a same set of buses for the active/reactive part of the apparent PI placement, while proposed approach presents two different sets for the active and reactive part of apparent PI placement that only differs for the second injection.

TABLE III
COMPARISON OF RESULTS FOR APPARENT PI PLACEMENT

Location №	IA [4]	Proposed method
	$P^{@bus}+jQ^{@bus}$ (kVA)	$P^{@bus}+jQ^{@bus}$ (kVA)
1	$1531.6^{@61}+j1638.7^{@61}$	$1828.6^{@61}+j1300.7^{@61}$
ΔP (kW)	22.62	23.17
2	$1498.8^{@61}+j1603.6^{@61}$	$1735.3^{@61}+j1239.0^{@61}$
	$450.0^{@17}+j481.4^{@17}$	$522.3^{@17}+j353.4^{@17}$
ΔP (kW)	7.25	7.20
3	$1415.5^{@61}+j1514.5^{@61}$	$1674.4^{@61}+j1195.5^{@61}$
	$424.7^{@17}+j454.4^{@17}$	$379.2^{@17}+j230.5^{@21}$
	$566.1^{@50}+j605.6^{@50}$	$494.3^{@11}+j374.8^{@11}$
ΔP (kW)	4.95	4.26

V. CONCLUSION

This paper presents an approach for optimal siting and sizing of power injections in distribution systems utilizing the benefits of powerful optimization solvers. The approach focuses on appropriate problem shaping instead of proper method shaping. There are several advantages to this approach, amongst which is the user's disencumbrance from complex mathematical formulations. Problem description is intuitive and simple and does not require any pre-processing, which is not the case in other methods. Equations and constraints are written in an understanding and readable way. The need for load flow sensitivity analysis before the optimization begins in order to detect suitable placement locations is completely eliminated through the utilization of a power system model represented with a system-subset of constraints. The approach presents better results in terms of power system losses compared to other methods that deal with the same problem.

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