Optimal Locations of Energy Storage Devices in Low-Voltage Grids

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Abstract – **In this paper, we present a methodology for optimal placement of storage devices in low voltage grids. We use CIGRE low voltage benchmark grid as a study case for solving the problem of optimal location and sizing of storage devices. The objective is to maximize the penetration of renewable energy sources (photovoltaic or wind) in a local residential area, while satisfying all grid constraints. The problem is defined as a mixed-integer quadratic programming, with an objective function equal to the total annual costs. The costs include annualized investments in storage devices, operation and maintenance costs and electricity cost separated into categories: bought/sold electricity and network losses. In the model we consider the variations in energy production and consumption by using daily variation curves for each month of the year. We show that the proper placement and sizing of storage devices enables maximum usage of renewable energy sources within the analyzed network keeping the costs at minimum even in cases with unfavorable feed-in tariffs.**

Keywords – **Storage devices, power systems, low-voltage grid, photovoltaic, optimization.**

I. Introduction

As climate changes become an issue [1], people are turning to embrace ideas for clean energy, such as wind and sun energy [2]. This idea is implementing on a large scale in transmission and distribution networks, as well as, in nowadays into low-voltage grids (LVG).

The main problem with renewable energy sources is their availability. Usually they are available partially throughout the day (for ex. sun energy), and may not match with the energy peaks, during the day. For that reason, we are trying to find a way to storage this "free" energy and use it in times when we need it the most. In that manner, energy storage devices become an attractive idea for storing surplus energy generated from renewable sources in order to be used when we have the greatest electrical or economic benefit.

As storage devices we may use batteries, gas compression facilities, liquid based storage, etc. In particular, Battery Storage (BS) in LVG is considered to be a promising technology for that matter [3]-[4].

The main idea of using storage devices in LVG is to make the consumers more independent from the main source and in that manner to allow lowering their electricity costs. This is particularly attractive in net-metering tariff schemes, as well as, in LVG with limited capacity. In some cases, they can be used for balancing short-term fluctuations and for alleviating grid congestion.

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II. Problem Formulation

A. Problem Statement

In this section we define the problem that we aim to solve. The basic assumption is that we already have the locations of the residential houses were the renewable sources are installed, and for that reason they are not part of this analysis. As we have stated before, the main goal is to find the best location for installing storage devices and define their size in LVG in such a manner that residential houses shall be more independent from the main energy source. In that way, they shall gain their energy "independence" and lower their energy bills, and also lowering grid losses.

Finding best location for placing and sizing of the elements in a network is a combinatorial problem and for that purpose we shall use OPF method to gain the solution. For the economics of the problem at hand we observe that large portion of the investment is associated with equipment costs, while remaining portions covers costs related to grid connection, foundations, buildings, losses, etc.

Assuming that this kind of projects are finances by debt, we should annualize the capital costs using an appropriate Capital Recovery Factor (CRF), which is defined as:

$$CRF(I,N)=\frac{I\cdot(1+I)^N}{(1+I)^N-1}, \tag{1}$$

where I is the interest rate in decimal fraction and N is the loan term in years.

The largest portion of the costs are for the equipment purchase, which we refer to as capital costs:

$$C_{\text{capital}}=\sum_{q=1}^{n_{loc}} Nbat_q\cdot C_{\text{battery}}\ [\$], \tag{2}$$

where $Nbat_q$ is the number of batteries installed at location q and C_{battery} is the cost for each battery. In each location we are allowed to install a certain number of batteries of predefined unit size. Because the project is financed by debt, we must take into consideration the loan L which is given as a percentage of the capital cost:

$$L=\frac{C_{\text{loan}}}{100}\cdot C_{\text{capital}}\ [\$], \tag{3}$$

with C_{loan} being the percentage of the capital costs payed by the loan.

Operations and maintenance costs (OM) include regular maintenance, repairs, insurance, administrations, etc. In general, OM costs depend not only of yearly equipment usage, but also from their age. To find a levelized cost estimation for the energy delivered by the batteries, we must divide the annual costs by annual energy delivery. To find annual cost, we must spread the capital costs out over the projected lifetime using an appropriate factor and then add in an estimate of annual OM. OM's costs are defined as:

$$OM = \frac{OM_{\text{econom}}}{100} \cdot C_{\text{capital}} \, [\$] \tag{4}$$

where OM_{econom} is OM rate of costs, expressed as a percentage.

One segment of the annual costs are the annual payments for the debt and they are defined as:

$$C_{\text{payment}} = L \cdot CRF(I, N) \, [\$] \, . \tag{5}$$

The equity return is defined as percentage of the difference between capital costs and the loan:

$$ER = \frac{ER_{\text{rate}}}{100} \cdot \left(C_{\text{capital}} - L \right) [\$], \tag{6}$$

where with ER_{rate} is defined the percentage rate for the desired equity return.

Other costs that must be taken into consideration, are the costs due to the power losses. This costs are defined by the network model (see subsection B). The total grid loss costs are defined for multi-period as hourly snapshot per day are taken into consideration, in period of one year:

$$C_{\text{loss}} = \sum_{l=1}^{ns} \left(\sum_{m=1}^{nt} \left(\sum_{k}^{nb} Grid\_loss_k \cdot C_{\text{loss\_cost}} \right)_m \right)_l [\$], \tag{7}$$

where with $Grid\_loss_k$ is defined the k-th branch losses (see subsection B) and C_{loss_cost} is the cost of the power losses expressed as \$/kWh. With nt and ns we define the number of periods per day, which in our case is nt = 24, and number of characteristics days/snapshots per year, which is ns = 12.

If the residential house is pulling energy from the main energy source, that will contribute to costs rise. This costs are defined as energy buy in each period of the day for all snapshots in the year:

$$C_{EB} = \sum_{l=1}^{ns} \left(\sum_{m=1}^{nt} \left(energy\_buy \right)_m \cdot snapDay \right)_l [\$]. \tag{8}$$

In certain time intervals when the renewable sources in the network are producing more than the energy needs of the load demand there will be a possibility to sell energy upstream into the medium voltage network, which shall introduce revenue for the residential houses where storage devices are installed. This revenue is defined by (9).

The energy_buy and energy_sell in Eqs. (8) and (9) are costs/revenue expressed in \$/kWh, for buying/selling energy from/into the grid.

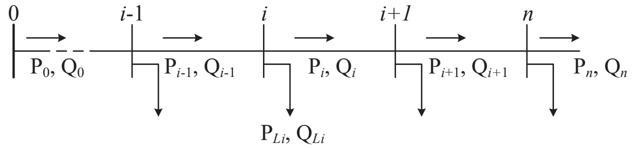


Fig. 1 One line diagram of radial grid

$$C_{ES} = \sum_{l=1}^{ns} \left(\sum_{m=1}^{nt} \left(energy\_sell \right)_m \cdot snapDay \right)_l [\$]. \tag{9}$$

Taking into consideration all above mentioned costs, the objective function that shall lead us to the solution of the location and sizing problem of storage devices is defined as:

$$\begin{aligned} C_{\text{annual}} &= C_{\text{payment}} + ER + OM + C_{\text{loss}} + C_{EB} + C_{ES} \, [\$] \\ F_{\text{obj}} &= \min C_{\text{annual}} \, [\$] \end{aligned} \tag{10}$$

B. Network Model

Low-voltage grids normally operates as radial networks, however, their configuration could be changed during operation for some reasons (load transfer, loss reduction, overloads relieves, etc.). Mainly, grid reconfiguration are done to lower the grid losses, so in some way grid model affects upon the total costs. Reconfiguring LVG was simply-made in the past, as they were treated as passive ones, but nowadays, they become more active as result of renewable source penetration.

To define the grid losses and accordantly to quantify their costs, we costs, we need to use a set of power flow equations, [5]. To illustrate them, consider the grid depicted in

Fig. 1 The lines impedances are represented by $z_i = r_i + jx_i$, and loads as constant power sinks, $S_{Li} = P_{Li} + jQ_{Li}$. The power flow in radial grid can be described by set of recursive equations, that use real power, reactive power and voltage magnitude at the sending end of a branch (P_i, Q_i, U_i) respectively to express the same quantities at the receiving end of the same branch as follows:

$$\begin{aligned} P_{i+1} &= P_i - r_i \frac{P_i^2 + Q_i^2}{U_i^2} - P_{Li+1} \\ Q_{i+1} &= Q_i - x_i \frac{P_i^2 + Q_i^2}{U_i^2} - Q_{Li+1} \\ U_{i+1}^2 &= U_i^2 - 2\left(r_i P_i + x_i Q_i \right) + \left(r_i^2 + x_i^2 \right) \frac{P_i^2 + Q_i^2}{U_i^2} \end{aligned} \, . \tag{11}$$

Hence, if the set of real power, reactive power and voltage magnitude are known for the first node (P_0, Q_0, U_0), then the same quantities could be calculated at the other nodes, by applying Eq. (11).

Having the network model, now we can express the power loss in terms of system variables, [6].

To set the proper grid configuration, that represents the condition of minimal grid losses, we need to minimize the total i^2r losses in the system, which can be calculated as follows:

$$Grid\_loss = \sum_{i=0}^{n-1} r_i \frac{P_i^2 + Q_i^2}{U_i^2} \quad (12)$$

The set defined by Eq. (11) could be simplified by noting that the quadratic terms in the equations represent the losses on the branches and hence they are much smaller than the branch power terms P_i and Q_i. Therefore, by dropping these second order terms we can get a new set of branch equations of the following form.

$$\begin{aligned} P_{i+1} &= P_i - P_{Li+1} \\ Q_{i+1} &= Q_i - Q_{Li+1} \\ U_{i+1}^2 &= U_i^2 - 2\left(r_i P_i + x_i Q_i\right) \end{aligned} . \quad (13)$$

Since the LVG are radial by nature, the solution for the simplified set of Eq. (13) can be obtained easily. For the given radial grid at

Fig. 1 the solution is defined as:

$$\begin{aligned} P_{i+1} &= \sum_{k=i+2}^{n} P_{Lk} \\ Q_{i+1} &= \sum_{k=i+2}^{n} Q_{Lk} \\ U_{i+1}^2 &= U_i^2 - 2\left(r_i P_i + x_i Q_i\right) \end{aligned} . \quad (14)$$

Now, the power losses on a branch can be approximated as:

$$Grid\_loss = \sum_{i=0}^{n-1} r_i \left(P_i^2 + Q_i^2\right), \quad (15)$$

as we used the fact that $U_i^2 \approx 1\ p.u.$

C. Constraints

The objective function given by Eq. (10) is presented by the sum of several costs: battery investment, operation and maintenance, grid losses, etc. The solution of Eq. (10) must meet all the models constraints. A set of constraints is deriving from the grid, that include real and reactive power limits of the generators (i=1,…,ng) (PV systems) as a function of weather conditions for the considered multi-period (nt-hourly level, ns-monthly level) Eq. (16), branch current constraints as no overload branches (k=1,…,nb) are allowed Eq. (17), node voltages $\left(U_j^2 = V_j\right), j = 1,...,nj$ must be in the defined range Eq. (18), as well as voltage constraints for the main supply bus Eq. (19) and the branches ($k = f$-t) Eq. (20).

$$\begin{aligned} 0 \le P_{i,nt,ns}^{PV_{gen}} \le P_{i_{\max},nt,ns}^{PV_{gen}} \\ 0 \le Q_{i,nt,ns}^{PV_{gen}} \le Q_{i_{\max},nt,ns}^{PV_{gen}} \end{aligned} . \quad (16)$$

$$-I_{k_{\max},nt,ns}^{branch} \le P_{k,nt,ns}^{branch} \le I_{k_{\max},nt,ns}^{branch} . \quad (17)$$

$$\begin{aligned} V_{j_{\min},nt,ns} \le V_{j,nt,ns} \le V_{j_{\max},nt,ns} \\ 0{,}9^2 \le V_{j,nt,ns} \le 1{,}1^2 \end{aligned} . \quad (18)$$

$$V_{1,nt,ns} = 1\ p.u.. \quad (19)$$

$$V_{t,nt,ns} = V_{f,nt,ns} - 2\cdot\left(r_k \cdot P_{k,nt,ns} + x_k \cdot Q_{k,nt,ns}\right). \quad (20)$$

Constraints written in vector-matrix form, given by Eqs. (21) and (22), defines the real and reactive power balance per node for the considered multi-period, while by Eq. (23) we define the supply node constraints which prohibit situations when the purchased ($\mathbf{P}_{EB}$) and sold ($\mathbf{P}_{ES}$) real power are simultaneous non-zero. **Cg** and **A** are connection matrices for the generator-bus and branch-bus, respectively and **bin** is vector-matrix variable with two stages 0 and 1.

$$\mathbf{P}_{\text{EB}} - \mathbf{P}_{\text{ES}} + \mathbf{C}_{\text{g}} \cdot \mathbf{P}_{\text{g}} - \mathbf{P}_{\text{load}} + \mathbf{P}_{\text{discharge}}^{\text{battery}} - \mathbf{P}_{\text{charge}}^{\text{battery}} = \mathbf{A} \cdot \mathbf{P}_{\text{branch}} . \quad (21)$$

$$\mathbf{Q}_{\text{EB}} + \mathbf{C}_{\text{g}} \cdot \mathbf{Q}_{\text{g}} - \mathbf{Q}_{\text{load}} = \mathbf{A} \cdot \mathbf{Q}_{\text{branch}} . \quad (22)$$

$$\begin{aligned} 0 \le \mathbf{P}_{\text{EB}} \le \mathbf{bin} \cdot 1000 \\ 0 \le \mathbf{P}_{\text{ES}} \le \left(1 - \mathbf{bin}\right) \cdot 1000 \end{aligned} . \quad (23)$$

Other set of constraints is derived from the storage devices and their element (battery) properties for energy storage and discharge, under the assumption that no storage devices will be installed at the main supply bus ($Nbat_1$ = 0). The first constraints given by Eq. (24) derived from available power for batteries charging, i.e. if power surplus appear into the grid it will be stored and if the storage device is empty it shall not be storing energy from the main supply bus.

$$\begin{aligned} P_{leftover}^{PV} &= \sum_{i=1}^{ng} P_{i,nt,ns}^{gen} - \sum_{j=1}^{nj} P_{j,nt,ns}^{load} \\ P_{leftover}^{PV} &\left(P_{leftover}^{PV} < 0\right) = 0 \end{aligned} . \quad (24)$$

The number of installed batteries per storage device is limited, Eq. (25). Conditions in which the batteries will simultaneously charging and discharging are not allowed, Eq. (26).

$$0 \le Nbat_i \le Nbat_{\max} . \quad (25)$$

$$\begin{aligned} 0 \le \mathbf{P}_{\text{charge}}^{\text{battery}} \le \mathbf{bin} \cdot Nbat_{\max} \cdot \mathbf{P}_{\text{charge, max}}^{\text{battery}} \\ \mathbf{P}_{\text{discharge}}^{\text{battery}} \le \left(1 - \mathbf{bin}\right) \cdot Nbat_{\max} \cdot \mathbf{P}_{\text{discharge, max}}^{\text{battery}} \end{aligned} . \quad (26)$$

We assume that the stored energy in the battery (SOE) at the end of the day, should be the same as the beginning of the day, Eq, (27) and SOE must be in the defined range, Eq. (28). The SOE for the first hour and the remaining period of the day are defined by Eqs. (29).

$$SOE_{nt} = SOE_1 . \quad (27)$$

$$Nbat \cdot SOE_{\min} \le SOE_{nt,ns} \le Nbat \cdot Bat\_size \quad (28)$$

$$\begin{aligned} SOE_1 &= Nbat \cdot SOE_{\min} + P_{\text{charge},1}^{\text{battery}} \cdot \eta_{\text{charge}} - \frac{P_{\text{discharge},1}^{\text{battery}}}{\eta_{\text{discharge}}} \\ SOE_{nt} &= SOE_{nt-1} + P_{\text{charge},nt}^{\text{battery}} \cdot \eta_{\text{charge}} - \frac{P_{\text{discharge},nt}^{\text{battery}}}{\eta_{\text{discharge}}} \end{aligned} . \quad (29)$$

And the last constraints derived by the maximum charge/discharge power of the batteries.

$$
\begin{aligned}
&0 \le \sum P^{\text{battery}}_{\text{charge},nt,ns} \le P^{\text{PV}}_{\text{leftover},ns,nt} \\
&0 \le P^{\text{battery}}_{\text{discharge},nt,ns} \le P^{\text{battery}}_{\text{discharge,max}} \cdot Nbat
\end{aligned}
\tag{30}
$$

III. Case Study

The proposed methodology is applied on CIGRE low voltage benchmark grid [7], that propose different approach than [4]. The method is based on the benefits of today's optimization solvers such as CPLEX [8] and algebraic modeling systems such as YALMIP [9].

Fig. 2a) and b) depicts the daily load profiles and PV production for 12 month period, respectively. The data for the load profiles are taken from OpenDSS [10], while the PV production are average of Meteonorm calculated irradiation data, [11]. The proposed method also takes into consideration the net-metering scheme for the purchased and sold energy, Fig. 2c). At Table I the input data for batteries and economic data are given.

TABLE I
BATTERY AND ECONOMIC INPUT DATA

Battery data	*Bat_size*, kWh	5	SOE_{min}, kWh	1
	$C_{battery}$, $	200	$P_{charge,max}$, kW	2
	η_{charge}	0,88	$P_{discharge,max}$, kW	2
	$\eta_{discharge}$	0,88	$Nbat_{max}$/per bus	20
Economic data	*I*, %	7	OM_{econom},%	3
	N, years	20	ER_{rate}, %	15
	C_{loan}, %	75	C_{loss_cost}, $/kWh	Fig. 2c)

As result of the optimization [8], the sizing of the storage devices and optimal locations ($Nbat^{@Bus}$) are presented by Table II, as well as the costs.

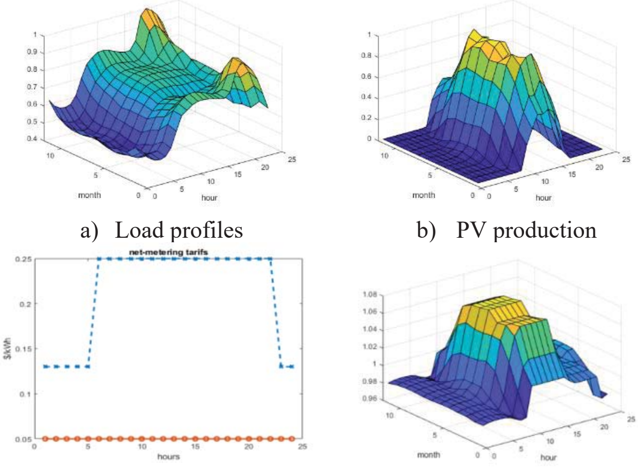
TABLE II
RESULTS FOR THE LOCATION AND SIZE OF THE BATTERIES

$Nbat^{@Bus}$								
$5^{@8}$	$9^{@9}$	$11^{@10}$	$3^{@13}$	$11^{@14}$	$12^{@15}$	$5^{@16}$	$11^{@17}$	$11^{@18}$
Costs, $								
$C_{capital} = 78\ 000$		ER = 2 925		$C_{payment} = 5{,}522 \cdot 10^3$		$C_{EB} = 2{,}0389 \cdot 10^4$		
L = 58 500		OM = 2 340		$C_{loss} = 4{,}8061 \cdot 10^3$		$C_{ES} = 1{,}9732 \cdot 10^4$		
$C_{annual} = 1{,}625 \cdot 10^4$								

The voltage profiles are depicted by Fig. 2d).

IV. Conclusion

In this paper we have presented a methodology for optimal location and sizing of storage devices (batteries) in LVG. The advantage of this approach is that the problem is modeled using multi-period optimization where we take into account daily variations in energy production and use for each month of the year. The problem is solved by using the mixed-integer quadratic programming solver CPLEX, while the model writing is facilitated by the use of algebraic modeling system YALMIP. In the model we consider the variations in energy production and consumption by using daily variation curves for each month of the year. We show that the proper placement and sizing of storage devices enables maximum usage of renewable energy sources within the analyzed network keeping the costs at minimum even in cases with unfavorable feed-in tariffs.



a) Load profiles b) PV production
c) Net-metering scheme d) Voltage profiles
Fig. 2 Data for optimization

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