Wavelet algorithm for denoising MEMS sensor data

Lachezar Hristov[1], Emil Iontchev[2], Rosen Miletiev[3] and Petar Kapanakov[4]

Abstract – **In this paper we propose a wavelet-based algorithm for denoising data acquired from MEMS based inertial sensors, in order to achieve accurate navigational information in terms of position, speed, acceleration and direction. It is well known that the low-cost MEMS inertial sensors output signals require a proper treatment in order to reduce the different random errors, consisting in the signal. The proposed algorithm is tested and the results are compared with the GPS data.**

Keywords – **Wavelet transform, denoising, inertial sensors.**

I. Introduction

MEMS integrated system, inertial sensor data includes large signal noise and sensor bias. The main error sources of the MEMS inertial sensors are recognized as: null offset error (bias) b_a, gain error - K, integral and differential nonlinearity and misalignment – T^p, specific force - F and output noise - e_n. [1] The influence of these errors may be written by the Eq. (1)

$$A = K(T^p)^{-1}F + b_a + e_n \tag{1}$$

Among them, sensor biases make more significant error on position result because of integration. Sensor biases are well compensated on good observable trajectory in general integrated navigation system. But sensor biases cannot exactly be compensated on low cost system. If the noise component could be removed, the overall inertial navigation accuracy is expected to improve considerably. The resulting position errors are proportional to the existing sensor bias and sensor noise.

In this paper, the wavelet denoising technique is implemented to eliminate the sensor noise for improving the performance of inertial sensor signals [2], [3]. The wavelet analysis has the advantage over other signal processing techniques in the capability of performing local analysis. It can decompose the signal to frequency component in local time. By using this characteristic, the wavelet denoising method shrinks the signal noise by eliminating the frequency component which contains only noise. Furthermore, the wavelet denoising method can also remove the signal bias by decomposing the signal and eliminating the low frequency component.

The wavelet thresholding method is verified using the collected real data from the field test. It is implemented to the MEMS integrated system, later shown in the experimental data section.

II. Wavelet Denoising Algorithm

A. Wavelet Transform

Wavelet transform is a signal transform technique popularly used in image and audio signal processing [4], [5], [6]. Compared by nature of the processing type, the traditional Fast Fourier Transform (FFT) has a fixed relationship between time and frequency while the wavelet does not have a fixed relationship between time and frequency. The wavelet analysis is based on a windowing technique with variable-sized windows shown in Fig. 1. The wavelet transform applies the wide window (long time intervals) to low frequency and the narrow window (short time intervals) to high frequency.

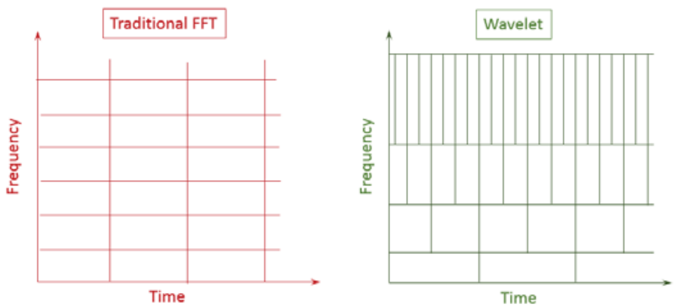

Fig. 1. Time-Frequency Domain Sampling

Discrete time wavelet transform is executed as Eqs. (1), (2) [7].

$$\phi(t) = \sum_k g_0[k]\phi(2t-k) \tag{2}$$

$$\Psi(t) = \sum_k g_1[k]\phi(2t-k) \tag{3}$$

Where ϕ is called the scale function and Ψ is called the wavelet function. Each g_0 and g_1 refers to the wavelet coefficient.

Wavelet function can be any function that satisfies the relationship of Eqs. (1) and (2). The scale function of upper level can be expressed as the convolution of the scale function and the wavelet function of lower level. It means that the low-frequency area can be decomposed to the high-frequency area and low-frequency area. Such relationship is shown in Fig. 2

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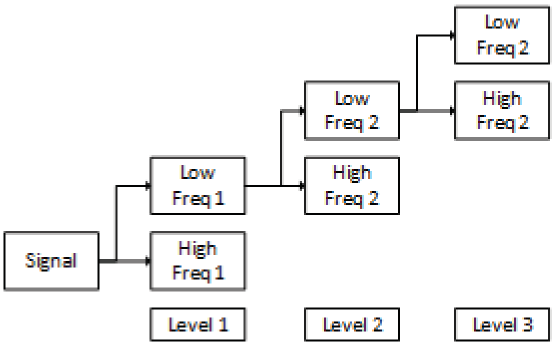

Fig. 2. Wavelet Transform

B. Wavelet Thresholding Technique

Wavelet thresholding technique is a signal estimation technique that exploits the capabilities of wavelet transform for signal denoising. It removes the noise by eliminating coefficients that are insignificant relative to some threshold. Researchers have developed various techniques for choosing denoising parameters none of which is best universal threshold determination technique. Wavelet thresholding technique assumes that the magnitude of the actual signal is greater than the noise level, and in general the noise is white noise.

The wavelet thresholding technique reduces the noise level with almost no distortion for the sudden change in signal. The result of the thresholding technique has almost no distortion and accurate, so that it sits right on the actual signal almost indistinguishably. Therefore, it can be used by preprocessing filter for the inertial sensor signal and overcomes the shortages of the existing low-pass filter.

The thresholding technique is classified into the hard thresholding and soft thresholding operation.

$$T_\lambda^{hard} = \begin{cases} u & if|u| \geq \lambda \\ 0 & otherwise \end{cases} \quad (3)$$

$$T_\lambda^{soft} = \begin{cases} (u - sign(u)\lambda) & if|u| \geq \lambda \\ 0 & otherwise \end{cases} \quad (4)$$

The Eq. (3) shows the hard thresholding function and the Eq. (4) shows the soft thresholding function. The wavelet coefficient means that the magnitude of a certain frequency component. Thus, wavelet coefficients in the band of noise become zero. The soft thresholding technique is known to have better performance than the hard thresholding technique, so the soft thresholding technique was used for the experiment [8], [9].

Important element influencing the performance in thresholding technique is how the standard value of λ is set [2], [8]. Generally, it is determined by the Eq. (5). Here, σ is the standard deviance of the signal, and *n* is the number of signal samples

$$\lambda = \sqrt{2 \log n}\, \sigma \quad (5)$$

The value determined by the formula, will not be the optimal result, so the appropriate λ must be determined through experimentation.

D. Wavelet Denoising Algorithm

The proposed denoising algorithm is executed as follows. Select wavelet based on predefined list. For the input signal, using interval-dependent thresholding method - obtain the maximum level of decomposition, define the intervals and interval-dependent thresholds, decompose the signal, replace original wavelet coefficients by the coefficients resulting from the thresholding operation and perform wavelet transform - apply the thresholds and reconstruct the signal. Integrate the wavelet denoised signal to get the speed and distance. Calculate the SNR and endpoint accuracy against the noisy signal speed and distance. If all predefined wavelets are used, compare the results and draw graphics. Estimate the most appropriate wavelet for denoising the MEMS based accelerometer data.

For the purpose of the experiment the denoising algorithm is implemented using MATLAB.

The process is shown in the flowchart on Fig. 3

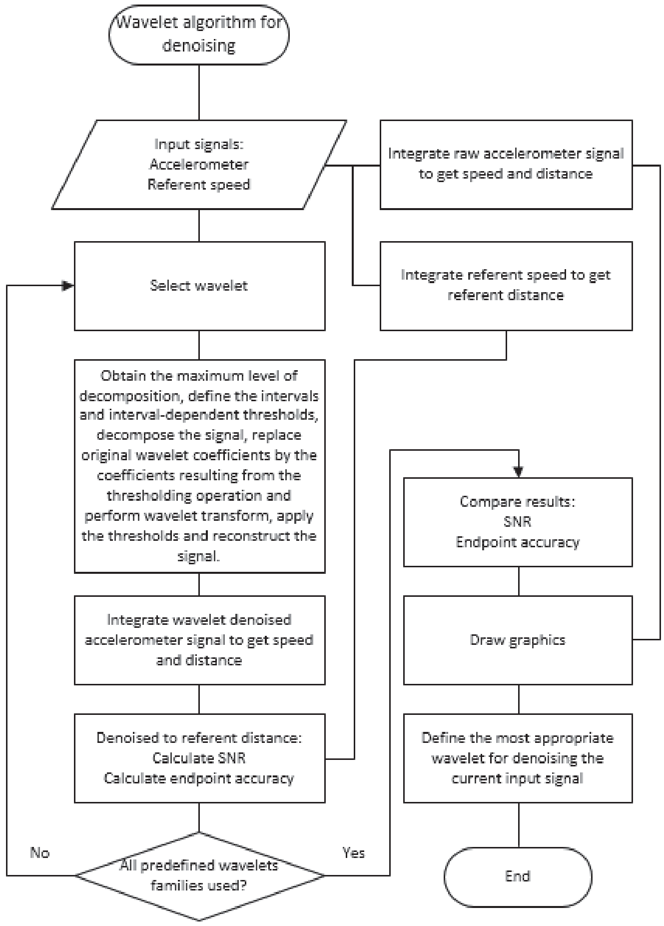

Fig. 3. Block diagram of the denoising

III. EXPERIMENTAL RESULTS

Collected MEMS sensor signal data includes the actual vehicle motion dynamics and the sensor noise as well as some other undesirable noise such as vehicle engine vibration.

The wavelet transform was carried out using predefined list of wavelets by function cmddenoise. Table I shows the experimental data results for each wavelet used – name, calculated maximum level of decompositions, calculated best number of intervals, calculated SNR of denoised signal compared to noise from signal data, deviation between calculated denoised signal last point distance and reference signal last point distance. Results are sorted based on the best SNR.

TABLE I
EXPERIMENTAL DATA

Signal	Wavelet	Max Lvl Deco mp	Best Nb of Int	SNR path [dB]	dev last point dist [%]
Accel.	db2	10	3	13.6674	2.8222
Accel.	sym2	10	3	13.6674	2.8222
Accel.	coif1	9	5	13.5842	3.1055
Accel.	db3	9	3	13.5464	3.2062
Accel.	sym3	9	3	13.5464	3.2062
Accel.	sym4	8	1	13.4629	3.4765
Accel.	sym6	8	1	13.4335	3.5684
Accel.	coif2	8	1	13.4139	3.6311
Accel.	db4	8	2	13.3311	3.8938
Accel.	db6	8	1	13.3110	3.9543
Accel.	sym8	7	1	13.2793	4.0551
Accel.	db7	8	2	13.2777	4.0643
Accel.	sym7	8	3	13.2765	4.0700
Accel.	coif3	7	1	13.2726	4.0765
Accel.	haar	11	1	13.2648	3.9585
Accel.	coif4	7	1	13.2502	4.1492
Accel.	sym5	8	1	13.2439	4.1706
Accel.	db8	7	1	13.2402	4.1836
Accel.	coif5	6	1	13.2126	4.2740
Accel.	db9	7	1	13.1922	4.3415
Accel.	db5	8	1	13.1892	4.3506
Accel.	db10	7	1	13.1862	4.3579

It is noticeable that “db2”, “sym2” and “coif1” wavelets have the best SNR ratio. The calculation of the last point deviation from reference distance confirm that the wavelets being on the top of the table is the most appropriate as they show less deviance from the last distance point and it is more accurate than the other wavelets in the table.

Figs. 4 and 5 shows a comparison between noisy signal and the top three best SNR ratio denoised signals. On the detailed Figure 5 we can see that “db2” and “sym2” wavelet results are almost identical and their lines are one over another.

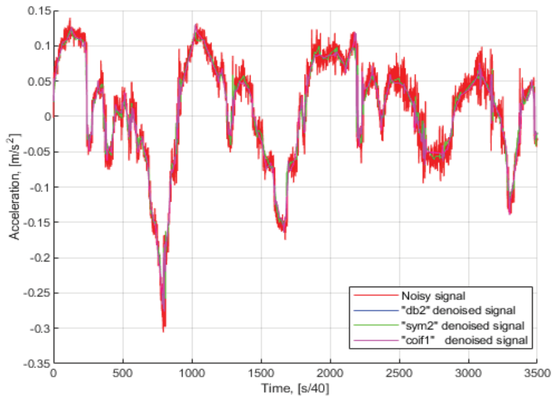

Fig. 4. Comparison between noisy signal and top three most effective wavelet denoised signals

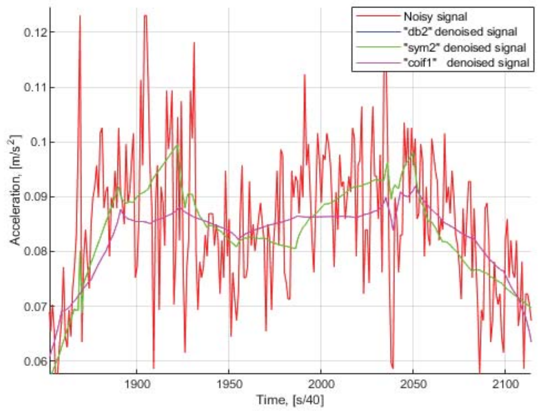

Fig. 5. Detailed comparison between noisy signal and top three most effective wavelet denoised signals

On Fig. 6 and more detailed on Fig. 7 we can see that denoised signal is following the reference speed curve most accurate.

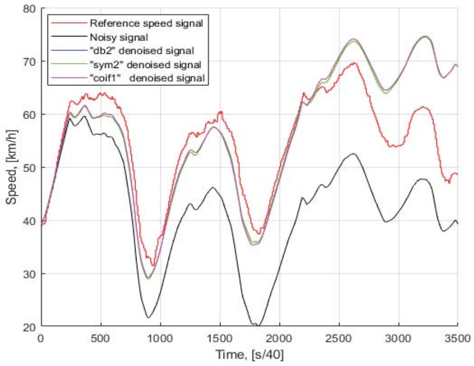

Fig. 6. Comparison between speed calculation results - reference speed signal, noisy signal and top three most effective wavelet denoised signals

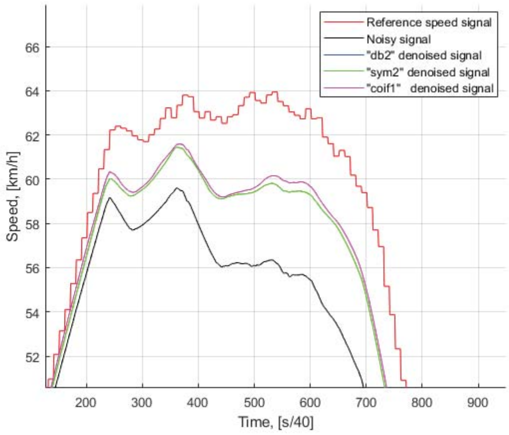

Fig. 7. Detailed comparison between speed calculation results - reference speed signal, noisy signal and top three most effective wavelet denoised signals

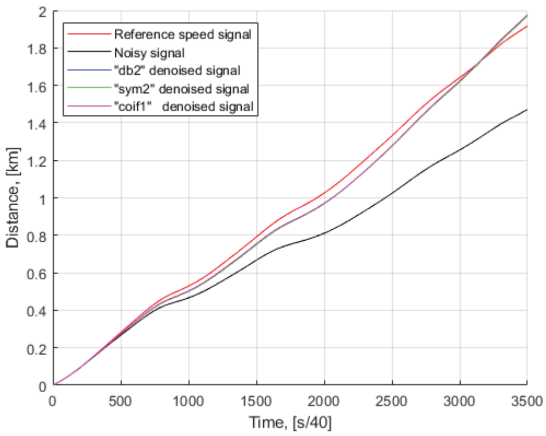

Fig. 8. Comparison between accumulated distance results - reference signal, noisy signal and top three most effective wavelet denoised signals

IV. Conclusion

In order to achieve accurate navigational information in terms of position, speed, acceleration and direction it is necessary to use proper denoising. The proposed algorithm will help us calculate the most accurate position, speed or distance by using raw signal acquired by MEMS Inertial Navigation Systems during the blind zones where GPS signal is weak or missing.

In the article we propose an algorithm for selecting the most appropriate wavelet for denoising MEMS sensor data, based on input noisy signal analysis, while searching for the best calculated SNR for each of the applied wavelet denoising.

Calculated the best SNR shows the most appropriate wavelet to be applied to the signal being processed. SNR is calculated by only using data from input noisy signal. Referent signal is used to show and confirm that the algorithm proposes the best wavelet which will approve the results on later calculations by having the smallest last point deviance as well.

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