Bi-isotropic Cylinder Placed in Homogeneous Electric Field Generated Using Plan-parallel Electrodes

Žaklina J. Mančić[1] and Zlata Ž. Cvetković[2]

Abstract – **In this paper, the influence of bi-isotropic body on the homogeneity of electric field, generated using plan-parallel cell of the first order (primary cell), is analyzed and a 2D view of its impact on the achieved homogeneity of the field is given.**

Keywords – **electrostatics, homogeneous field, image theorem, bi-isotropic material of Tellegen type.**

I. Introduction

The problem of generating homogeneous fields is relatively old [1,2] but still attractive research area [3-10]. For homogeneous field generation, various approaches may be used. For instance, rings are used for homogeneous field generation in [1, 3, 4, 6], plan-parallel systems are exploited in [2,5, 8-10] whereas conical electrodes are considered in [4, 7]. Also, the influence of conducting body placed in homogeneous electric field is analyzed in the case of ring electrodes [6], biconical electrodes [7] and plan-parallel electrodes [8, 9]. Complex plan-parallel systems are modelled and the primary *N*-th order cell, whose electrodes are positioned on an imaginary cylindrical surface, was defined in [8]. Furthermore, the stability and efficiency of such systems in the case of small deviations of some geometric parameters is analyzed in [9].

One of the major issues in the analysis of generated homogeneous electric fields is to determine to what extent an external body, placed into the field, affects the achieved homogeneity. There are several types of external bodies that are of interest. Classification is made depending on their material, so that conducting body, bodies made of homogeneous dielectric, anisotropic dielectric and bi-isotropic material of Tellegen type can be considered as the most significant. In the case when plan-parallel electrodes are used for field generation, the influence of all aforementioned types of external bodies is analyzed in [10].

In the recent decades, bi-isotropic materials have gained increasing attention [11-20]. This paper is extension of the research presented in [10] and its aim is to illustrate how a bi-isotropic body influences the generated homogeneous field and, above all, to give a 2D view of the bi-isotropic body's impact on the achieved homogeneity.

[1]Žaklina Mančić is with the Faculty of Electronic Engineering at University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia, E-mail: zaklina.mancic@elfak.ni.ac.rs.

[2]Zlata Cvetković is with the Faculty of Electronic Engineering at University of Niš, Aleksandra Medvedeva 14, 18000Niš, Serbia. E-mail: zlata.cvetkovic@elfak.ni.ac.rs.

II. External Body in the Homogeneous Electric Field

In Fig. 1 is shown a primary cell [10] for generating homogeneous electrostatic field in the case of plan-parallel systems. The cell consists of two pairs of plan-parallel, straight linear lines of a circular cross-section. In the case of the coordinate system used in Fig. 1, symmetry plane is $y=0$, i.e. $\theta=\frac{\pi}{2}$ for the zero potential. It should be noted that conductors are on the same absolute potential. An external cylindrical body of radius a is placed into the origin of coordinate system.

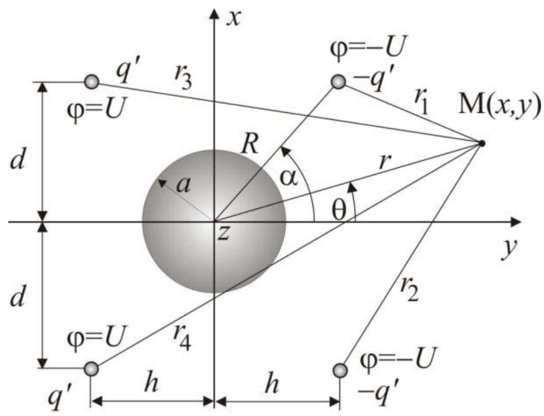


Fig. 1. Primary cell and external body.

Charges densities per unit length q' and $-q'$ are on conductors. Potential in the point $\mathrm{M}(x,y)$ of the adopted coordinate system, generated by charges on conductors in the presence of air is:

$$\varphi(\theta,r)=\frac{q'}{2\pi\varepsilon_0}\ln\left(\frac{r_1r_2}{r_3r_4}\right) \tag{1}$$

A detailed analysis of plan-parallel systems of *N*-th order is presented in [10] and it is demonstrated that a primary cell of the first order have dimensions $h=0.5R$ and $d=0.8660254R$, which we adopt here for the homogeneous field generation. It has been also shown that due to geometry of the system, the usage of cylindrical coordinate system would be an advantage for the purpose of calculating field in the case of cylindrical external body [10].

Image theorem can be used to determine how and to what extent the material from which the body is made, "spoils" the homogeneity of the field. This theorem significantly simplifies calculation process and it provides low complex solutions in mathematical terms. It has been shown that the images on the basis of which is determined the electric scalar potential for charges per unit length outside the external body are positioned along the cylindrical surface of the radius $D = a^2/R$ in the case of perfectly conducting external body [8]. In previous equation for calculating radius D, parameter a represents the radius of the conducting body, whereas R is the radius of the imaginary cylinder, upon which are placed the plan-parallel electrodes for generating a homogeneous electrostatic field.

III. Image theorem for bi-isotropic materials

Constitutive relations between vectors **E**, **D**, **B** and **H** for bi-isotropic Tellegen medium are defined as [11]:

$$\mathbf{D} = \varepsilon\mathbf{E} + \xi\mathbf{H}\,,\ \ \mathbf{B} = \mu\mathbf{H} + \xi\mathbf{E} \tag{1}$$

where $\varepsilon = \varepsilon_r \varepsilon_0$ (ε_r is relative permittivity and $\varepsilon_0 = 8.8541878 \cdot 10^{-12}$ F/m is vacuum permittivity) and $\mu = \mu_r \mu_0$ (μ_r is relative magnetic permeability and magnetic permeability of free space is $\mu_0 = 4\pi \cdot 10^{-7}$ H/m).

According to Tellegen, such materials are polarized and magnetized simultaneously if condition $\varepsilon\mu \neq \xi^2$ is satisfied and if they are placed into electric or magnetic field [11-14]. Starting from Maxwell's equations and constitutive relation (1), Poisson's equation for electric scalar potential φ can be obtained:

$$\Delta\varphi = -\rho/\varepsilon_e \tag{2}$$

where is $\varepsilon_e = \varepsilon\left(1 - \xi^2/(\varepsilon\mu)\right)$.

Poisson's equation for magnetic scalar potential φ_m is:

$$\Delta\varphi_m = \frac{\xi}{\varepsilon_e \mu}\rho\,. \tag{3}$$

Once determined φ and φ_m, electric and magnetic field are calculated as:

$$\mathbf{E} = -\mathrm{grad}(\varphi) \text{ and } \mathbf{H} = -\mathrm{grad}(\varphi_m) \tag{4}$$

whereas remaining fields (**B** and **D**) are calculated from the constituent relation (1).

In the case when the external body is bi-isotropic cylinder of Tellegen type (Fig. 2), unknown constants are [10]:

$$\mathrm{A} = -\mathrm{B} = \frac{(\varepsilon - \varepsilon_0)(\mu + \mu_0) + \xi^2}{(\varepsilon + \varepsilon_0)(\mu + \mu0) - \xi^2}\,, \tag{5}$$

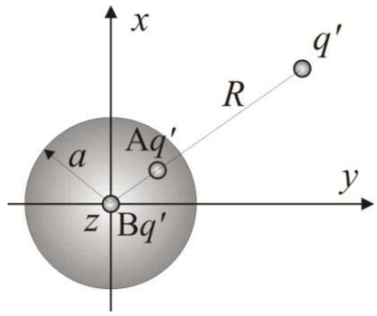


Fig. 2. Charges per unit length and its images in the bi-isotropic cylindrical mirror.

IV. Bi-isotropic external body placed into a homogeneous electrostatic field

Fig. 3 shows normalized electric field in the case of bi-isotropic external body presence with the weak ($\xi^2/(\varepsilon\mu) = 0.1$) and strong bi-isotropy ($\xi^2/(\varepsilon\mu) = 0.9$). By observing Fig. 3, it can be noticed that bi-isotropic body with a strong bi-isotropy slightly distrupts the homogeneity of the field. Furthermore, it can be seen that bi-isotropic body with a weak bi-isotropy disrupts homogeneity to the larger extent than the body with a strong bi-isotropy, whereas conducting body disrupts the homogeneity of the field most of all. It has been demonstrated in [10] that a bi-isotropic body increasingly disrupts achieved homogeneity by increasing ε_r for a given $\xi^2/(\varepsilon\mu)$, so that for $\varepsilon_r > 40$ the body begins to behave similarly to the conductive body of the same radius a.

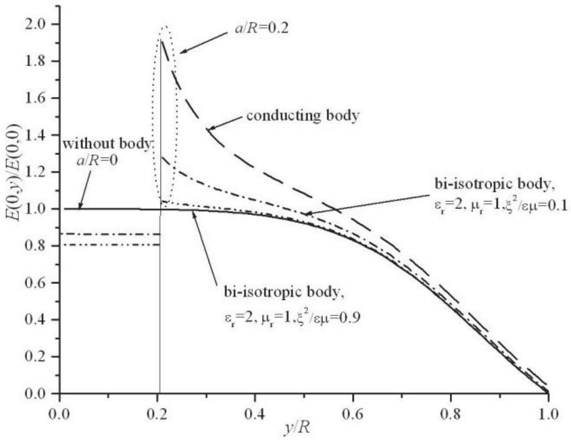
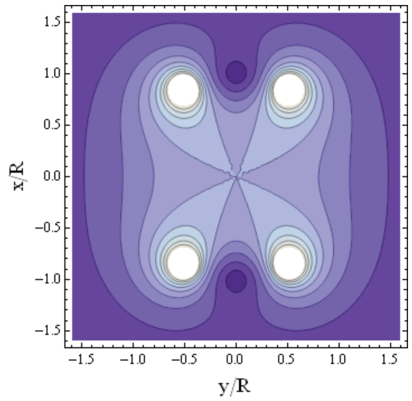


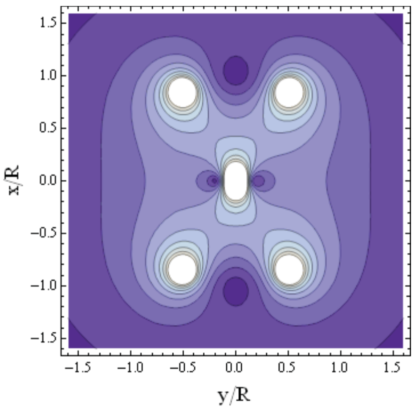
Fig. 3. Normalized electric field along y-axis using one primary cell with and without external bi-isotropic and conducting body,[10].

V. 2D PRESENTATION OF THE UNIFORM ELECTRIC FIELD FOR PRIMARY CELL OF THE FIRST ORDER WITH AND WITHOUT EXTERNAL OBJECTS

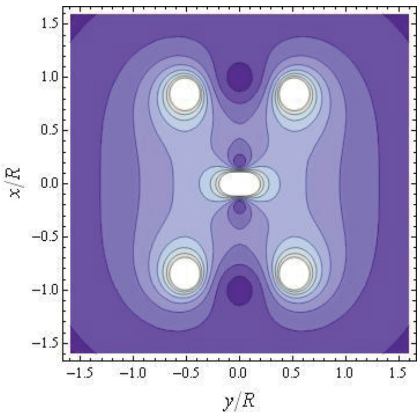
In Fig. 4 it is qualitatively presented electric field of the primary cell without external cylindrical body (Fig. 4.a), with external cylindrical conducting body of radius $a = 0.2R$, Fig. 4.b), and bi-isotropic cylinder inserted as an external body, Fig. 4.c) − Fig. 4.f), by using the *Wolfram's Mathematica* ContourPlot function.



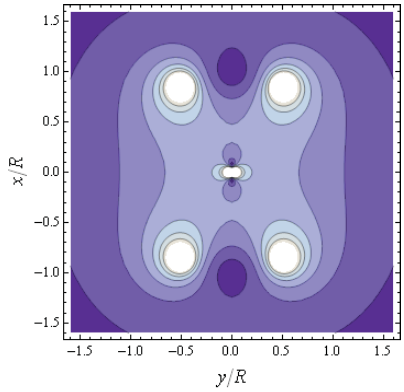
a) Without external body



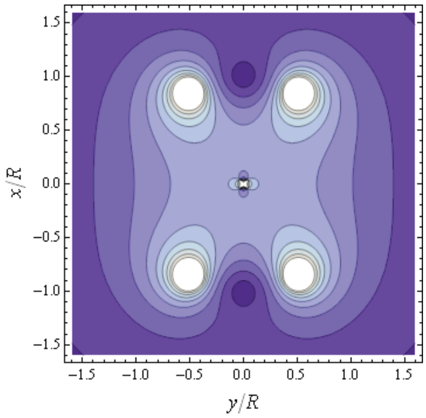
b) With conducting external body



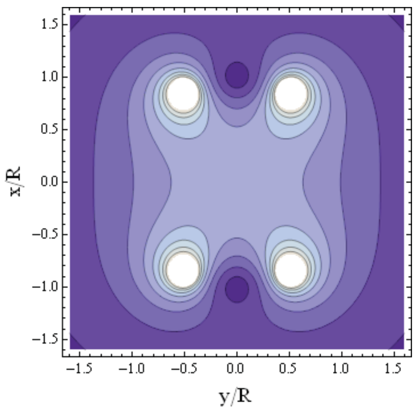
c) Bi-isotropic external body,weak bi-isotropy, $\varepsilon_r = 2,\ \mu_r = 1, \xi^2/\varepsilon\mu = 0.1$



c) Bi-isotropic external body, bi-isotropy of medium strength, $\varepsilon_r = 2,\ \mu_r = 1, \xi^2/\varepsilon\mu = 0.5$



e) Bi-isotropic external body, strong bi-isotropy $\varepsilon_r = 2,\ \mu_r = 1, \xi^2/\varepsilon\mu = 0.9$



f) Bi-isotropic external body, extremely strong bi-isotropy, $\varepsilon_r = 2,\ \mu_r = 2, \xi^2/\varepsilon\mu = 0.99$

Fig. 4. 2D presentation of the uniform electric field for a primary cell a) without body, b) with conducting body and with c) bi-isotropic external body with weak bi-isotropy; e) bi-isotropy of medium strength, e) strong bi-isotropy and f) extremely strong bi-isotropy.

It can be noticed that the external body with stronger bi-isotropy ($\varepsilon_r = 2,\ \mu_r = 1, \xi^2/\varepsilon\mu = 0.9$) less disrupts the homogeneity of the field than the body with weak bi-isotropy ($\varepsilon_r = 2,\ \mu_r = 1, \xi^2/\varepsilon\mu = 0.1$) whereas in the case of extremely strong bi-isotropy, Fig. 4.f), the body practically does not disrupt the homogeneity of the field.

VI. Conclusion

In this paper it is presented the influence of bi-isotropic external body on the achieved homogeneity of electric field generated by using plan-parallel system of the first order (primary cell). The body is modelled using a cylinder whose axis coincides with the axial axis of the system. Field calculations are performed using the Image theorem and we have observed bodies made by bi-isotropic materials of various strengths, conducting body as well as the case without a body.

The results show that the bi-isotropic body with a high degree of bi-isotropy $\xi^2/\varepsilon\mu > 0.8$ disrupts the homogeneity of the field in its vicinity to the least extent, comparing to the bi-isotropic body with a low degree of bi-isotropy $\xi^2/\varepsilon\mu < 0.2$. Furthermore, we have demonstrated that in the case of very strong bi-isotropy, external body does not disrupt homogeneity of the field whereas a conducting body disrupts the homogeneity to the largest extend comparing to the all other observed cases.

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