SYNTHESIS OF SOLUTIONS IN TRANSPORT TESTING IN MATLAB SOFTWARE ENVIRONMENT

Plamen Dqnkov, Stefan Kazakov
"Konstantin Preslavsky" University of Shumen
Republic of Bulgaria

Abstract: ***The task is a special type of linear optimization task. It refers to the allocation of quantities between a source group and a group of destinations in such a way as to minimize the total cost of this allocation.***

Keyword: ***linear, optimization, transport, costs***

I. INTRODUCTION

The solution for the transport task is based on a modified Simplex method for solving the Line Optimization Problem [1,2], called the transport simplicity method. The development of computer technology and the growth of its computing power, especially in the last decade, allows the study of complex objects and phenomena in real-time mode. The Matlab programming environment is a typical representative of this field [6]. In the MATLAB program environment [3,4], commands are available to solve the classical linear limiting task with limitations: linprog, bintprog, but in this case for a transport task, there is no such command. Here is considered a method for solving a Transport task by reducing it to a task of linear optimization.

Formulation of the transport task

The introduction of engineering design techniques into the engineering practice allows to move from the traditional method of creating a model of the developed equipment to modeling with the help of personal computers. [7]

Let m be given by sources Ai, which offer quantities a_i $i = 1..m$, and the destinations Bj are n in number, need for quantities bj, $j = l...n$. Let matrix C be given for the transport cost of one distribution unit from the i source to j destination:

$$C = \begin{cases} c_{11}\ c_{12} \ldots .\ c_{1n} \\ c_{21}\ c_{22} \ldots .\ c_{2n} \\ c_{m1}\ c_{m2} \ldots .\ c_{mn} \end{cases} \qquad (1)$$

Let x_{ij} be unknown units of distribution from i source to j, $i = 1.m, j = 1 \ldots n$.

The goal is to minimize the total cost of allocating these quantities.

Transport task:

Minimize the function of total transport costs:

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (2)$$

subject to restrictions:

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1 \ldots .m, \qquad (3)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1 \ldots .n \qquad (4)$$

and limitations for the non-negative of the unknown variables $x_{ij} > 0$.

The Matrix X of the Unknown is called a transport matrix.

$$X = \begin{pmatrix} x_{11}\ x_{12} \ldots .\ x_{1n} \\ x_{21}\ x_{22} \ldots .\ x_{2n} \\ \ldots \ldots \ldots \ldots . \\ x_{m1}\ x_{m2} \ldots .\ x_{mn} \end{pmatrix} \qquad (5)$$

A transport matrix satisfying the system constraints (2) - (3) is called an acceptable solution to the transport task. For convenience, the prices, availability and needs of the transport task are given in Table 1:

TABLE I
AVAILABILITY AND NEEDS

	B_1	B_2		B_n	a_i
A_1	c_{11}	c_{12}	………………......	c_{1n}	a_1
A_2	c_{21}	c_{22}	…………………..	c_{2n}	a_2
.					
.					
A_m	c_{m1}	c_{m2}	…………………..	c_{mn}	a_m
b_j	b_1	b_2	…………………	b_n	

Specific features of the transport task

If the unknown variables are treated as a pillar vector derived from the X matrix rows, then the Transport task is recorded as a Line Optimization task:

$$\min Z = c_{11}x_{11} + c_{12}x_{12} + \ldots .c_{1n}x_{1n} + c_{21}x_{21} + c_{22}x_{22} + \ldots + c_{m1}x_{m1} + c_{m2}x_{m2} + \ldots c_{mn}x_{mn} \qquad (6)$$

subject to restrictions:

$$\begin{pmatrix} 1\ 1 \ldots 1\ 0\ 0 \ldots\ 0\ 0\ 0 \ldots\ 0 \\ 0\ 0 \ldots 0\ 1\ 1 \ldots 1\ 0\ 0 \ldots\ 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0\ 0 \ldots 0\ 0\ 0 \ldots\ 0\ 1\ 1 \ldots\ 1 \\ 1\ 0 \ldots 0\ 1\ 0 \ldots\ 0\ 1\ 0 \ldots\ 0 \\ 0\ 1 \ldots 0\ 0\ 1 \ldots\ 0\ 0\ 1 \ldots\ 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0\ 0 \ldots 0\ 0\ 0 \ldots\ 0\ 0\ 0 \ldots\ 0 \\ 0\ 0 \ldots 1\ 0\ 0 \ldots 1\ 0\ 0 \ldots\ 1 \end{pmatrix} \begin{pmatrix} x_{11} \\ \vdots \\ x_{1n} \\ . \\ x_{m1} \\ . \\ . \\ x_{mn} \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \qquad (7)$$

and limitations of non-negativity $h_{ij} \geq 0$.

The transport task has the following specific features:

- The task is to minimize the target function;
- All restrictions are of the "equality" type;
- The matrix elements of the coefficients before the unknowns in the limitations are zeros or units;
- In each pillar of the matrix of coefficients in front of the unknown in the limits there are two units, ie each variable is involved only in two equations with a coefficient unit.

Theorem 1: The transport task (1) - (3) has an acceptable solution then and only when the class is fulfilled:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (8)$$

Theorem 2: The rank of the equation system (2) equals $m + n\text{-}1$.

Consequence: Any acceptable solution X of the transport task contains at most $m + n\text{-}1$ positive components x_{ij}. The remaining components are zero.

Definition 1: An acceptable solution to the transport task, in which there are exactly $m + n \text{-}1$ positive components h_{ij}, is called unconditionally.

Definition 2: An acceptable solution to the transport task in which there are less than $m + n\text{-}1$ positive components h_{ij} is called degenerate.

Types of transport tasks

• Balanced and unbalanced transport tasks

According to whether the condition of Theorem 1 is met, transport tasks are two types: balanced and unbalanced. If the condition (4) is met, the transport task is called a balanced one, or in the opposite case:

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \qquad (9)$$

the task is unbalanced. There are two possible scenarios:

1. Offering is greater than demand. In this case, a fictitious destination (user) is entered, with zeros for transport costs and the required amount - the difference between total stocks and needs:

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j \qquad (10)$$

2. Demand is greater than supply. In this case, a fictitious source (manufacturer) is introduced, the transport costs are zero and the quantity produced - the difference between total needs and stocks:

$$\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i \qquad (11)$$

• A blocked transport task (with bans)

When there is no transport link between a source and destination, the transport task is "blocked shipments". In this case, a very positive number $M >> 0$ (large M method) is assumed for transport expense.

Another case where the use of the M-method is required is when the Transport task is unbalanced and the user B_K is required to obtain the requested amount in full. In this case, after introducing a fictitious source A_s, M $(M >> 0)$ is applied to transport expense c_{sk}, ensuring that B_k obtains the necessary quantities only from real sources.

An analogous situation occurs when the task is unbalanced, and in addition a producer wants to distribute all of his available quantity. Then, after introducing a dummy user (destination) B_K, M $(M >> 0)$ is applied for transport expense c_{sk}, thus ensuring that A_s can distribute its available quantities only to real users (destinations).

Program implementation

The Transport Troubleshooting Program is a MATLAB function with the following arguments:

Entry arguments:

• Matrix C of transport costs;

• Vector a - quantities offered;

• Vector b - Queries;

Output arguments:

• Z - value of the target function;

• X - matrix of the solution of the Transport task.

After input values are entered, the command is set:

[Z,X]=transport(C,a,b)

In the event of an unbalanced transport task, the program automatically balances it.

When solving a blocked transport task(Fig. 1, Fig. 2), the corresponding element in transport matrix C is assigned a large enough number, of the order of*1e+009*.

```
function [Z,X]=transport(C,a,b)
[m,n]=size(C);
[ma,na]=size(a);
[mb,nb]=size(b);
if ma<na
    a=a';
    [ma,na]=size(a);
end
if mb<nb
    b=b';
    [mb,nb]=size(b);
end
if (ma~=m)|(mb~=n)
    error('некоректен број елементи на a или b')
end
suma=sum(a);
sumb=sum(b);
Iflag=0;
if sumb>suma
```

Fig. 1. Blocked transport task

```
    Iflag=1;
    nulb=zeros(1,length(b));
    C=[C;nulb];
    a=[a;sumb-suma];
end
if sumb<suma
    Iflag=2;
    nula=zeros(length(a),1);
    C=[C,nula];
    b=[b;suma-sumb];
end
B=[a;b];
[m,n]=size(C);
f=reshape(C',m*n,1);
A1=zeros(m,m*n);
A2=eye(n);
A3=eye(n);
for k=1:m
    A1(k,((k-1)*n+1):(k*n))=1;
end
for k=1:(m-1)
    A2=[A2,A3];
end
A=[A1;A2];
lb=zeros(m*n,1);
options = optimset('LargeScale', 'off', 'Simplex', 'on');
[X,Z]=linprog(f,[],[],A,B,lb,[],[],options);
X=reshape(X,n,m);X=X';
if Iflag==1
    X(end,:)=[];
elseif Iflag==2
    X(:,end)=[];
end
```

Fig. 2. Blocked transport task

Numerical experiments

The program for solving a transport task has been tested multiple times with different numerical data. Here are three examples of balanced, unbalanced tasks and a Transport task with blocked shipments (Fig. 3, Fig. 4).

No.1	B_1	B_2	B_3	a_i
A_1	4	4	2	30
A_2	5	3	1	20
A_3	3	6	4	40
b_j	45	35	10	

No.2	B_1	B_2	B_3	a_i
A_1	1	2	4	80
A_2	4	2	6	70
A_3	6	1	3	50
b_j	50	80	50	

No.3	B_1	B_2	B_3	a_i
A_1	20	45	40	60
A_2	–	30	25	60
A_3	–	15	20	30
b_j	20	10	30	

```
>> Пример 1: балансирана транспортна задача
>> C=[4 4 2;5 3 1;3 6 4];
>>a=[30 20 40];
>> b=[45 35 10];
>> [Z,X]=transport(C,a,b)
Z =
   280

X =
     5    25     0
```

Fig. 3. Transport task with blocked shipments

```
     0    10    10
    40     0     0
```

```
>> Пример 2: небалансирана транспортна задача
>> C=[1 2 4;4 2 6;6 1 3];
>> a=[80 70 50];
>> b=[50 80 50];
>> [Z,X]=transport(C,a,b)
Z =
   360
X =
    50    10     0
     0    70     0
     0     0    50
```

```
>> Пример 3: небалансирана транспортна задача с блокирани превози
>> C=[20 45 40;1e9 30 25;1e9 15 20];
>>a=[60 60 30];
>> b=[20 10 30];
>> [Z,X]=transport(C,a,b)
Z =
        1200
X =
    20     0     0
     0     0    10
     0    10    20
```

Fig. 4. Transport task with blocked shipments

II. CONCLUSION

The Transport Problem Solving Program is widely applied in science and practice and is successfully applied to the study of Optimization Methods.

REFERENCES

[1] Караколева, Ст. Investigation of Operations, Part 1, Angel Kanchev University, 2002.

[2] Pavlov, V. Mathematical Methods in Economics, "Angel Kanchev" University, 2006.

[3] Tonchev, Y., Matlab 7, Part 3, Technique, 2009.

[4] Matlab Optimization Toolbox documentation: www.mathworks.com/access/helpdesk/help/toolbox/optim/

[5] Образовательнь математический сайт http://matlab.exponenta.ru/optimiz/index.phply applied to the study of Optimization Methods.

[6] Lalev H., Zhelev S., Tsankov Ts. Analyzing and monitoring the frequency response of the circuit. Scientific Session of the “Vasil Levski” National Military University, “Artillery, Air Defense and CIS” Faculty, Shumen, ISSN 1314-1953 (in Bulgarian).

[7] Lalev H., Tsankov Ts. Synthesis of simulink modules. Scientific Conference, dedicated to the 105th anniversary of the birth of the pioneers of computing John Atanasoff and John von Neumann, Shumen, 2008, ISBN 978-954-577-540-6. (in Bulgarian).

[8] Антон Антонов, Транспортна логистична система за екологичен морфологичен анализ на отпадъци, Университетско издателство "Епископ Константин Преславски", гр. Шумен 2016

[9] Антон Антонов, Логистични методи и транспортна схема в преработката на био-разградими отпадъци. Third international scientific conference science, education, innovation, shumen, Bulgaria 21 - 23 MAY 2014. Vol. 2, 2014

The present study is conducted with the financial help of Project № РД-08-89/31.01.2019г., fund “Scientificstudies” of Konstantin Preslavsky University of Shumen.