# Analysis of the precision of neural network designed with analog methods 

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#### Abstract

By the solution of many problems,arised during the research process, a lot of time and means can be saved by means of modeling of neural networks. The analogue realization of neural network is usually prefered, because: -they are more similar to biological neural networks, -they are simple and can easy be realized, -they don't lose the parallel processing of information. In this paper results of two-layer neural networks, modeled by means of analogue equivalent scheme, are presented. Two-layers neural networks,described here, are compared with single-layer neural networks of the same type, considered in [2] and [3].

In each area and in every systems the accuracy is very important property, especially if we talk about neural networks and their precision.In the presented work the dependence between the step of the input signal,the number of layers and the precision have been examined.


Keywords—Neural network, Analogue methods, Analogue models.

## I. Introduction

Usually analogue neural networks are preferred, because they are more similar to biological neural networks, it is easier to be realized and they don't lose the parallel processing of information. It is well-known that by the increase of the numbers of layers the decision function becomes more complicated. A single-layer neural network can't decide a more complicated problem. That is the reason, the influence of the number of the layers upon the accuracy to be investigated. The step of the input signals is also very important.

## II. Neural cell mathematical model

In the discussed paper a two-layer neural network with two neurons in the input layer, two neurons in the hidden layer and one neuron in the output layer, is presented fig. (1). In [2] depicts a model of neurone, presented as equivalent electrical circuit. The circuit consist of: independent voltage source $E$, independent current source $I$, capacitor $C$, resistors $R_{z, j}$ and $R_{y, j}$, linear voltage controlled current source and nonlinear voltage controlled current source.

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Fig. 1. Two-layer neural network architecture.

The linear current sources are controlled by means of the voltages of the neighbour cells. The respective relations are:

$$
\begin{align*}
& I_{x y, i j}=B_{i j} \cdot U_{x, i j}  \tag{1}\\
& I_{y x, i j}=A_{i j} \cdot U_{y, i j} \tag{2}
\end{align*}
$$

where the coefficients $A_{i j}$ and $B_{i j}$ are denoted respectively as feedback operator and control operator. These coefficients represent the relatively weight of the connection between $i$ and $j$ neurons. The voltage, obtained at the output of neuron $i$ controls the current source of neuron $j$, denoted as $U_{i j}$. The indexes $x$ and $y$ are denoting respectively the input or the output.

The signal, obtained at the output of the cell, is passed to the nonlinear voltage controlled current source. It's characteristic is described by the equation:

$$
\begin{equation*}
I_{y, j}=\frac{1}{R_{y, j}} \cdot f\left(U_{z, j}\right) \tag{3}
\end{equation*}
$$

where:

$$
\begin{equation*}
f\left(U_{z, j}\right)=\frac{1}{1+e^{-U_{z, j}}} \tag{4}
\end{equation*}
$$

The processes in the body of cell with $n$ inputs and $m$ outputs are presented by means of the following equation 5:

$$
\begin{equation*}
C \frac{d U_{z, j}(t)}{d t}=-\frac{1}{R} \cdot U_{z, j}(t)+\sum_{i=1}^{n} B_{i j} U_{x, i j}+\sum_{i=1}^{m} A_{i j} U_{y, i j}+I, \tag{5}
\end{equation*}
$$

$U_{z, j}(t)$ is the neuron body voltage.
If the output voltage of the neural cell is to be found, it is necessary $U_{z, j}(t)$. to be known. After that the equation 5 has been solved, the voltage of the body of the neural cell becomes:

$$
\begin{equation*}
U_{z, j}(t)=R \cdot\left(e^{\frac{t}{R C}}-1\right) \cdot\left(\sum_{i=1}^{n} B_{i j} U_{x, i j}+\sum_{i=1}^{m} A_{i j} U_{y, i j}+I\right) \tag{6}
\end{equation*}
$$

Assuming:

$$
\begin{equation*}
\phi_{T}=R \cdot\left(e^{\frac{t}{R C}}-1\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi_{I}=\left(\sum_{i=1}^{n} B_{i j} U_{x, i j}+\sum_{i=1}^{m} A_{i j} U_{y, i j}+I\right) \tag{8}
\end{equation*}
$$

we can write:

$$
\begin{equation*}
U_{z, j}(t)=\phi_{T} \cdot \phi_{I} \tag{9}
\end{equation*}
$$

Finally after the transforms the output voltage becomes:

$$
\begin{equation*}
U_{y, j}=f\left(U_{z, j}\right)=\frac{1}{1+e^{-\phi_{T} \cdot \phi_{I}}} . \tag{10}
\end{equation*}
$$

From Fig. 2 to Fig. 6 are shown the forms of the decision function by step of the input signal varying from $0,5 \mathrm{~V}$ to $0,01 \mathrm{~V}$ in the neural cell mathematical model.


Fig. 2. Decision function by input signal step $0,5 \mathrm{~V}$.


Fig. 3. Decision function by input signal step $0,2 \mathrm{~V}$.


Fig. 4. Decision function by input signal step $0,1 \mathrm{~V}$.


Fig. 5. Decision function by input signal step $0,05 \mathrm{~V}$.


Fig. 6. Decision function by input signal step $0,01 \mathrm{~V}$.

## III. A SINGLE-LAYER NEURAL NETWORK ANALYSIS

For single-layer neural network we can write:

$$
\begin{equation*}
U_{z, 3}(t)=R \cdot\left(e^{\frac{t}{R C}}-1\right) \cdot\left(B_{13} U_{x, 13}+B_{23} U_{x, 23}+I\right) \tag{11}
\end{equation*}
$$

From Fig. 2 to Fig. 6 are shown the forms of the decision function by step of the input signal varying from $0,5 \mathrm{~V}$ to $0,01 \mathrm{~V}$ in the single-layer neural network.

## IV. Two-LAYER NEURAL NETWORK ANALYSIS

In this way we can define the output voltage of neural network, presented on fig. 1. For the neuron body in the output layer the relation:

$$
\begin{equation*}
U_{z, 5}(t)=R \cdot\left(e^{\frac{t}{R C}}-1\right) \cdot\left(B_{35} U_{x, 35}+B_{45} U_{x, 45}+I\right) \tag{12}
\end{equation*}
$$

is valid. A perceptron is analyzed, therefore $A_{i j}=0$. In 12 is substituted with:

$$
\begin{equation*}
U_{x, 35}=U_{y, 3}=f\left(U_{z, 3}\right)=\frac{1}{1+e^{-U_{z, 3}}}, \tag{13}
\end{equation*}
$$

where:

$$
\begin{equation*}
U_{z, 3}(t)=R \cdot\left(e^{\frac{t}{R C}}-1\right) \cdot\left(B_{13} U_{x, 13}+B_{23} U_{x, 23}+I\right)=\phi_{T} \cdot \phi_{3} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
U_{x, 45}=U_{y, 4}=f\left(U_{z, 4}\right)=\frac{1}{1+e^{-U_{z, 4}}} \tag{15}
\end{equation*}
$$

where:


Fig. 7. A single-layer neural network architecture.


Fig. 8. Decision function by input signal step $0,5 \mathrm{~V}$.
$U_{z, 4}(t)=R \cdot\left(e^{\frac{t}{R C}}-1\right) \cdot\left(B_{14} U_{x, 14}+B_{24} U_{x, 24}+I\right)=\phi_{T} \cdot \phi_{4}$
Then:

$$
\begin{equation*}
U_{x, 35}=\frac{1}{1+e^{-\phi_{T} \cdot \phi_{3}}}, \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
U_{x, 45}=\frac{1}{1+e^{-\phi_{T} \cdot \phi_{4}}} . \tag{18}
\end{equation*}
$$

therefore:

$$
\begin{equation*}
U_{z, 5}(t)=\phi_{T} \cdot\left(B_{35} \frac{1}{1+e^{-\phi_{T} \cdot \phi_{3}}}+B_{45} \frac{1}{1+e^{-\phi_{T} \cdot \phi_{4}}}+I\right) \tag{19}
\end{equation*}
$$

The output voltage of neural network is:

$$
\begin{equation*}
U_{y, 5}=f\left(U_{z, 5}\right) \tag{20}
\end{equation*}
$$

when we substitute in equation 19 it can be solved:

$$
\begin{equation*}
U_{y, 5}=\frac{1}{1+e^{-\phi_{T} \cdot\left(B_{35} \frac{1}{\left.1+e^{-\phi_{T} \cdot \phi_{3}}+B_{45} \frac{1}{1+e^{-\phi_{T} \cdot \phi_{4}}}+I\right)}\right.} \text {. }} \tag{21}
\end{equation*}
$$

In this way the output voltage of the neural network, presented on fig. 1, can be analytical determined.

From Fig. 2 to Fig. 6 are shown the forms of the decision function by step of the input signal varying from $0,5 \mathrm{~V}$ to $0,01 \mathrm{~V}$ in the two-layer neural network.


Fig. 9. Decision function by input signal step 0,2V.


Fig. 10. Decision function by input signal step $0,1 \mathrm{~V}$.

## V. Conclusion

* Investigations, concerning the analogue neural networks and their's application in signal processing have been done.
* The decision functions have been analytical developed.
* By the decrease of the step value of the input signal a better approach of the obtained form of the decision function to the ideal form has been observed.
* The increase of the number of layers leads to an increase of the slope of the decision function characteristic.
* The obtained simulation results confirm the expected theoretical dependencies.


## REFERENCES

[1] T. Kirova, Neural Networks. Main arhitectures and learning algorithm, Softeh. Sofia 1995.
[2] A. Bekiarsky, L. Docheva, "Neural Network design through analog methods" Communication, electronic computer systems, Vol. 1,pp 112-117. Sofia 2000
[3] Docheva L., A. Bekiarsky, "Neural Networks modeling through analog equivalent scheme", Energy and information systems and technologies, Vol. 2, pp 471-475. Bitola 2001.
[4] L.O. Chua and L. Yang, "Cellular Neural Networks: Theory", IEEE Transactions Circuit and systems, CAS-35, pp. 1257-1272, 1988.
[5] L.O. Chua and L. Yang, "Cellular Neural Networks: Applications", IEEE Transactions Circuit and systems. CAS-35, pp. 1257-1272,1988..


Fig. 11. Decision function by input signal step $0,05 \mathrm{~V}$.


Fig. 12. Decision function by input signal step $0,01 \mathrm{~V}$.


Fig. 13. Decision function by input signal step $0,5 \mathrm{~V}$.


Fig. 14. Decision function by input signal step $0,2 \mathrm{~V}$.


Fig. 15. Decision function by input signal step $0,1 \mathrm{~V}$.


Fig. 16. Decision function by input signal step $0,05 \mathrm{~V}$.


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