Direct Detection Receiver Decision Statistics in Optically Amplified Communication Systems

Ivan B. Djordjevic and Bane V. Vasic

Abstract-- **An approach for finding the exact direct detection receiver decision statistics (that is the probability density function at the decision circuit input) is proposed. It is independent on optical and electrical filter choice, pulse shape and on modulation scheme. The additive noise composed of the amplifier spontaneous emission, multi-path interference and other additive noise sources is modeled as a stationary process with an arbitrary autocorrelation function. The comparison with the Gaussian approximation of decision statistics is given and various aspects of the application of the proposed method in long haul-haul systems are discussed.**

Index Terms-- **Optically amplified communication systems, Direct detection receiver statistics, Statistical communication theory**

I. Introduction

Additive white Gaussian noise (AWGN) is commonly used to describe the amplifier spontaneous emission (ASE) noise [1-5], [11-12]. In contrast to terrestrial communication links, a typical undersea fiber communications system operates at a signal power below 0 dBm (even less than -3 dBm) per channel for Nx10 Gb/s systems, with relatively short amplifiers spacing (less then 45 km) and properly chosen dispersion compensated fiber pairs to minimize the influence of the fiber nonlinearities and dispersion [1]. The validity of the AWGN fiber channel model in considering ASE noise was confirmed for such applications through experiments [1]. However, in most terrestrial optical communication systems the AWGN assumption is not completely accurate. For example in terrestrial long-haul wavelength division multiplexing (WDM) systems, due to the interaction of fiber nonlinearities and dispersion, the WDM carriers can act as a set of pumps, and the amplifier spontaneous emission (ASE) noise spectral components can be selectively amplified. In other words the noise enhancement is much higher in certain spectral regions. This gain introduced by these effects is known as a parametric gain [5]. In this case the ASE noise is neither white nor Gaussian. Moreover, an optical filter colors the white (ASE) noise in every amplifier stage, even in the absence of parametric gain. Apart from ASE noise, in long-haul communication systems, especially those with Raman amplifiers, multipath interference (MPI) becomes an important factor in performance degradation. In a Raman amplifier based long-haul communications MPI is even more important factor in performance degradation. Due to the fact that double Rayleigh-back scattering (DRBS), which is a main source of MPI, occurs in an optical fiber due to small inhomogeneities or microscopic variations in the refractive index, that reflections may occur on splices and poor connectors [6] and the fact that these sources are independent of each other, MPI can be modeled as a stationary Gaussian process with an autocorrelation function determined from the measured power spectral density (PSD) function.

Therefore, in order to characterize the receiver performance and to design an optimal receiver it is of great importance to determine the statistics of samples at the input of the decision device. In this paper we tackle the problem of determining such statistics in the presence of colored Gaussian noise at the receiver input and in the presence of intersymbol interference (ISI) caused by filtering. A number of models were proposed recently [2-5]. Unfortunately, all these models lack generality. They are either restricted to a specific modulation scheme or applicable to a narrow class of optical/electrical filters. Some of them do not even consider the influence of optical filter [11-12]. The problem of properly modeling of direct detection receiver is still an open issue.

Although the formal procedure for finding the decision statistics is well-known [4-5], [7] the probability density function (PDF) of decision statistics has been determined only a rectangular/Lorentzian optical filter transfer function and for integrate-and-dump electrical filter [4].

We propose a universal method to determine the decision statistics independent of the optical and electrical filter choice, the pulse shape or modulation scheme. The ASE noise is modeled as a stationary process with an arbitrary autocorrelation function. The model takes into account the intersymbol interference as well. The proposed method is also applicable to other types of additive noises that accompany the ASE noise, such as multi-path interference. To compare the proposed method for finding PDF with frequently used Gaussian approximation of decision statistics, we use the skewness and the kurtosis coefficients, defined in [8-10]. The proposed method is illustrated for the case when the optical filter is modeled as a super-Gaussian filter and the electrical filter is modeled as a Gaussian filter. In this case the PDF of decision statistics can be determined in a closed form.

II. Model Description

A typical direct detection receiver, which follows an amplifier chain, consists of a polarization filter, an optical filter, a photodiode, an electrical filter, a sampler and a decision circuit, as shown in Fig.1. The electrical field in fiber at the optical filter input can be written as

$$r(t) = s(t) + n(t),\ s(t) = \sum_{n=-\infty}^{\infty} \sqrt{b_n P}\, p_n\left(t - nT_b\right), \tag{1}$$

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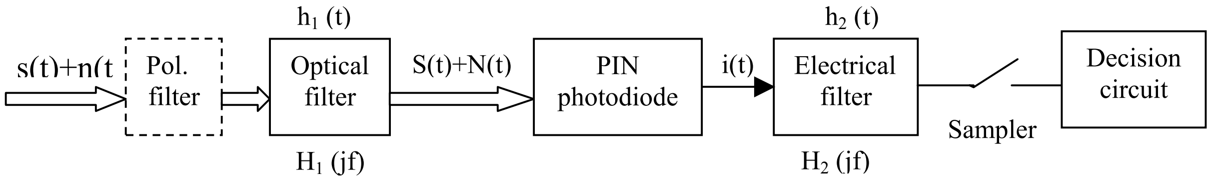

Fig. 1 Block scheme of receiver following an amplifier chain

where $s(t)$ is the ouput of the chain of (optical) amplifiers, $p_n(t)$ is the n^{th} bit pulse shape, P the peak power, and b_n is the user symbol $b_n \in \{r,1\}$, with r being the extintion ratio, $0 \le r < 1$. The additive noise component $n(t)$ is assumed to be a zero-mean wide sense stationary (ASE, MPI, etc.) with the autocorrelation function $R_n(\tau)$. $r(t)$, $s(t)$ and $p_n(t)$ are in fact the complex envelopes of corresponding analytical signals [4-5], [7].

Let $h_1(t)$ and $h_2(t)$ be optical filter and electrical filter impulse responses, respectively. $h_2(t)$ can be considered as the impulse response of whole receiver electronics, while $h_1(t)$, as an inverse Fourier transform of the demultiplexer (e.g., the AWG) transfer function of the observed channel. Since the optical filter is a linear susbsystem, there is no interaction between signal and noise, and both optical filter output signal $S(t)$ and noise $N(t)$ can be written as a convolution of the impulse response and corresponding filter input, that is

$$S(t) = \int_{-\infty}^{\infty} h_1(\tau) s(t-\tau) d\tau \, , N(t) = \int_{-\infty}^{\infty} h_1(\tau) n(t-\tau) d\tau \, . \quad (2)$$

The electrical filter output $I(t)$ is a convolution of the photodiode current $i(t)$

$$i(t) = |S(t) + N(t)|^2 \quad (3)$$

and the electrical filter impulse response $h_2(t)$

$$I(t) = \int_{-\infty}^{\infty} h_2(\tau) |S(t-\tau) + N(t-\tau)|^2 \, d\tau =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(u,v)[s(t-u)+n(t-u)][s^*(t-v)+n^*(t-v)]dudv \quad (4)$$

(Photodiode responsivity is omitted without loss of generality).

$$K(u,v) = \int_{-\infty}^{\infty} h_1(u-\tau) h_2(\tau) h_1^*(v-\tau) d\tau \quad (5)$$

is the kernel of the transformation (filtering-photodetection-filtering). As $K^*(v,u) = K(u,v)$ the kernel is symmetric. Furthermore, since the kernel is continuous, it can be expanded into a series using orthogonal functions

$$K(u,v) = \sum_{i=1}^{\infty} \frac{\phi_i(u)\phi_i^*(v)}{\lambda_i} \, , \quad (6)$$

where λ_i and $\phi_i(x)$ are the eigenvalues and eigenfunctions satisfying the following equaiton

$$\phi_i(x) = \lambda_i \int_{-\infty}^{\infty} K(x,y)\phi_i(y) dy, \quad x \in \{u,v\}. \quad (7)$$

Note that we use the inverse kernel definition for relation (6) [7]. Substituting (6) into (4) the electrical filter output becomes

$$I(t) = \sum_{i=1}^{\infty} \frac{|s_i(t) + n_i(t)|^2}{\lambda_i} \, , \quad (8)$$

where

$$s_i(t) = \int_{-\infty}^{\infty} s(t-\tau)\phi_i(\tau) d\tau \, , \; n_i(t) = \int_{-\infty}^{\infty} n(t-\tau)\phi_i(\tau) d\tau \, . \quad (9)$$

A set of the orthonormalized functions is chosen in such a way that the coefficients n_i are uncorrelated [7], i.e., $m_1(n_i^2) = R_n(0)$, with m_1 being the first-order moment of the noise process [7].

Following the similar procedure described in [4-5], [7], we can write the characteristic function of the electrical filter output signal as follows

$$C_I(j\Omega) =$$

$$= \prod_{i=1}^{\infty} \frac{1}{1-(2j\Omega R_n(0)/\lambda_i)} \exp\left[\frac{|s_i|^2}{2R_n(0)} \frac{2j\Omega R_n(0)}{\lambda_i - 2j\Omega R_n(0)} \right] . \quad (10)$$

The natural logarithm of the characteristic function can be expanded using Taylor series formula

$$\ln[C_I(j\Omega)] = \sum_{n=1}^{\infty} \frac{\kappa_n}{n!} (j\Omega)^n \, , \quad (11)$$

where the coefficients in the expansion are

$$\kappa_n(t) = (-j)^n \frac{d^n}{d\Omega^n} \{\ln[C_I(j\Omega)]\}\Big|_{\Omega=0} = (2R_n(0))^n n! \cdot$$

$$\cdot \left[\frac{1}{R_n(0)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(t-u) K^{(n)}(u,v) s^*(t-v) dudv + \right.$$

$$\left. + \frac{1}{n} \int_{-\infty}^{\infty} K^{(n)}(u,u) du \right] . \quad (12)$$

$K^{(n)}(u,v)$, the n^{th} order kernel, is defined iteratively by

$$K^{(n)}(u,v) = \int_{-\infty}^{\infty} K^{(n-1)}(u,w) K(w,v) dw, \quad n \ge 2$$

$$K^{(1)}(u,v) \equiv K(u,v) \, . \quad (13)$$

Finally, the probability density function of decision statistics $w(x)$ can be found using the following definition expression

$$w(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} C_I(j\Omega)\exp[-j\Omega x]d\Omega\,. \qquad (14)$$

Although the derivation of the model is certainly not straightforward, it is conceptually very simple. The difficulties arise only in numerical calculations of certain integrals.

If the optical filter is modeled as a super-Gaussian filter, and electrical filter modeled as a Gaussian filter, and NRZ signals are observed, the coefficient of Taylor expansion can be determined in a closed form

$$\kappa_n \approx \frac{\left(2bR_n(0)B_1\right)^n (n-1)!}{\left(\sqrt{b^2+2}+b\right)^n - \left(\sqrt{b^2+2}-b\right)^n}\cdot$$

$$\cdot\left[1+\frac{P_{av}}{2R_n(0)B_1}\sqrt{\frac{\left(\sqrt{b^2+2}+b\right)^n - \left(\sqrt{b^2+2}-b\right)^n}{\left(\sqrt{b^2+2}+b\right)^n + \left(\sqrt{b^2+2}-b\right)^n}\frac{\sqrt{b^2+2}}{b}}\right] \qquad (15)$$

where B_1 is the equivalent optical filter bandwidth

$$B_1 = \int_{-\infty}^{\infty}\left|H_1(j\omega)\right|^2 d\omega\,, \qquad (16)$$

with $H_1(j\omega)$ being the optical filter transfer function. b is the equivalent electrical filter bandwidth

$$B_2 = \int_{0}^{\infty}\left|H_2(j\omega)\right|^2 d\omega \qquad (17)$$

over the equivalent optical filter bandwidth, with $H_2(j\omega)$ being now the electrical filter transfer function. (P_{av} is the observed bit average optical power).

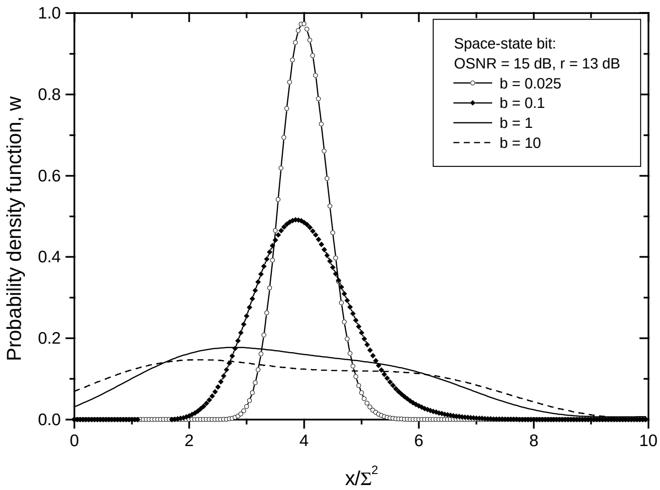

Fig. 2. Probability density function of decision statistics for a space state bit

III. NUMERICAL RESULTS

To illustrate the proposed model, a PDF of the decision circuit input noise for a space-state bit is shown in Fig. 2. The curves in Fig. 2 are obtained under the assumption that the optical filter is modeled as a super-Gaussian filter, that the electrical filter is modeled as Gaussian and that the signal is NRZ. It is evident that for a space-state bit, the PDF curve is like Gaussian just as in the case when the ratio $b = 2B_2 / B_1$ is close to zero (B_2-electrical filter bandwidth, B_1-optical filter bandwidth).

To assess the validity of Gaussian approximation of decision statistics, we define, similarly as in [8-10], the skewness (asymmetry coefficient) α and the kurtosis (flatness coefficient) β respectively as

$$\alpha = M_3 / \sqrt{M_2^3} \quad \text{and} \quad \beta = M_4 / M_2^2 - 3\,, \qquad (18)$$

where M_n is the central moment of the nth order [7]. The skewness describes the symmetry ($\alpha = 0$) or asymmetry ($\alpha \neq 0$) of PDF curve with respect to the center mass axis, while the kurtosis describes whether the PDF curve is more narrow with higher peak ($\beta > 0$) than Gaussian distribution ($\beta = 0$) or vice versa ($\beta < 0$). The skewness and the kurtosis versus optical signal-to-noise ratio (OSNR, defined in B_1 bandwidth) are shown in Figs. 3-4. For the mark-state bit and OSNR grater than 20 dB the flatness coefficient is close to zero for any electrical-filter bandwidth-to-optical filter bandwidth ratio (*b*), while the coefficient of asymmetry tends to zero only when *b* converges to zero. For the space-state bit the PDF is always different from Gaussian. Strictly speaking the decision statistics is never Gaussian.

For a typical the electrical filter bandwidth of 0.65 R_b (R_b is the bit rate) and optical filter bandwidth region ($2R_b$, $5R_b$) the ratio $2B_2/B_1$ is in an interval (0.26, 0.6) and the decision statistics is not Gaussian.

The cumulative distribution function for a mark-state bit for different values of ratio *b* is shown in Fig. 5, while the bit-error rate is shown in Fig. 6. For the bit-error rate (BER) of 10^{-12} and a typical value of ratio b = 0.6 the approximation error obtained when the exact PDF is approximated by Gaussian with the mean value and the standard deviation determined from exact PDF is 0.45 dB.

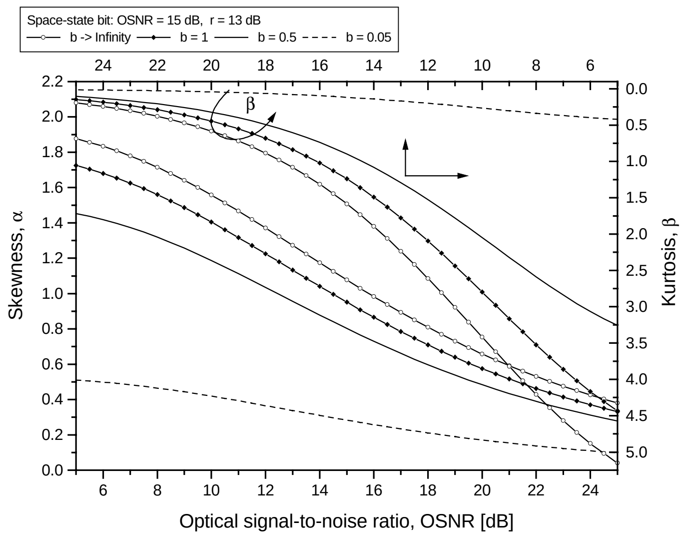

Fig. 3. Skewness and kurtosis for a space-state bit

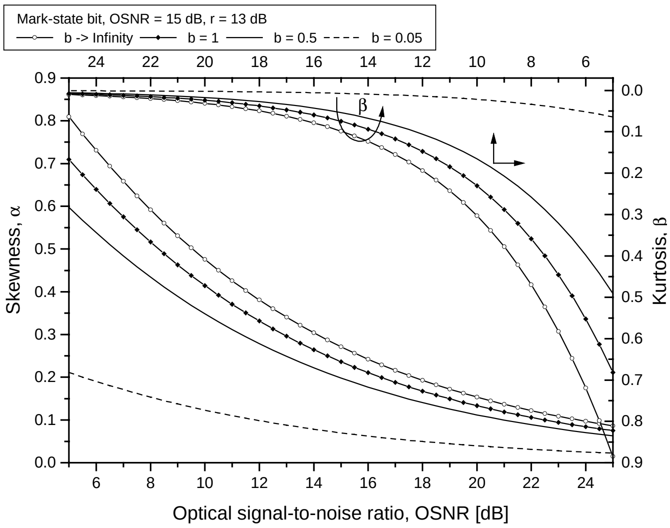

Fig. 4. Skewness and kurtosis for a mark-state bit

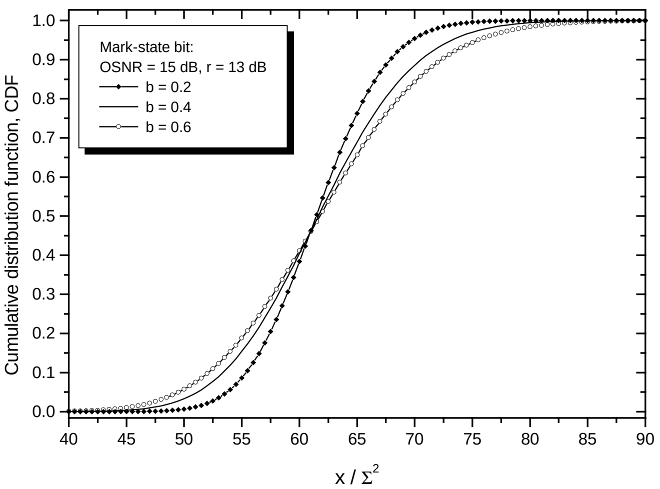

Fig. 5. Cumulative distribution function for a mark-state bit

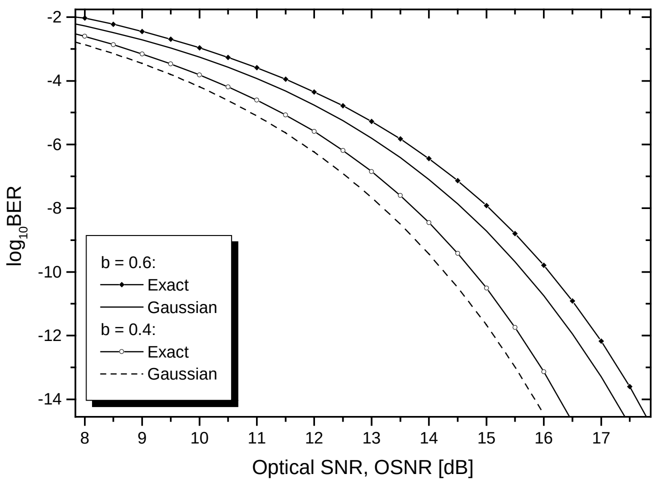

Fig. 6. Bit-error rate vs. optical signal-to-noise ratio

IV. CONCLUSION

An advanced method to determine the exact decision statistics of direct detection receiver is proposed in this paper. It is independent of the choice of optical and electrical filters and independent of the modulation scheme. The ASE noise is modeled as a stationary process with the autocorrelation function that is obtained experimentally. The comparison with the Gaussian approximation of decision statistics is given and limits of its application are pointed out. The proposed method is also applicable for other type of additive noises that accompany the ASE noise, MPI in long-haul communications for example, that can be modeled as a stationary normal process with arbitrary autocorrelation function.

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