The Data Crosstalk as a Performance Limiting Factor of IM-DD Systems

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Abstract- **In this paper, the signal propagation along the fiber, in either absence or presence of the crosstalk interference appearing at different places along the fiber, for both dispersive regime cases, is considered. The optical signal which appears at transmitter output has the envelope in Gaussian form. The crossltalk appears at the transmitter output or along the fiber. The pulse shape at the receiver input is determined using Schrödinger equation. The noise sources are the photodetector and resistance in the receiver. The bit error probability of intensity modulation and direct detection (IM-DD) system in the presence of the crosstalk, quantum and thermal noise is determined.**

Keywords- **Optical fiber, Data crosstalk interference, Interferometric noise, nonlinear Schrödinger equation, Bit error probability.**

I. INTRODUCTION

In the field of fiber communications, IM-DD techniques are popular and have been used widely for high bit-rate data transmission. This is because IM-DD technique is simple and it enables lower cost for system implementation than any other technique. Recently, optical amplifiers have proven successful in supporting long-haul communications. With optical amplifiers used in IM-DD systems, bit-rate length product becomes greater than ever before.

The following noises appear in the receiver: the photodiode quantum noise, photodiode working resistance thermal noise and amplifier resistance thermal noise.

In the push to develop even more powerful optical communication networks, interferometric noise, which is the result of data crosstalk interference, has frequently been cited as the key performance-limiting factor. The crosstalk appears at the transmitter output or along the fiber.

The pulse shape at the receiver input is determined by solving the nonlinear Schrödinger equation [1], when the influence of nonlinear and dispersive effects is balanced. The starting point, for solving the propagation equation is the useful signal's electrical field envelope and the total field envelope (useful signal and crosstalk) where the crosstalk appears.

In this paper, the bit error probability of IM-DD systems, in the presence of interferometric, thermal noise and quantum noise is determined. The bit error probability as a function of SNR (signal-to-noise ratio) for different SIR (signal-to-interference ratio) values, was used as the system performance rate. The system was analyzed for the following cases:(1) signal interference is absent, (2) is present at the input, (3) in the middle and (4) at the output of the fiber.

The obtained results can be used for IM-DD systems design.

II. THE NONLINEAR SHRÖDINGER EQUATION

Mathematical description of the propagation of optical pulses requires solving of the propagation equation in dispersive and nonlinear medium. To begin with, we observed the propagation equation[1]:

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A+\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2}=i\gamma|A|^2 A \tag{1}$$

satisfying slowly varying pulse function $A(z,t)$ in presence of group-velocity dispersion (GVD) and self-phase modulation (SPM) having β_2 and γ given as

$$\beta_2=\frac{\partial^2\beta}{\partial\omega^2}\Big|\ \omega=\omega_0\ , \qquad \gamma=\frac{2\pi n_2}{\lambda A_{eff}} \tag{2}$$

where γ is the nonlinear coefficient, n_2 is the nonlinear-index refractive coefficient, A_{eff} is the effective core area and β_2 and γ are governed by the effects of GVD and SPM, respectively.

The propagation equation is sufficiently accurate for the description the picosecond pulses and can be used in many practical cases. It is useful to observe Eq. (1) in normalized form and then we can use following normalized parameters:

$$\tau=\frac{t-\beta_1 z}{T_0},\ \zeta=\frac{z}{L_D},\ U=\frac{A}{\sqrt{P_0}} \tag{3}$$

where T_0 is the half width(at 1/e-intensity point) of pulse , P_0 is the peak power of the incident pulse and L_D is the dispersion length i.e.

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$$L_D = \frac{T_0^2}{|\beta_2|} \tag{4}$$

Than for α=0, Eq. (1) takes form:

$$i\frac{\partial U}{\partial \zeta} = \mathrm{sgn}(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} - N^2|U|^2 U \tag{5}$$

where $sgn(\beta_2)$ takes values +1 or –1 in dependence of dispersive regime (β_2>0-normal dispersion regime, β_2<0-anomalous dispersion regime). The propagation equation in the form of Eq. (5) is known as Nonlinear Shrödinger equation (NSE).

The parameter N is defined as

$$N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|} = \frac{L_D}{L_{NL}} \tag{6}$$

and it represents nondimensional combination of the pulse and fiber parameters. Dispersion dominates for $N<<1$, while SPM dominates for $N>>1$. For values of N~1, both SPM and GVD play an equally important role during pulse evolution. In Eq. (6) parameter L_{NL} is the nonlinear length and it is defined as

$$L_{NL} = \frac{1}{\gamma P_0} \tag{7}$$

III. INTERFEROMETRIC NOISE AS A PHENOMENON IN OPTICAL TRANSMISION

The prevalence of interferometric noise is a consequence of the large number of generation mechanisms of parasitic crosstalk, which follows multiple transparent paths before adding the data.

It is simply a requirement that crosstalk and data arise from the same source-the typical case, or from distinct sources closely aligned in wavelength.

Levels of crosstalk thought to be innocuous with an intersymbol interference (ISI) mind-set may generate unacceptable quantities of interferometric noise because the crosstalk add on a field amplitude basis as a for other interference manifestations.

Consider the simplest optical network, an optical link comprising a laser connected to a fiber patchcord which is turn is connected to a photodetector.

Light reflected at the connector nearest to the detector is partly reflected at the other connector and passes as parasitic crosstalk to the detector. On square-law detection the photocurrent is given by [2,3]:

$$i\vartheta \underbrace{P_d}_{data} + \underbrace{P_x}_{crosstalk} + \underbrace{2\sqrt{P_d P_x}\cos(relative\,phase)\,\underline{p_d}\,\underline{p_x}}_{interferometric\ noise} \tag{8}$$

where P_d and P_x are the instantaneous optical power, $\underline{p_d}$ and $\underline{p_x}$ polarization vectors of the data and crosstalk, respectively. ϑ is reciprocal modulation depth. The data can be seen to be corrupted not only by the additive crosstalk P_x, as would be predicted by a sum of intensities (ISI) approach, but also by the mixing term that exhibits a cosinuisodal dependence on the relative phase of the data and crosstalk. When this relative phase fluctuates randomly interferometric noise arises.

For example, in the worst-case of a aligned polarization if Pd=10 (arbitrary units), P_x=1, the interferometric noise varies by ±6.3 (assuming a relative phase spanning (0,2π)) -the eye opening is greatly reduced.

IV. THE INFLUENCE OF CROSSTALK ON THE PULSE PROPAGATION ALONG THE FIBER

The propagation equation (5) is a nonlinear partial differential equation. To solve this equation we can use one of many numerical methods. They can be divided into two broad categories known as the finite difference methods and the pseudospectral methods.

Generally speaking, pseudospectral methods are faster by an order of magnitude or more, to achieve the same accuracy. having this in mind, we can use the split-step Fourier method from the categories of pseudospectral methods to solve the pulse-propagation problem in nonlinear-dispersive media[1,4].

In this paper we consider the input pulse whose envelope have Gaussian form, i.e.

$$U(0,\tau) = a\exp\left(-\tau^2/2\right) \tag{9}$$

where the value of parameter a depend on sent information (1 or 0). The interference signal which can appear along the fiber also has the Gaussian envelope. Because of its nature, the interference signal which appears in the same place along the fiber together with the useful signal can "walk" along the the useful signal, i.e. it can be time shifted with the respect to the useful signal. Additional, it can be phase shifted. The useful signal at the input of an optical system can be defined as:

$$\begin{gathered} s(0,\tau) = U(0,\tau)\cos\omega_n\tau \\ U(0,\tau) = a\exp\left(-\tau^2/2\right) \end{gathered} \tag{9}$$

where $\omega_n = \omega_0 T_0$ is normalized frequency.

The interference signal is defined as:

$$\begin{gathered} s_i(\zeta_i,\tau) = U_i(\zeta_i,\tau)\cos(\omega_n\tau + \varphi) \\ U_i(\zeta_i,\tau) = a_i\exp\left(-(\tau - b)^2/2\right) \end{gathered} \tag{10}$$

where b represents time shift of the interference signal in relation to the useful signal. ζ_i is the distance where the interference signal appears, φ represents phase shift of the interference signal with the respect to the useful signal.

The total signal envelope at the place where the interference signal appears, is determined as [5]

$$U_r(\zeta_i,\tau) = (U^2(\zeta_i,\tau) + U_i^2(\zeta_i,\tau) + \\ + 2U(\zeta_i,\tau)U_i(\zeta_i,\tau)\cos\varphi)^{1/2} \quad (11)$$

The total signal phase at the place where the interference signal appears is determined as [5]

$$\psi_r(\zeta_i,\tau) = arctg\frac{U_i(\zeta_i,\tau)\sin\varphi}{U(\zeta_i,\tau)+U_i(\zeta_i,\tau)\cos\varphi} \quad (12)$$

This envelope and phase is then used for solving NSE.

V. THE DETERMINATION OF BIT ERROR PROBABILITY

The conditional bit error probability $P_{e/\varphi,b}$ was determined on the basis of the square of field envelope and variances of the IM-DD receiver noises.

The conditional bit error probability is determined using Gaussian approximation.

The decision is done on the basis of the signal [6]

$$z = k\,n + y \quad (13)$$

where n is the number of electrons emitted by the photodiode, and y represents Gaussian noise accumulated in resistance and amplifiers in the receiver:

$$p(y) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{y^2}{2\sigma^2}\right) \quad (14)$$

where σ^2 is the variance of Gaussian noise. The conditional probability density function of the signal z is determined as:

$$p(z/n) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{(z-kn)^2}{2\sigma^2}\right) \quad (15)$$

where n has Poisson probability function:

$$p(n) = \frac{\lambda^n}{n!}\exp(-\lambda) = \\ = \frac{\left|U_r(\xi_{end},\tau)\right|^2}{n!}\exp\left(-\left|U_r(\xi_{end},\tau)\right|^2\right) \quad (16)$$

$U_{1r}(\xi_{end},\tau)$ and $U_{0r}(\xi_{end},\tau)$ represent the normalized total envelope at input of receiver in dependence of sent information (1 or 0, respectively).

The conditional likelihood functions are

$$p_0(z/\varphi,b) = \sum_{n=0}^{\infty} p_o(n)p_0(z/n,\varphi,b) \quad (17)$$

$$p_1(z/\varphi,b) = \sum_{n=0}^{\infty} p_1(n)p_1(z/n,\varphi,b) \quad (18)$$

For between equal variance σ_1^2 and σ_0^2 , the threshold of decision is determined as:

$$V_p = \left(\overline{z_1} + \overline{z_0}\right)/2 \quad (19)$$

For $P(H_0)$=$P(H_1)$=1/2, the conditional bit error probability is determined as [7,8]:

$$P_{e/\varphi,b} = \frac{1}{2}\left[\int_{V_p}^{+\infty} p_0(z/\varphi,b)dz + \int_{-\infty}^{V_p} p_1(z/\varphi,b)dz\right] \quad (20)$$

We will calculate P_e - the bit error probability for the worse case when $p(b)$ and $p(\varphi)$ are uniformly distributed [9,10].

$$P_e = \int_{-1/2}^{1/2}\int_{-\pi}^{\pi} P_{e/\varphi,b}\, p(\varphi)p(b)\,db\,d\varphi \quad (21)$$

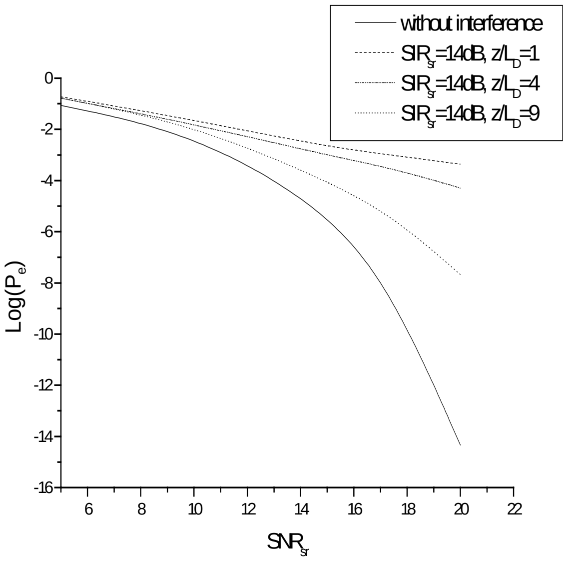

Fig. 1. The bit error probability in the function of SNR_{sr} for SIRsr=14dB in the anomalous dispersion-regime

Fig. 1. shows the bit error probability as a function of SNR_{sr} for SIR_{sr}=14 dB in the anomalous dispersion-regime in cases when (1)the interference is absent, (2)when interference signal appears at the input of fiber (z/L_D=1), (3) when the interference signal appears in the middle of the fiber (z/L_D=4) and (4) when the interference signal appears at the end of fiber (z/L_D=9). We can see from Fig. 1. that the bit error probability decreases with the increase of SNR_{sr} in all four cases. Also, the bit error probability decreases as the distance where interference appears increases.

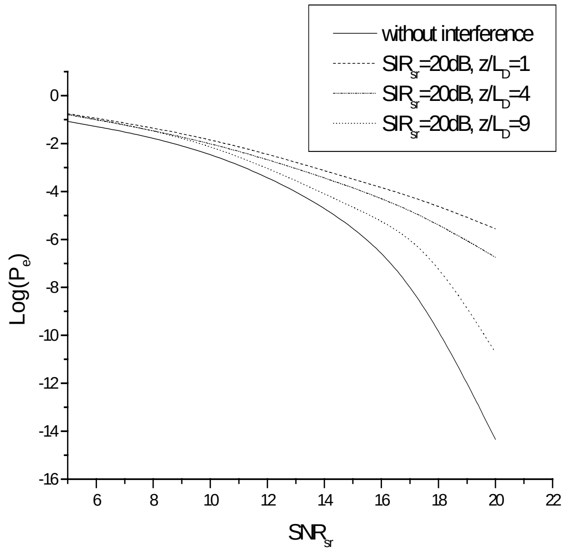

Fig. 2. The bit error probability in the function of SNR_{sr} for SIRsr=20dB in the anomalous dispersion-regime.

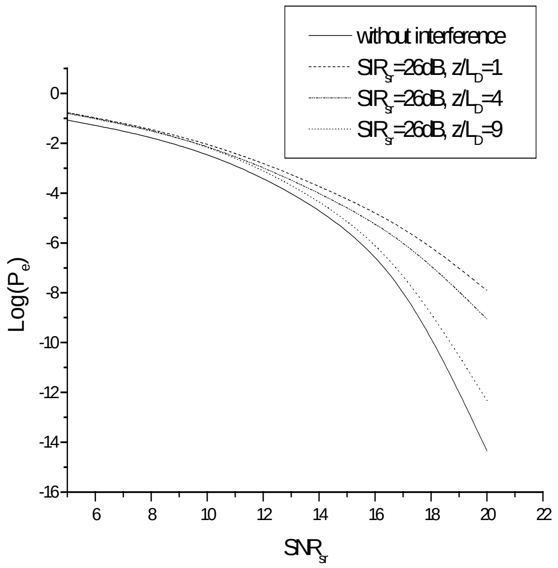

Fig. 3. The bit error probability in the function of SNR_{sr} for SIRsr=26dB in the anomalous dispersion-regime.

Figs. 2. and 3. show bit error probability as a function of SNR_{sr} but for SIR_{sr}=20 dB and SIR_{sr}=24 dB, respectively, with same conditions as for Fig. 1. One can see that bit error probability decreases with the increase of SIR_{sr}.

VI. Conclusion

We can see from figures that the influence of crosstalk interference on the pulse shape is greater in normal dispersion-regime than in anomalous regime. The bit probability error decreases with the increase of SNR_{sr} and SIR_{sr} for all cases, and the decrease is the most significant when the crosstalk appears at the end of the fiber. We can also see that the system performance is better for higher SIR values.

VII. References

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