Probability of Error for IM-DD Optical Communication System in the Presence of Quantum Noise, Thermal Noise in the Receiver and Disturbances in the Fiber

Dragan Lj. Draca[1], Ivana Tosic[2]

Abstract. **In this work we will consider optical digital communication system with Intensity Modulation and Direct Detection (IM-DD), in the presence of quantum noise formed in the photo-diode, thermal noise formed in the receiver and interference in the fiber. For this system, we will determine likelihood functions and compute probability of error.**

I. INTRODUCTION

Optical digital communication systems with Intensity Modulation and Direct Detection (IM-DD) have found a very wide deployment area for the last couple of years. These systems are used in Wavelength Division Multiplexing (WDM) systems. WDM systems are applied for transmission of great amounts of information at long distances. In this case, gain control is used for attenuation compensation, and compensation fiber for dispersion compensation. Capacity of these systems is around 100Gb/s and distance between transmitter and receiver up to 10000km.

For IM-DD systems we have determined likelihood functions and probability of error, considering quantum noise formed in the photo-diode, thermal noise formed in resistors and amplifiers, and interference. Quantum noise has Poisson distribution. In some cases, one can consider that quantum noise magnitude values have Gaussian probability density distribution. Thermal noise, formed in resistors, also has Gaussian probability density distribution.

In optical communication IM-DD systems interferences can appear in the transmitter, optical fiber, and receiver. They cause the appearance of interference noise. They can be modeled with one or several sinusoids with constant magnitudes and random phases. Discrete phase values can have uniform distribution or Gaussian probability density distribution. Interferences can have the same wavelength as the information signal, or they can have different wavelength. In addition, interferences can be coherent or non-coherent.

In this work, performances will be calculated for the case of disturbances in the fiber.

In most of the cases, this kind of disturbances appears as a result of various reflections on fiber splices. Likewise, we will consider the fiber as linear, without nonlinear characteristics. Likelihood functions and probability of error will be shown as functions of signal-to-thermal noise ratio. The obtained results can be used in designing IM-DD optical systems.

[1]Dragan Lj. Draca is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia
[2] Ivana Tosic is a student at the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia, e-mail: ivanat@medianis.net

II.PERFORMANCES

The receiver of optical IM-DD system makes the decision based on the signal z, in the presence of quantum noise, thermal noise and disturbances on the line. This signal is:

$$z = cn + y \tag{1}$$

where n is the number of electrons emitted from the photo-diode, and y represents thermal noise with variance σ_1^2. Conditional probability density distribution of a random variable z, in this case is:

$$p(z/n) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(z-cn)^2}{2\sigma_1^2}} \tag{2}$$

The probability of number of electrons emitted from the PIN photo-diode is:

$$p\ (n,T) = \frac{\lambda^n}{n!} e^{-\lambda} \tag{3}$$

where λ is the intensity of light, given by:

$$\lambda_0 = a(A_0 + i)^2 \text{ for } H_0$$

$$\lambda_1 = a(A_1 + i)^2 \text{ for } H_1 \tag{4}$$

The disturbance i is characterized by one cosine wave:

$$i = D_1 \cos\varphi_1 \tag{5}$$

Replacing (5) into (4), we get:

$$\lambda = a(A_0 + D_1 \cos\varphi_1)^2 \quad \text{for } H_0,$$

and

$$\lambda = a(A_1 + D_1 \cos\varphi_1)^2 \quad \text{for } H_1 \tag{6}$$

The number of electrons emitted from the PIN photo-diode is:

$$p(n,T) = \frac{a^n (A_0 + D_1 \cos\varphi_1)^{2n}}{n!} e^{-a(A_0 + D_1 \cos\varphi_1)^2}$$

for H_0, and

$$p(n,T)=\frac{a^n(A_1+D_1\cos\varphi_1)^{2n}}{n!}e^{-a(A_1+D_1\cos\varphi_1)^2}$$

for H_1. (7)

Likelihood functions of these systems are:

$$p_0(z)=\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}d\varphi_1\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(z-cn)^2}{2\sigma_1^2}}\frac{a^n(A_0+D_1\cos\varphi_1)^{2n}}{n!}e^{-a(A_0+D_1\cos\varphi_1)^2}\frac{1}{\sqrt{2\pi}\sigma_{\varphi_1}}e^{-\frac{\varphi_1^2}{\sqrt{2\pi}\sigma_{\varphi_1}}}$$

$$p_1(z)=\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}d\varphi_1\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(z-cn)^2}{2\sigma_1^2}}\frac{a^n(A_1+D_1\cos\varphi_1)^{2n}}{n!}e^{-a(A_1+D_1\cos\varphi_1)^2}\frac{1}{\sqrt{2\pi}\sigma_{\varphi_1}}e^{-\frac{\varphi_1^2}{\sqrt{2\pi}\sigma_{\varphi_1}}} \quad (8)$$

Probability of error of the system is:

$$P_e=p(H_0)\int_{z_T}^{\infty}dz\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}d\varphi_1\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(z-cn)^2}{2\sigma_1^2}}\frac{a^n(A_0+D_1\cos\varphi_1)^{2n}}{n!}e^{-a(A_0+D_1\cos\varphi_1)^2}\frac{1}{\sqrt{2\pi}\sigma_{\varphi_1}}e^{-\frac{\varphi_1^2}{\sqrt{2\pi}\sigma_{\varphi_1}}}+$$

$$+p(H_1)\int_{-\infty}^{z_t}dz\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}d\varphi_1\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(z-cn)^2}{2\sigma_1^2}}\frac{a^n(A_1+D_1\cos\varphi_1)^{2n}}{n!}e^{-a(A_1+D_1\cos\varphi_1)^2}\frac{1}{\sqrt{2\pi}\sigma_{\varphi_1}}e^{-\frac{\varphi_1^2}{\sqrt{2\pi}\sigma_{\varphi_1}}} \quad (9)$$

III.NUMERICAL RESULTS AND GRAPHS

We have calculated and shown likelihood functions for three values of A_0: 0.1, 0.2 and 0.3, and changed values of A_1 for each case, to see the changing of threshold. Probability of error is calculated for each case. The surface under the graph is calculated to show the accuracy of calculation. At the end, we have shown probability of error as a function of signal-to-thermal noise ratio.

1.

$$c=1;\ a=1;\ A_0=0.1;\ D_1=0.09;$$
$$\sigma_1=0.1;\ \sigma_{\varphi_1}=\pi/5$$

Surface under the graph for $p_0(z)$ is: 1.0064

for $p_1(z)$ is: 1.00602 ($A_1=1$)

1.00616 ($A_1=2$)

1.00615 ($A_1=3$)

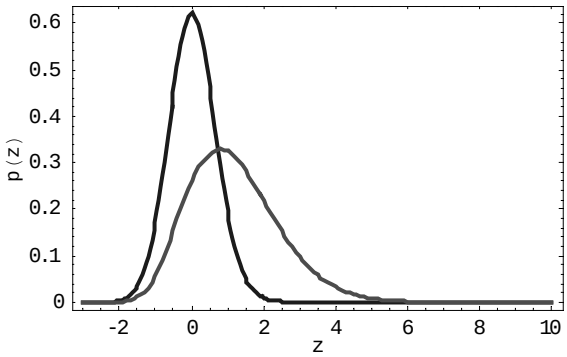

Fig.1.

$A_1=1$

Optimal threshold: 0.74069

Probability of error: 0.26935

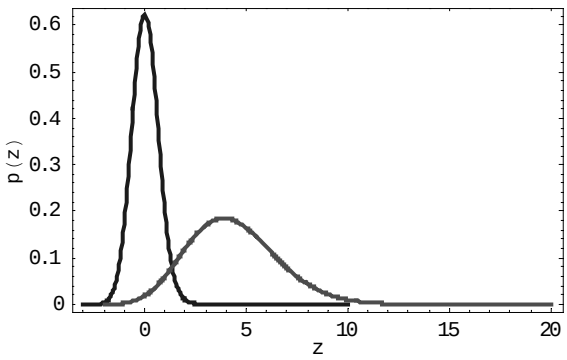

Fig.2.

$A_1=2$

Optimal threshold: 1.35161

Probability of error: 0.04869

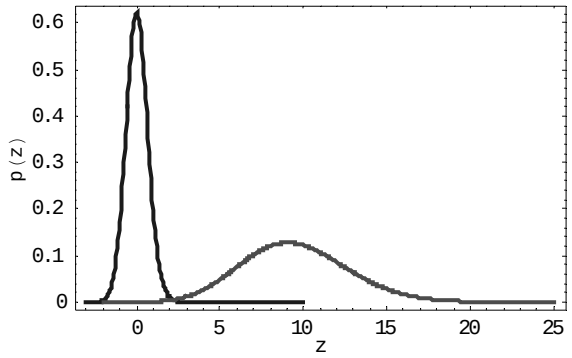

Fig 3. $A_1 = 3$
Optimal threshold: 2.20154
Probability of error: 0.002675

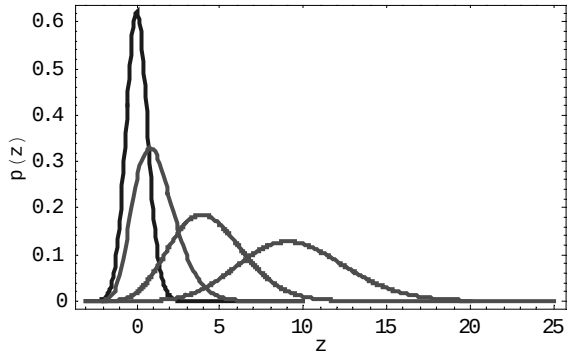

Fig.4. $A_1 = 1, 2, 3$

2.

$c = 1; a = 1; A_0 = 0.2; D_1 = 0.18;$

$\sigma_1 = 0.2; \sigma_{\varphi_1} = \pi/5$

Surface under the graph for $p_0(z)$ is: 0.999616

for $p_1(z)$ is: 0.999292 ($A_1 = 1$)

0.999353 ($A_1 = 2$)

0.999363 ($A_1 = 3$)

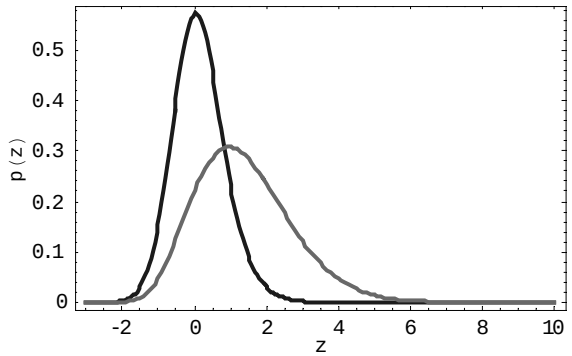

Fig.5. $A_1 = 1$
Optimal threshold: 0.83203
Probability of error: 0.26771

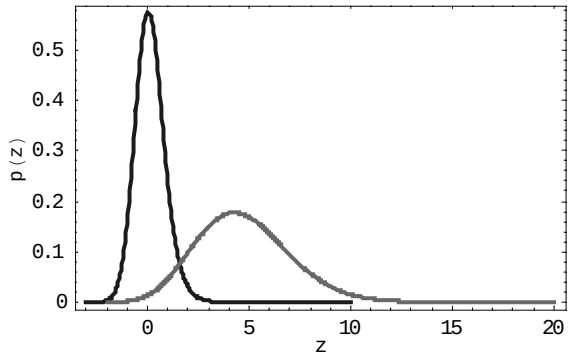

Fig.6. $A_1 = 2$
Optimal threshold: 1.59694
Probability of error: 0.05172

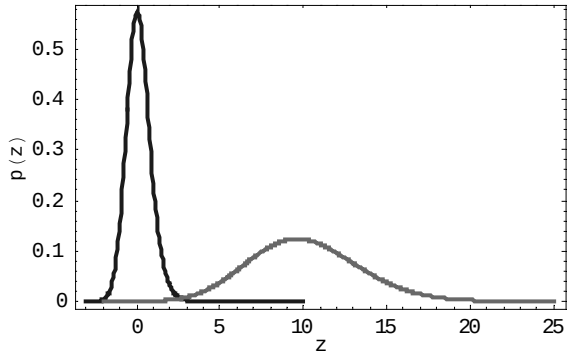

Fig.7. $A_1 = 3$
Optimal threshold: 2.66013
Probability of error: 0.003378

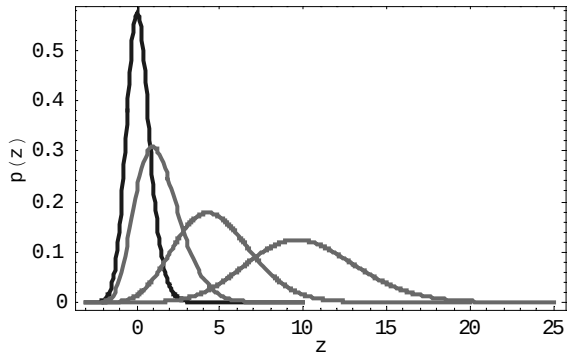

Fig.8. $A_1 = 1, 2, 3$

3.

$c = 1; a = 1; A_0 = 0.5; D_1 = 0.4;$

$\sigma_1 = 0.5; \sigma_{\varphi_1} = \pi/5$

Surface under the graph for $p_0(z)$ is:1.00009

for $p_1(z)$ is: 0.999925 ($A_1 = 1$)

0.99994 ($A_1 = 2$)

0.999731 ($A_1 = 3$)

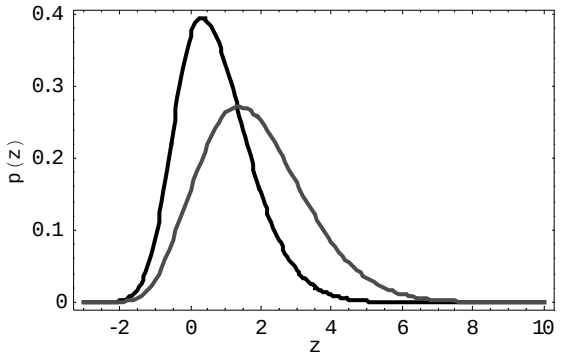

Fig.9. $A_1 = 1$

Optimal threshold: 1.33514

Probability of error: 0.33236

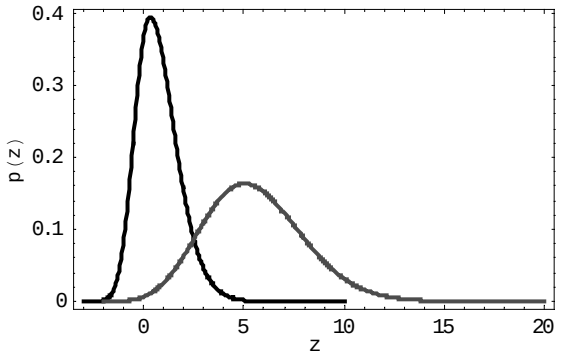

Fig.10. $A_1 = 2$

Optimal threshold: 2.50658

Probability of error: 0.08111

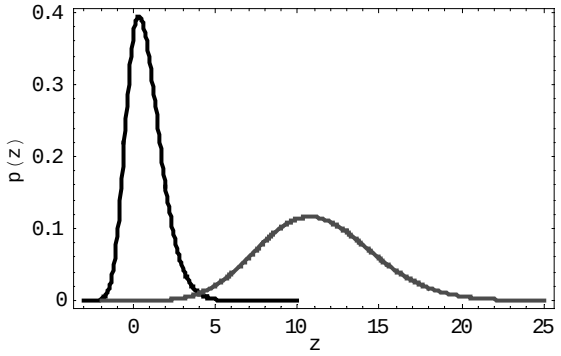

Fig.11. $A_1 = 3$

Optimal threshold: 4.00821

Probability of error: 0.00786

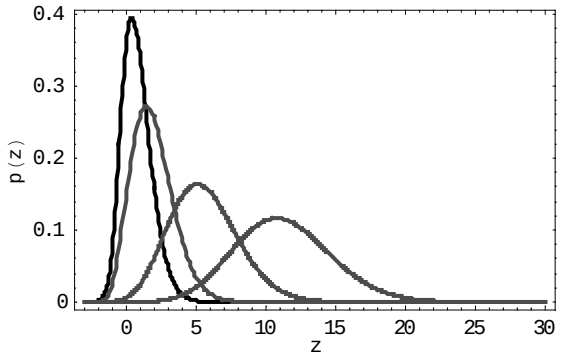

Fig.12. $A_1 = 1, 2, 3$

Probability of error as a function of signal-to-thermal noise ratio for $\sigma_{\varphi_1} = \frac{\pi}{5}$: $P_e = f\{(A_1 - A_0)/\sigma_1\}$

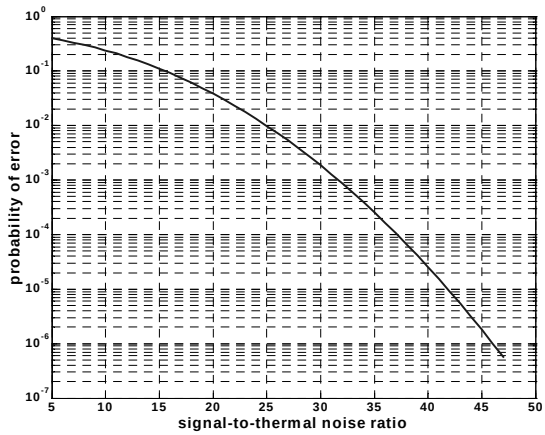

Fig.13. Probability of error

IV.CONCLUSION

Calculation of performances of IM-DD optical communication systems in the presence of quantum noise, thermal noise in the receiver and disturbances in the fiber is of a great importance in designing IM-DD optical systems, and further for their deployment in WDM systems. The obtained results show that probability of error becomes very low with increasing the signal-to-noise ratio.

REFERENCES

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